



P. S. Arya

Department of Mathematics,
Central University of Kerala,
Tejaswini Hills,
Kasaragod, Kerala 671325, India
e-mail: arya.p.s@cukerala.ac.in

Manikandan Rangaswamy¹

Department of Mathematics,
Central University of Kerala,
Tejaswini Hills,
Kasaragod, Kerala 671325, India
e-mail: mani@cukerala.ac.in

A. Krishnamoorthy

Department of Mathematics,
Central University of Kerala,
Tejaswini Hills,
Kasaragod, Kerala 671325, India;
Centre for Research in Mathematics,
C.M.S. College,
Kottayam, Kerala 686001, India
e-mail: achyuthacusat@gmail.com

S. Dharmaraja

Department of Mathematics,
Indian Institute of Technology Delhi,
New Delhi 110016, India
e-mail: dharmar@maths.iitd.ac.in

Optimizing Battery Maintenance and Reliability in Ground Control Station for Tethered High-Altitude Platforms

This paper presents an integrated system for ensuring uninterrupted power supply to tethered high-altitude platform systems (HAPS) by strategically managing the repair and replenishment of batteries in a k -out-of- n : G , COLD system. We assume that the batteries are identical and their lifetimes are independent of each other and exponentially distributed. The batteries deteriorate independently due to failures and await repair. The repair facility is activated when the number of working batteries decreases to L ($L < n$), and a replenishment order for $n - k + 1$ batteries is placed when the number of operational batteries falls to N ($N < L$). We derive an explicit solution for the system-state probability and analyze key performance measures. Furthermore, we employ the particle swarm optimization (PSO) algorithm to determine the optimal cost for the proposed optimization problem and use the Morris method for sensitivity analysis. The results provide insights into efficient battery management strategies for HAPS, ensuring reliable power supply while minimizing costs. [DOI: 10.1115/1.4067545]

Keywords: tethered HAPS, k -out-of- n : G system, COLD system, repair, lead time, maintenance

1 Introduction

Terrestrial and satellite networks are considered two well-established and dominant technologies in the telecommunications industry. High-altitude platform systems (HAPS) have recently emerged as a novel option based on quasi-stationary aerial platforms in the stratosphere. Due to their exceptional qualities, HAPS—a revolutionary alternative to terrestrial and satellite communications systems, have attracted much attention from researchers. HAPS are aircraft positioned above 20 km altitude in the stratosphere (layer of the atmosphere where the temperature starts to rise with altitude) to conduct remote sensing or build a telecommunications network for either civilian or military purposes. These aircraft could be manned or unmanned or include planes, airships, or balloons, as mentioned in d'Oliveira et al. [1].

This paper focuses on the tethered HAPS (see Fig. 1). The tethered HAPS positioned 200–400 m above land, discussed in Jain et al. [2]. Compared to terrestrial stations or satellite stations, tethered HAPS have several advantages. It can complement the current wired and wireless network by offering rapid coverage and high capacity, capable of servicing highly populated cities, suburban regions, and rural and remote places. With the advantages of delivering high

capacity accessible from terrestrial systems and comprehensive coverage similar to that available from satellites, tethered HAPS have the potential to offer a significant change in system performance and availability. Aragon-Zavala et al. [3] quoted that tethered HAPS complement and integrate the current technologies rather than completely replace them. Due to their ease of deployment, tethered HAPS are also frequently employed in disaster situations in the affected area. An extensive and comprehensive overview of networked tethered flying platforms and a general overview of tethered flying platforms, including different types and their components, characteristics, applications, advantages, challenges, and regulations, are given in Belmekki and Alouini [4]. For a brief overview of the development of unmanned tethered telecommunication platforms, see Vishnevsky and Meshcheryakov [5]. They explore the architecture, scientific and technical issues, and design principles of platforms used to augment ground-based electronic warfare capabilities.

One of the key obstacles in using HAP to enhance the existing wireless networks is the restricted onboard energy and flight time, see in Arum et al. [6]. A tethered HAP frequently works with power served by lithium-ion batteries (LIBs), solar panels, and ground-based energy sources where a thin cable rope transmits the energy. Alam et al. [7] discussed that the expected energy density of a new lightweight LIB increases by 3% annually. LIBs have established dominance in the energy storage systems market owing to their

¹Corresponding author.

Manuscript received October 18, 2024; final manuscript received December 27, 2024; published online February 4, 2025. Assoc. Editor: Lechang Yang.

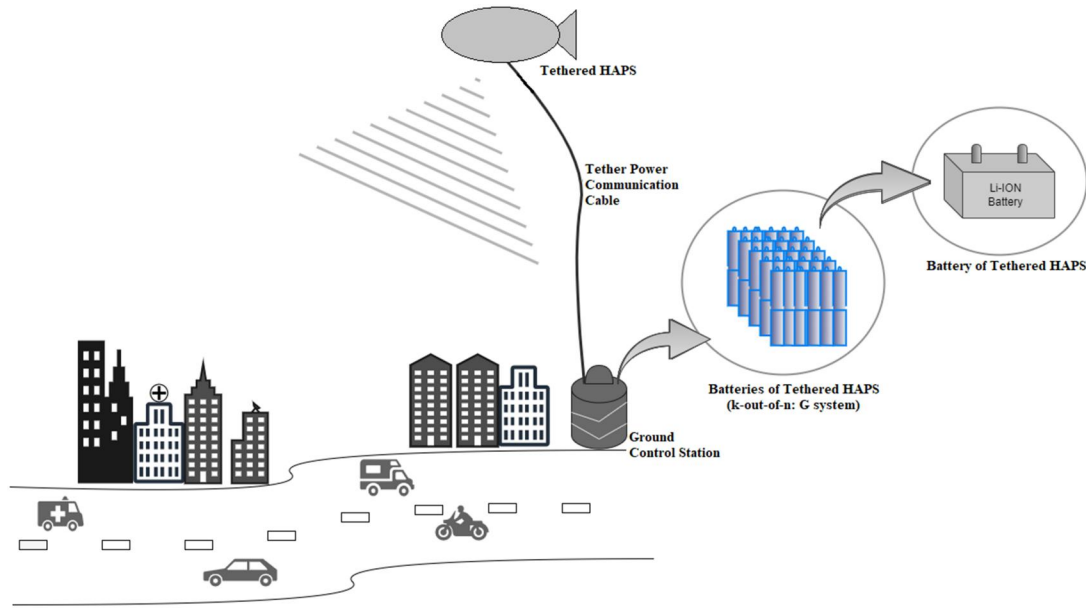


Fig. 1 An overview of tethered HAPS

notable advantages, including high energy density, high working voltage, and efficiency. For further details, refer to Choi and Wang [8]. The study of the necessity to merge tethered HAPS for communication was presented in Perelomov et al. [9]. They considered HAPS unification, standardization strategies, and the norms and techniques for structural optimization. They demonstrated that combining requirements, constituent parts, and composite systems improves communications networks' efficiency. They established a justification for using logical-dynamic models to solve unification issues. Various studies are available to describe and demonstrate the system's reliability using a queueing system.

The reliability of the flight module operation of tethered HAPS is studied extensively by Vishnevsky et al. [10] focusing on factors that could impact its performance and operational stability under various conditions. Considering the flight modules as a complex circular k -out-of- $n : F$ model with a nonrepairable HOT standby, they obtained analytical expressions and algorithms for calculating the reliability function, the mean time to the first system failure, and other reliability measures. The reliability of a k -out-of- n system with repair has been studied exhaustively by several researchers; a few such contributions can see in Li et al. [11], Yam et al. [12], and Li et al. [13]. Kozyrev et al. [14] proposed a stochastic model using a multidimensional Markov process applied for the evaluation of the reliability characteristics of a tethered multirotor HAP based on a hexacopter. The model considers both the location of the failed components and the increase in functional load that occurs when one component fails, which impacts the other working components. The analysis of the reliability of the flight module of a tethered multirotor high-altitude platform consisting of n elements operating in a random environment is done by Nguyen and Kozyrev [15]. Recently, the reliability of tethered HAPS using a repairable k -out-of- $n : G$ system model with redundant rotors is evaluated by Nikita et al. [16]. Their study offers enhanced accuracy in assessing system failure probabilities through a semi-Markov approach as an alternative to traditional Markov models.

Further, focusing on the tether that connects the tethered HAP to the ground, Dharmaraja et al. [17] propose a Markov model design for determining the tether's reliability. They looked at the reliability of the tethered HAP system by supposing that the tether consists of a number of wires, the failure rates of which depend on the number of failed wires. The numerical computation of the performance measures provides the optimal number of wires. The power

management of LIBs based on tethered HAP systems for wireless communications service is analyzed in Jain et al. [2]. A stochastic model based on a multidimensional Markov process was applied to evaluate the power consumption characteristics of the batteries of tethered HAPS. The specified model considered the increment in the load on the system's functioning after the power consumption in batteries. Krishnamoorthy [18] analyzed a reliability-inventory problem under the N -policy of replenishment of components by considering a k -out-of- $n : G$ system with COLD standby with all the components identical and their lifetimes independent of each other.

This paper primarily addresses the development of an integrated system designed to ensure a continuous power supply to tethered HAPS, where the focus is on strategically managing and maintaining the repair and replenishment of batteries to achieve uninterrupted power delivery. Accordingly, we design a k -out-of- $n : G$, COLD system that incorporates both the replenishment of components (batteries) and the repair of failed batteries. In this system, batteries that fail and are awaiting repair are placed in a queue. Therefore, the system operates as a k -out-of- $n : G$ configuration where we place a replenishment order for new batteries and a queue for the failed batteries awaiting repair.

For any system, reliability is critical. In particular, for emergency systems, it is of utmost importance. To attain the highest reliability, we must order when N ($N < n$) is close to n , adding to the holding cost of spare batteries in stock. We can reduce this holding cost by bringing N closer to k . However, the risk of the system failing before the realization of the order will loom large. Thus, finding the optimal value of N is one problem of interest. Another problem is that rather than placing an order, we can start the repair of failed batteries. This is how we implement the concept of repair of failed batteries. Repaired batteries are immediately reintegrated into the operational system, ensuring no spares remain. Thus, the advantage is that we can avoid the holding cost of the spare batteries. However, the continuous repair of failed batteries may cause an increase in repair costs. In order to reduce the repairing cost, it is important to find the optimal value of L ($L < N$). The cost of the system breakdown can be very high. Thus, the main objective of this model is to find the optimal pair (L^*, N^*) that yields the minimal cost.

The rest of the paper is organized as follows. The stochastic model of the problem is stated and analyzed in Sec. 2. The long-run system-state probability is explicitly computed. In Sec. 3, we obtained several important system performance measures. Section 4 presents

a comprehensive numerical and sensitivity analysis of performance measures and an optimization problem to justify the obtained results. Finally, Sec. 5 concludes with a future plan.

2 Problem Formulation and Model Description

The flight module of the tethered HAPS receives power from batteries kept in the ground control station (GCS) through a connecting wire. Therefore, to ensure a highly reliable system, we prioritize strategically maintaining the batteries and consider them as the components of a k -out-of- n : G system. The lifetime of n batteries are independently distributed random variables exponentially distributed with parameter λ . The system operates continuously, and when the number of operational batteries hits $k-1$, the system fails. The system considered is assumed to be a COLD system. That is, until the system is back into operation by replacing or repairing failed batteries, the operational batteries do not deteriorate further at the time of system failure.

Repair of failed batteries starts when the number of working batteries falls to L ($L < n$) due to failures. Failed batteries form a queue waiting for repair until the number of working batteries falls to L . The repair of failed batteries starts when the queue length is $(n-L)$. Failed batteries are served/repared on a first-in-first-out basis. The service/repair rate of failed batteries follows an exponential distribution with parameter γ . The repaired units are assumed to be as good as new, and repairing failed units takes place one at a time. Further, when the number of working batteries falls to N ($N < L$) due to failures, then an order of $n-k+1$ batteries is placed. The above policy is termed as a (L, N) policy. On replenishment, all failed batteries are replaced by new arrivals. The ordering lead time follows an exponential distribution with parameter β . The repair of failed batteries will be halted only when all failed batteries become operational by repair or replenishment against the order materializes or whichever occurs first. If replenishment occurs during the repair, we discard the batteries waiting for repair. However, the outstanding replenishment order is canceled if replenishment does not occur until all the n units return to work after repair.

After placing an order for replenishment, there are three possible events to consider. One case is when a failure occurs, the number of working batteries goes down to $N-1$, and the order against

replenishment materializes. In this case, the number of spare batteries available is $N-k$. Another event that can occur is the repair of a failed battery, which increases the number of working batteries from N to $N+1$ (where the state is represented as $(N+1, 1, 1)$, indicating that the repair process is currently active and replenishment order has been initiated), and then the order against replenishment materializes. In this case, the number of spare batteries available is $N-k+2$. Finally, the replenishment order materializes when the number of working batteries is N itself. In this case, the number of spare batteries available is $N-k+1$. These three scenarios represent different outcomes after a replenishment order with varying levels of spare batteries available for future use. Now, let us see the various possibilities if the order realization does not occur until the number of working batteries reaches L through repair. If, through repairs, the number of working batteries increases from L to $L+1$ (here, the state transition is from $(L, 1, 1)$ to $(L+1, 1, 1)$ as the repair is ON and replenishment order is placed), and then the order realization occurs and then the available spare batteries would be $L-k+2$. Alternatively, if a failure occurs, the number of working batteries drops to $L-1$; once the order realization happens, the number of available spares would reach $L-k$. Another possibility is that the order realization happens when the number of working batteries already reaches L through repair. Here, the available spare batteries would be $L-k+1$.

On replenishment, the number of operational batteries can go above n , which are kept as excess and then used for replacing failed batteries. The possible number of excess batteries is $0, 1, 2, \dots, N-k+1, \dots, n-k$. The minimum possible value for excess batteries immediately after replenishing and replacing failed batteries is zero. This happens if replenishment occurs when the system is in a failed state. Thus, all $n-k+1$ failed batteries are immediately replaced by the replenished batteries. The possible maximum value for excess batteries is $n-k$. Hence, when the number of operational batteries is N , the number of excess batteries after replenishment and replacement is $N-k+1 = (N+n-k+1-n)$. Thus, the proposed model is a k -out-of- n : G , COLD system with repair and replenishment of failed batteries with all the distributions involved assumed to be exponentially distributed. For a better understanding of the model, refer to Figs. 2 and 3 for the state transition diagram for the proposed model.

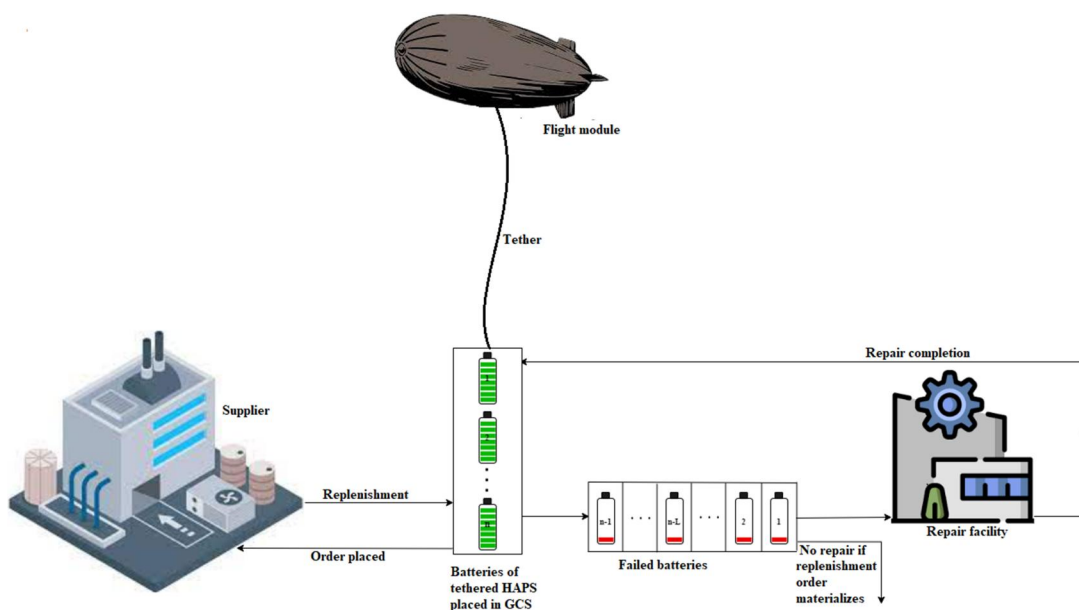


Fig. 2 Schematic representation of the model

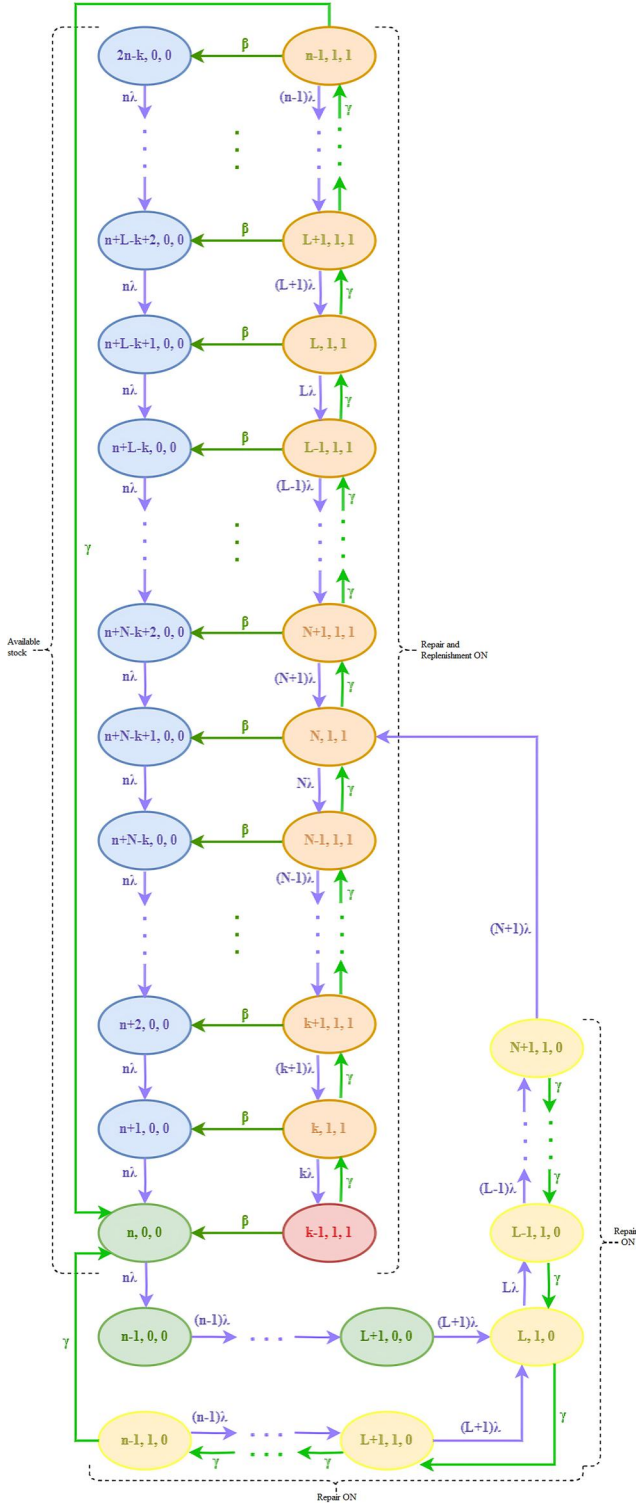


Fig. 3 State transition diagram of the model

2.1 Analysis of the Model. Let $X(t)$ be defined as the total number of batteries, including spares available, at time t , $Y(t)$ be the repairing state, and $Z(t)$ be the replenishment state, with

$$Y(t) = \begin{cases} 1, & \text{when repair is ON} \\ 0, & \text{when repair is OFF} \end{cases} \quad \text{and}$$

$$Z(t) = \begin{cases} 1, & \text{when replenishment order is placed} \\ 0, & \text{when replenishment order is not placed} \end{cases}$$

Thus, we have a three-dimensional continuous-time Markov chain (CTMC), $\{(X(t), Y(t), Z(t)) : t \geq 0\}$ with state space

$$S = \{(i, 0, 0) : L+1 \leq i \leq 2n-k\} \cup \{(i, 1, 0) : N+1 \leq i \leq n-1\} \cup \{(i, 1, 1) : k-1 \leq i \leq n-1\}$$

The transition rates between states in the Markov chain are as given below:

- (i) $(i, 0, 0) \rightarrow (i-1, 0, 0)$: rate is $i_{\min}\lambda$, where $i_{\min} = \min\{i, n\}$, for $i = L+2, \dots, 2n-k$.
- (ii) $(i, 0, 0) \rightarrow (i-1, 1, 0)$: rate is $i\lambda$, for $i = L+1$.
- (iii) $(i, 1, 0) \rightarrow (i-1, 1, 0)$: rate is $i\lambda$, for $i = N+2, N+3, \dots, n-1$.
- (iv) $(i, 1, 0) \rightarrow (i+1, 0, 0)$: rate is γ , for $i = n-1$.
- (v) $(i, 1, 0) \rightarrow (i+1, 1, 0)$: rate is γ , for $i = N+1, \dots, n-2$.
- (vi) $(i, 1, 0) \rightarrow (i-1, 1, 1)$: rate is $i\lambda$, for $i = N+1$.
- (vii) $(i, 1, 1) \rightarrow (i-1, 1, 1)$: rate is $i\lambda$, for $i = k, k+1, \dots, n-1$.
- (viii) $(i, 1, 1) \rightarrow (i+1, 0, 0)$: rate is γ , for $i = n-1$.
- (ix) $(i, 1, 1) \rightarrow (i+1, 1, 1)$: rate is γ , for $i = k-1, k, \dots, n-2$.
- (x) $(i, 1, 1) \rightarrow (i+n-k+1, 0, 0)$: rate is β , for $i = k-1, k, \dots, n-1$.

All other transition rates are zero.

Let $\mathbf{g}$ be the steady-state probability vector and $\mathcal{P}$ be the transition rate matrix for the underlying Markov chain. $\mathcal{P} = [p_{xy}]$ is a matrix of order $4n-2k-L-N$ and is given by

$$\mathcal{P} = [p_{xy}] = \begin{cases} -n\lambda, & \text{if } y = x, \\ & \text{where } x = (2n-k, 0, 0), \dots, (n, 0, 0) \\ n\lambda, & \text{if } x = (2n-k, 0, 0), \dots, (n, 0, 0) \\ & \text{and } y = (2n-k-1, 0, 0), \dots, (n-1, 0, 0) \\ -i\lambda, & \text{if } y = x, \\ & \text{where } x = (n-1, 0, 0), \dots, (L+1, 0, 0) \\ i\lambda, & \text{if } x = (n-1, 0, 0), (n-1, 1, 0), (n-1, 1, 1), \\ & \dots, (L+1, 0, 0), (L+1, 1, 0), (L+1, 1, 1), \\ & (L, 1, 0), (L, 1, 1), \dots, (N+1, 1, 0), \\ & (N+1, 1, 1), (N, 1, 1), \dots, (k, 0, 1) \text{ and} \\ & y = (n-2, 0, 0), (n-2, 1, 0), (n-2, 1, 1), \\ & \dots, (L+1, 0, 0), (L+1, 1, 0), (L+1, 1, 1), \\ & (L, 1, 0), (L, 1, 1), \dots, (N+1, 1, 0), \\ & (N+1, 1, 1), (N, 1, 1), \dots, (k-1, 1, 1) \\ -(i\lambda + \gamma), & \text{if } y = x, \\ & \text{where } x = (n-1, 1, 0), \dots, (N+1, 1, 0) \\ -(i\lambda + \gamma + \beta), & \text{if } y = x, \\ & \text{where } x = (n-1, 1, 1), \dots, (k, 1, 1) \\ \gamma, & \text{if } x = (n-1, 1, 0), (n-1, 1, 1), \dots, \\ & (N+1, 1, 0), (N+1, 1, 1), (N, 1, 1), \\ & \dots, (k-1, 1, 1) \text{ and } y = (n, 0, 0), \\ & (n-1, 1, 0), (n-1, 1, 1), \dots, (k, 1, 1) \\ \beta, & \text{if } x = (n-1, 1, 1), \dots, (k-1, 1, 1) \\ & \text{and } y = (2n-k, 0, 0), \dots, (n, 0, 0) \\ -(\beta + \gamma), & \text{if } y = x, \text{ where } x = (k-1, 1, 1) \\ 0, & \text{elsewhere} \end{cases}$$

The CTMC $\{(X(t), Y(t), Z(t)) : t \geq 0\}$ we analyze has finite state space. It is irreducible because all states are mutually accessible (in either just one transition or in a finite number (>1) of transitions). Further, it is aperiodic. So the limiting system-state probabilities

exist. Thus, the steady-state probability is obtained by solving $qP = 0$ with $\sum_{i \in S} q_i = 1$. As a result, we obtain the steady-state probabilities for the system as follows:

$$q_{i+1,0,0} = \begin{cases} q_{i,0,0} - \left(\frac{\beta}{n\lambda}\right) q_{i-(n-k+1),1,1}; & \text{for } i = n+1, \dots, 2n-k-1 \\ q_{i,0,0} - \left(\frac{\beta}{n\lambda}\right) q_{i-(n-k+1),1,1} - \left(\frac{\gamma}{n\lambda}\right) [q_{i-1,1,0} + q_{i-1,1,1}]; & \text{for } i = n \\ \left(\frac{i\lambda}{(i+1)\lambda}\right) q_{i,0,0}; & \text{for } i = L+1, L+2, \dots, n-1 \\ \left(\frac{i\lambda+\gamma}{(i+1)\lambda}\right) q_{i,1,0} - \left(\frac{\gamma}{(i+1)\lambda}\right) q_{i-1,1,0} - q_{i+1,1,0}; & \text{for } i = L \end{cases}$$

$$q_{i+1,1,0} = \begin{cases} \left(\frac{i\lambda+\gamma}{(i+1)\lambda}\right) q_{i,1,0} - \left(\frac{\gamma}{(i+1)\lambda}\right) q_{i-1,1,0}; & \text{for } i = N+2, \dots, L-1, \\ & L+1, \dots, n-2 \\ \left(\frac{i\lambda+\gamma}{(i+1)\lambda}\right) q_{i,1,0}; & \text{for } i = N+1 \\ \left(\frac{i\lambda+\beta+\gamma}{(i+1)\lambda}\right) q_{i,1,1} - \left(\frac{\gamma}{(i+1)\lambda}\right) q_{i-1,1,1} - q_{i+1,1,1}; & \text{for } i = N \\ \left(\frac{\gamma}{(i+1)\lambda+\gamma}\right) q_{i,1,0}; & \text{for } i = n-2 \end{cases}$$

$$q_{i+1,1,1} = \begin{cases} \left(\frac{i\lambda+\beta+\gamma}{(i+1)\lambda}\right) q_{i,1,1} - \left(\frac{\gamma}{(i+1)\lambda}\right) q_{i-1,1,1}; & \text{for } i = k, \dots, N-1, N+1, \dots, n-2 \\ \left(\frac{\beta+\gamma}{(i+1)\lambda}\right) q_{i,1,1}; & \text{for } i = k-1 \\ \left(\frac{\gamma}{(i+1)\lambda+\beta+\gamma}\right) q_{i,1,1}; & \text{for } i = n-2 \end{cases}$$

The system-state probabilities can be expressed in terms of $q_{k-1,0,1}$. Then, $q_{k-1,0,1}$ can be computed by the normalizing condition/total probability argument. Hence, we have obtained the explicit analytical expression for the system-state probabilities.

3 Performance Measures

In this section, the important system performance measures and their expressions for the proposed model are listed as follows.

3.1 Characteristics of the System

- (1) Expected number of working batteries when the repair is ON, $E(X)$

$$E(X) = \sum_{i=N+1}^{n-1} i q_{i,1,0} + \sum_{i=k-1}^{n-1} i q_{i,1,1}$$

- (2) Expected number of working batteries when the repair is OFF, $E(Y)$

$$E(Y) = \sum_{i=L+1}^n i q_{i,0,0} + \sum_{i=n+1}^{2n-k} n q_{i,0,0}$$

- (3) Mean number of operational batteries when the system is in working mode (excluding spares, if any)

$$\text{OCW} = \sum_{i=n+1}^{2n-k} n q_{i,0,0} + \sum_{i=N+1}^{n-1} i q_{i,1,0} + \sum_{i=k}^{n-1} i q_{i,1,1} + \sum_{i=L+1}^n i q_{i,0,0}$$

- (4) Mean number of spare batteries

$$\text{MSC} = \sum_{i=n+1}^{2n-k} (i-n) q_{i,0,0}$$

- (5) Effective repair rate

$$\text{ERT} = \gamma \left[\sum_{i=k-1}^{n-1} (n-i) q_{i,1,1} + \sum_{i=N+1}^{n-1} (n-i) q_{i,1,0} \right]$$

- (6) Fraction of time system is down (that is, in failed state)

$$\text{FTD} = q_{k-1,1,1}$$

- (7) Fraction of time system is working (that is, system reliability)

$$\text{FTU} = 1 - q_{k-1,1,1}$$

3.2 The Expected Length of Cycle, $E(W)$. Let W be the cycle length (starting from state $(N, 1, 1)$, returning to it for the first time). We have to compute the distribution of W and then $E(W)$. Here, the cycle time is computed as the time duration from an epoch at which state $(N, 1, 1)$ is reached to the next epoch at which that state is

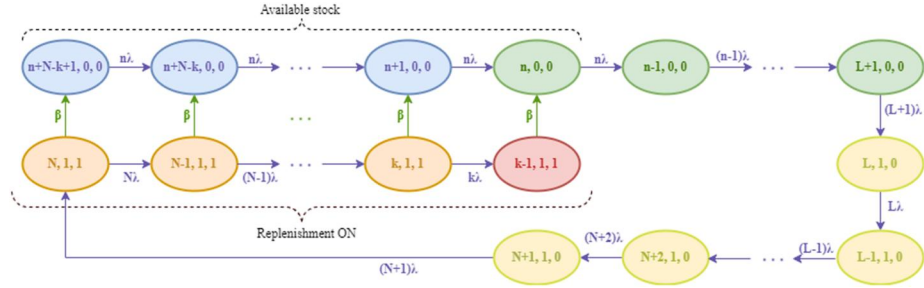


Fig. 4 Transition diagram of the replenishment cycle

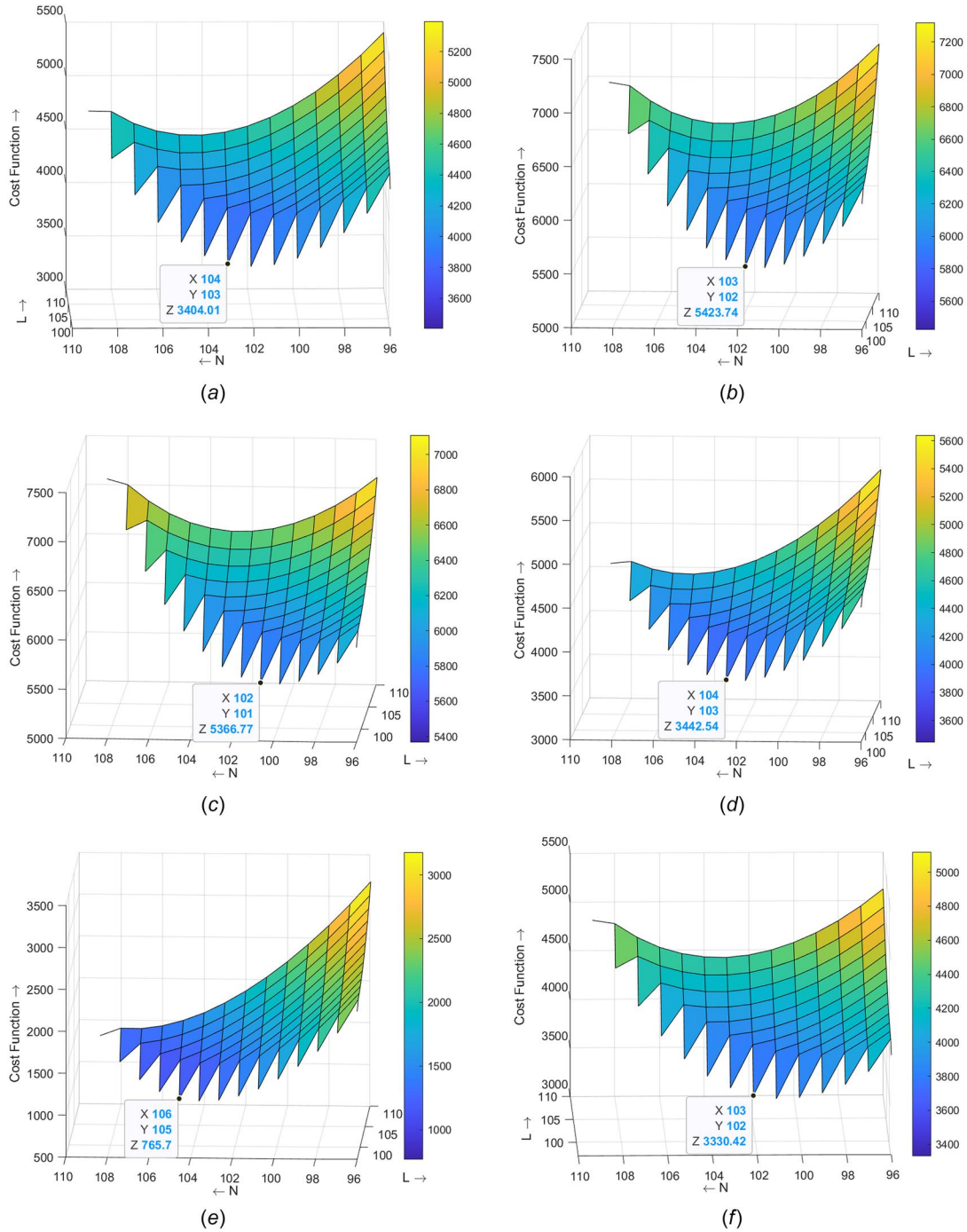


Fig. 5 Cost function obtained by varying L and N for various choices of λ , β , and γ : (a) $\lambda = \beta = \gamma$, (b) $\lambda > \beta > \gamma$, (c) $\lambda > \gamma > \beta$, (d) $\beta > \lambda > \gamma$, (e) $\beta > \gamma > \lambda$, (f) $\gamma > \lambda > \beta$, (g) $\gamma > \beta > \lambda$, (h) $\lambda > \beta$ with $\beta = \gamma$, (i) $\beta > \lambda$ with $\lambda = \gamma$, (j) $\gamma < \beta$ with $\beta = \lambda$, (k) $\lambda < \beta$ with $\gamma = \beta$, (l) $\beta < \lambda$ with $\lambda = \gamma$, and (m) $\gamma < \beta$ with $\lambda = \beta$

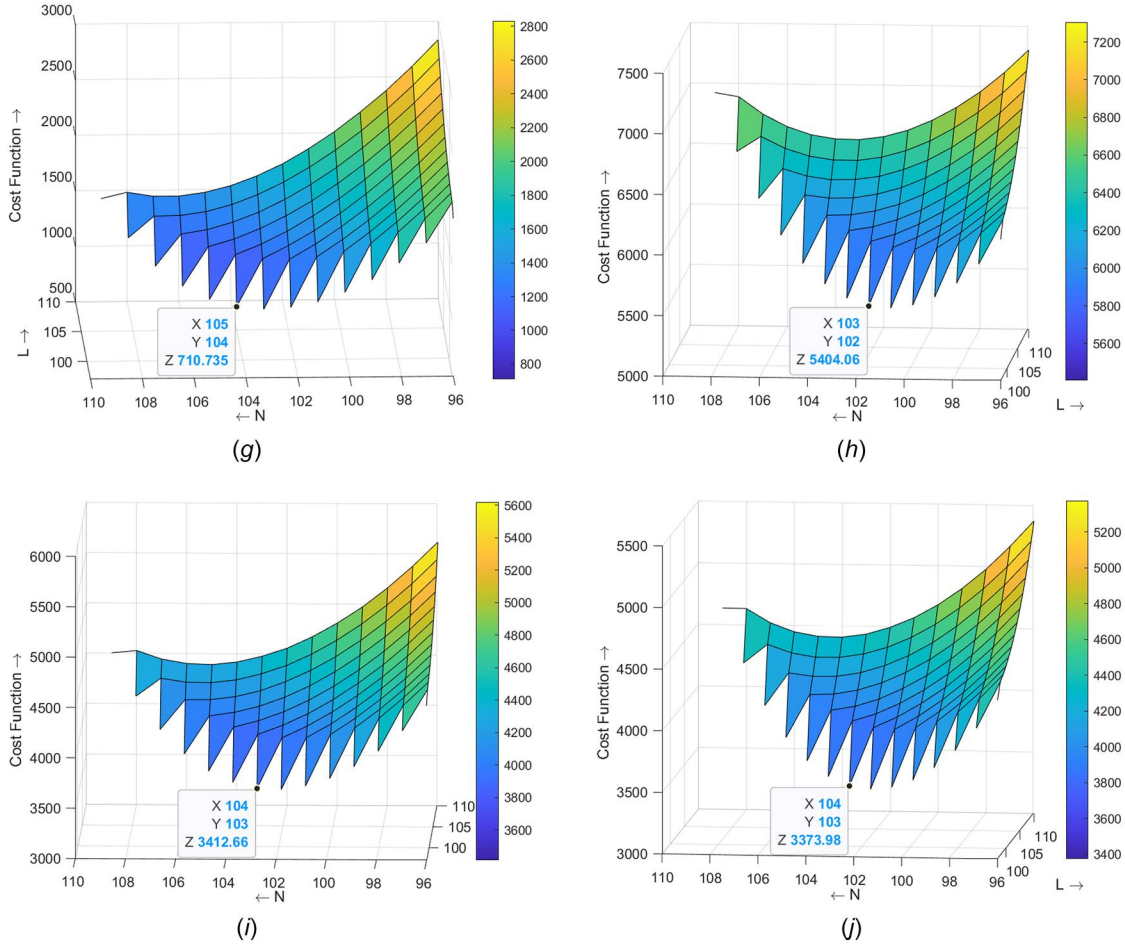


Fig. 5 (Continued)

revisited. A vital modification is made in the state space's order. An additional element is added to the state space for the computation of the distribution of W . Thus, the state space becomes

$$\Omega = \{(N, 1, 1), (N-1, 1, 1), \dots, (k-1, 1, 1), (n+N-k+1, 0, 0), \\ (n+N-k, 0, 0), \dots, (n, 1, 0), (n, 0, 0), (n-1, 0, 0), \dots, \\ (L+1, 0, 0), (L, 1, 0), (L-1, 1, 0), \dots, (N+1, 1, 0), D\}$$

The state transition diagram is shown in Fig. 4.

Here, D is an absorbing state, and the remaining states are transient. D is same as the state $(N, 1, 1)$. Although this D is identical to the state $(N, 1, 1)$, employing a different notation is meant to show that the state $(N, 1, 1)$ is revisited. Thus, starting from $(N, 1, 1)$ returning to $(N, 1, 1)$ for the very first time after the next replenishment at the same level or a lower level followed by battery deterioration. We compute the distribution of the time duration elapsed. The infinitesimal generator of the corresponding CTMC is given as follows:

$$Q = \begin{bmatrix} T & T^* \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad \text{where} \quad T = \begin{bmatrix} A_{11} & A_{12} \\ & A_{22} & A_{23} \\ & & A_{33} \end{bmatrix} \quad \text{and}$$

T^* is a column vector with $(N+1)\lambda$ in the last position.

$A_{11} = [a_{xy}]$ where

$$[a_{xy}] = \begin{cases} -(\beta + i\lambda), & \text{if } y = x, \text{ if } x = (N, 1, 1), \\ & (N-1, 1, 1), \dots, (k, 1, 1) \\ i\lambda, & \text{if } x = (N, 1, 1), \dots, (k, 1, 1) \text{ and} \\ & y = (N-1, 1, 1), \dots, (k-1, 1, 1) \\ -\beta, & \text{if } y = x, \text{ where } x = (k-1, 1, 1) \\ 0, & \text{elsewhere} \end{cases}$$

$A_{12} = [a_{xy}]$ where

$$[a_{xy}] = \begin{cases} \beta, & \text{if } x = (N, 1, 1), (N-1, 1, 1), \dots, (k-1, 1, 1) \\ & \text{and } y = (n+N-k+1, 0, 0), \dots, (n, 0, 0) \\ 0, & \text{elsewhere} \end{cases}$$

$A_{22} = [a_{xy}]$ where

$$[a_{xy}] = \begin{cases} -n\lambda, & \text{if } y = x, \text{ if } x = (n+N-k+1, 0, 0), \\ & \dots, (n, 0, 0) \\ n\lambda, & \text{if } x = (n+N-k+1, 0, 0), \dots, (n, 1, 0) \\ & \text{and } y = (n+N-k, 0, 0), \dots, (n, 0, 0) \\ 0, & \text{elsewhere} \end{cases}$$

$A_{23} = [a_{xy}]$ where

$$[a_{xy}] = \begin{cases} n\lambda, & \text{if } x = (n, 0, 0) \text{ and } y = (n-1, 0, 0) \\ 0, & \text{elsewhere} \end{cases}$$

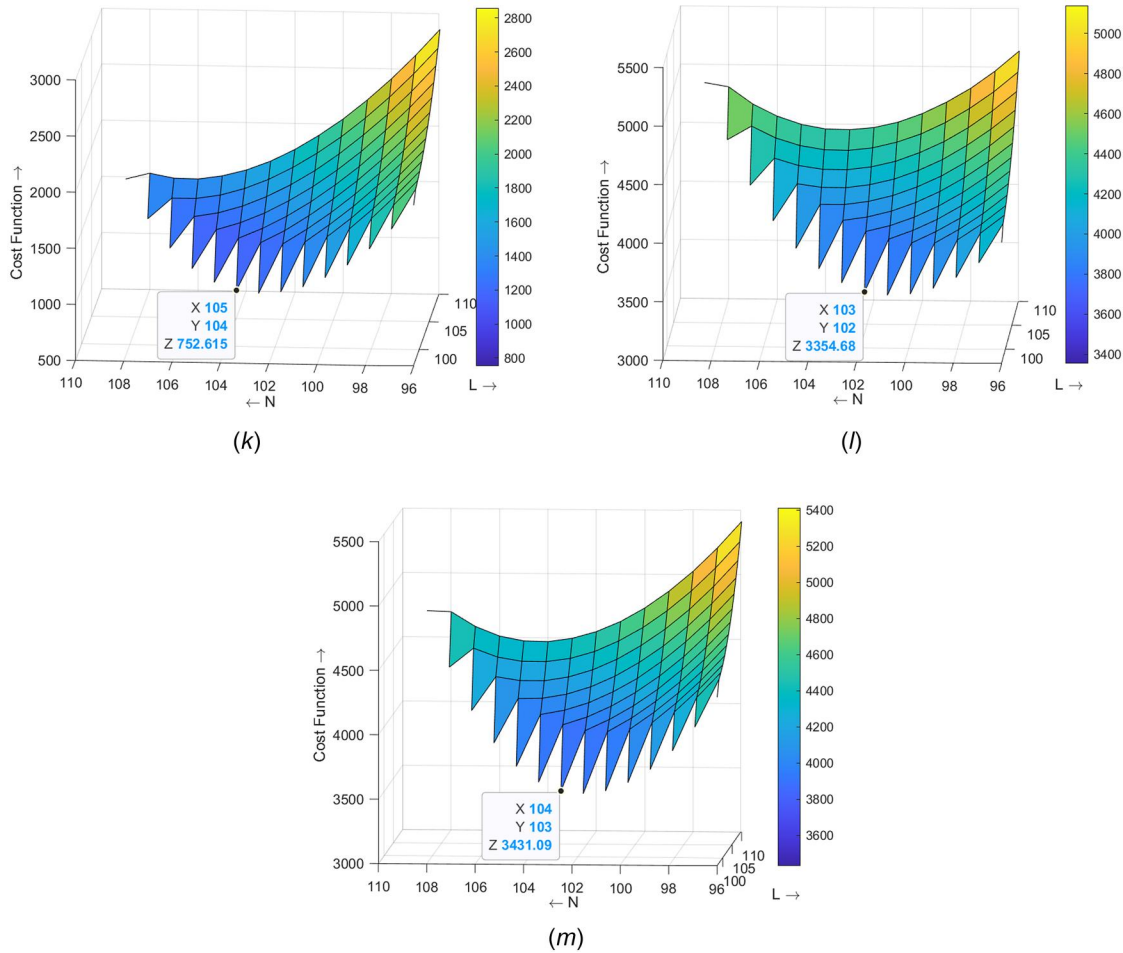


Fig. 5 (Continued)

Table 1 Pseudocode for cost optimization

Pseudocode for cost optimization

Step 1. Importing libraries:

- numpy
- matplotlib.pyplot
- pyswarms
- mpl_toolkits.mplot3d

Step 2. Define functions:

- `callexe(L, N)`:
 - Initialize parameters and ranges.
 - Generate valid combinations of states ($M1$, $M2$) based on the given problem.
 - Populate a dictionary $D1$ with costs based on specific rules.
 - Return $D1$.
- `calculate_cost(L, N)`:
 - Convert L and N to integers.
 - Call `callexe(L, N)` to generate the dictionary $D1$.
 - Calculate individual cost components ($f1$, $f2$, $f3$, $f4$) using the cost function and $D1$.
 - Return the total cost.
- `cost_optimum_function(positions)`:
 - For each particle's position (L , N), compute the cost using `calculate_cost(L, N)`.

Step 3. Setup bounds and PSO optimization:

- Define the lower and upper bounds for L and N .
- Initialize the PSO optimizer with bounds, number of particles, dimensions, and control parameters ($c1$, $c2$, w).

Step 4. Optimize:

- Call the optimizer to minimize the `cost_optimum_function` over a specified number of iterations.
- Extract the optimal cost and positions.

Step 5. Output results:

- Print the optimal cost and corresponding values of L and N .

Table 2 Optimal cost obtained by varying L and N for different choices of λ , β , and γ

λ	β	γ	L	N	Optimal costs
5	5	5	103	102	3330.417221
7	5	3	103	102	5423.737678
7	3	5	102	101	5366.765683
5	7	3	104	103	3442.539785
3	7	5	106	105	765.6998094
5	3	7	103	102	3330.417221
3	5	7	105	104	710.7352907
7	5	5	103	102	5404.057678
5	7	5	104	103	3412.663785
5	5	7	104	103	3373.975065
3	5	5	105	104	752.6152907
5	3	5	103	102	3354.677221
5	5	3	104	103	3431.093065

The lowest optimal cost obtained for different choices of parameters are in bold. $A_{33} = [a_{xy}]$ where

$$[a_{xy}] = \begin{cases} i\lambda, & \text{if } x = (n-1, 0, 0), \dots, (L+1, 0, 0), \\ & (L, 1, 0), \dots, (N+2, 1, 0) \text{ and} \\ & y = (n-2, 0, 0), \dots, (L+1, 0, 0), \\ & (L, 1, 0), \dots, (N+1, 1, 0) \\ -i\lambda, & \text{if } y = x, \text{ where} \\ & x = (n-1, 0, 0), \dots, (L+1, 0, 0), \\ & (L, 1, 0), \dots, (N+1, 1, 0) \\ 0, & \text{elsewhere} \end{cases}$$

The time until absorption to D has the Coxian distribution with representation (α, T) where α is the initial probability vector with 1 at the first position, and the remaining elements are 0's, and T is of order $(n + N - 2k + 3)$. Therefore, $E(W) = \alpha T^{(-1)} \mathbf{e}$.

3.3 Distribution of Time Until the First Failure of System. We aim to concentrate on the duration during which the system consistently remains in the transient states of the Markov chain. Thus, we narrow our focus to the intervals during which the system persists in transient states, excluding the state $(k-1, 1, 1)$ from our state space S . As a result, following a replenishment, the spare battery level cannot be zero. Initially, we define the initial probability vector ϕ_1 of the Markov chain, assigning probability 1 to the state $(n, 0, 0)$ and 0 to all other states. This setup signifies the system's commencement from the state $(n, 0, 0)$, setting the stage for our investigation into determining the distribution of time required for the system to reach the state $(k-1, 1, 1)$ for the first occurrence.

Our consideration of the state space, denoted as S_1 , encompasses three distinct subsets: $(i, 0, 0)$ for $L+1 \leq i \leq 2n-k$, $(i, 1, 0)$ for $N+1 \leq i \leq n-1$, and $(i, 1, 1)$ for $k \leq i \leq n-1$.

The distribution governing the time elapsed until the system reaches the state $(k-1, 1, 1)$ for the first time adheres to a

phase-type distribution, characterized by the representation (ϕ_1, T_1) . Here, T_1 is derived by eliminating the last row and column from the infinitesimal generator corresponding to the Markov chain with state space S_1 . The dimension of T_1 is determined to be $4n - L - N - 2k - 1$. Consequently, the anticipated duration until failure is computed as $-\phi_1 T_1^{-1} \mathbf{e}$, where $\mathbf{e}$ denotes a vector of ones. This formulation encapsulates the expected time until the occurrence of the first system failure.

3.4 Distribution of the Time During Which the Repair Facility Is Continuously Available. To derive the distribution characterizing the duration during which the repair facility remains continuously active, we adopt an assumption where the replenishment order does not materialize. This consideration leads us to analyze a specific Markov chain comprising states $\{(L, 1, 0), (L-1, 1, 0), \dots, (N+1, 1, 0), (k-1, 1, 1), (k, 1, 1), \dots, (N-1, 1, 1), (N, 1, 1), (N+1, 1, 1), \dots, (n-1, 1, 1), (n, 0, 0)\}$, where $(n, 0, 0)$ signifies the absorbing state. Notably, the state $(k-1, 1, 1)$ is excluded from our analysis.

In this Markov chain, the matrix T_2 , derived by omitting the last row and column from the corresponding infinitesimal generator matrix, plays a pivotal role in determining the time distribution until absorption. Notably, this distribution adheres to a phase-type distribution, characterized by the representation (ϕ_2, T_2) . Here, ϕ_2 represents the probability vector, with a value of 1 assigned to the entry corresponding to the initial state $(L, 1, 0)$, while all other entries remain zero. This strategic formulation facilitates a comprehensive understanding of the temporal dynamics governing the repair facility's uninterrupted activity.

4 Optimization Problem and Sensitivity Analysis

In this section, we delve into the formulation and solution of the optimization problem, followed by a comprehensive sensitivity analysis of the performance measures. The objective is to optimize the cost function and identify the optimal pair (L^*, N^*) , where L^* represents the optimal L to start the repair, and N^* denotes the optimal N to place the replenishment order. We aim to enhance the system's efficiency and performance by determining these optimal values.

The section is structured into three subsections. In Sec. 4.1, we present the mathematical formulation of the cost function and employ optimization techniques to find the optimal pair (L^*, N^*) . In Sec. 4.2, we conduct a detailed numerical analysis of the performance measures based on the optimized values obtained, whereas Sec. 4.3 discusses the sensitivity analysis using the Morris method. This approach allows us to identify optimal settings and understand the system's robustness, sensitivity, and adaptability under different conditions.

4.1 Optimization Problem. As discussed in SoftBank R & D [19], while the primary objective for most aircraft is to maximize

Table 3 Pseudocode for sensitivity analysis

Pseudocode for sensitivity analysis

- **Step 1.** Import libraries: Import `numpy` and `expm` from `scipy.linalg`.
- **Step 2.** Initialize variables: Define the main function `callexe(La, B, G)` with fixed constants N, n, k, L and an empty state list `r1`.
- **Step 3.** Generate state tuples: Populate `r1` using nested loops and conditions based on N, n, k , and L .
- **Step 4.** Transition rates dictionary: Create a dictionary `D1` by calculating transition rates between states using `La, B`, and `G`.
- **Step 5.** Transition matrix: Construct a matrix (`matrixtrm`) using state pairs and fill it with values from `D1` or zeros for undefined transitions.
- **Step 6.** Steady-state probabilities: Define `compute_steady_state(matrixtrm)` to solve a linear system for steady-state probabilities and normalize them.
- **Step 7.** Performance metrics: Calculate metrics using steady-state probabilities:
 - $E(X), E(Y)$: Expected working components when repair ON and OFF.
 - OCW, MSC, ERT: Operational components, spare components, and effective repair rate.
 - FTD, FTU: System downtime and uptime probabilities.
- **Step 8.** Output results: Print probabilities and metrics, verifying their correctness.
- **Step 9.** Iterate for `La, B`, and `G` individually: Vary `La, B`, and `G` individually over a range and compute results for each value.

flight speed and distance, HAPS are designed for optimized circling flight or loitering. In particular, we know that the flight module of tethered HAPS is connected to the GCS using the tether. So, in this mode, the aircraft is supposed to remain in a specific airspace for extended periods. Consequently, the air-frame is engineered to minimize propulsive energy consumption, akin to the design principles of human-powered aircraft, high-performance gliders, and competition model flight modules. Thus, assuming an average power consumption of 14 kW for a HAPS flight module, resulting in a daily power demand of 336 kWh, and with each battery having a capacity of 3.5 kWh, the system requires a minimum of 96 batteries to remain operational. For a detailed sensitivity analysis, we assume that the system contains 112 batteries in total (excluding spares), forming a 96-out-of-112 : G , COLD system. Thus, the study analyzes the average cost associated with maintaining a stockpile of 129 batteries (including spares) within the GCS, ensuring successful flight module operation. Now, we construct a cost function incorporating the relevant costs associated with the model, and as mentioned in Sec. 1, we aim to determine the optimal pair (L^*, N^*) that corresponds to the minimal cost. Thus, we primarily focus on optimizing the cost function in this section.

We consider the cost function as the average cost per unit time when the replenishment order for spare batteries is placed, with the number of operating batteries dropping to N . For constructing the cost function, the following relevant costs are considered:

- C_{repair} —Repairing cost per unit battery.

- C_{fixed} —The fixed cost of placing an order for replenishment.
- C_{cancel} —Order cancellation cost.
- C_{purchase} —Purchase cost per unit battery.
- C_{hold} —Holding cost per excess unit held per time.
- C_{penalty} —Penalty cost/time when the system is down.

The cost function is given by

$$F(N, L) = \frac{C_{\text{fixed}} + C_{\text{purchase}}(n - k + 1) + C_{\text{cancel}}}{\beta E(W)} + C_{\text{repair}} \left(\sum_{i=k-1}^{n-1} (n-i)q_{i,1,1} + \sum_{i=N+1}^{n-1} (n-i)q_{i,1,0} \right) + C_{\text{hold}} \left(\sum_{i=n+1}^{2n-k} (i-n)q_{i,0,0} \right) + C_{\text{penalty}} q_{k-1,1,1}$$

where the expected length of the cycle, $E(W)$, is given in Sec. 3.2. The decision variables are N and L , explicitly mentioned in the cost function.

In this study, we fix the per unit time costs as follows: $C_{\text{fixed}} = \$100$, $C_{\text{purchase}} = \$400$, $C_{\text{cancel}} = \$44$, $C_{\text{repair}} = \$300$, $C_{\text{hold}} = \$80$, and $C_{\text{penalty}} = \$21,000$. The objective is to determine the optimal cost and the corresponding optimal pair (L^*, N^*) for various values of λ , β , and γ . The different choices of parameters reflect practical situations in which these conditions may occur. Our analysis reveals

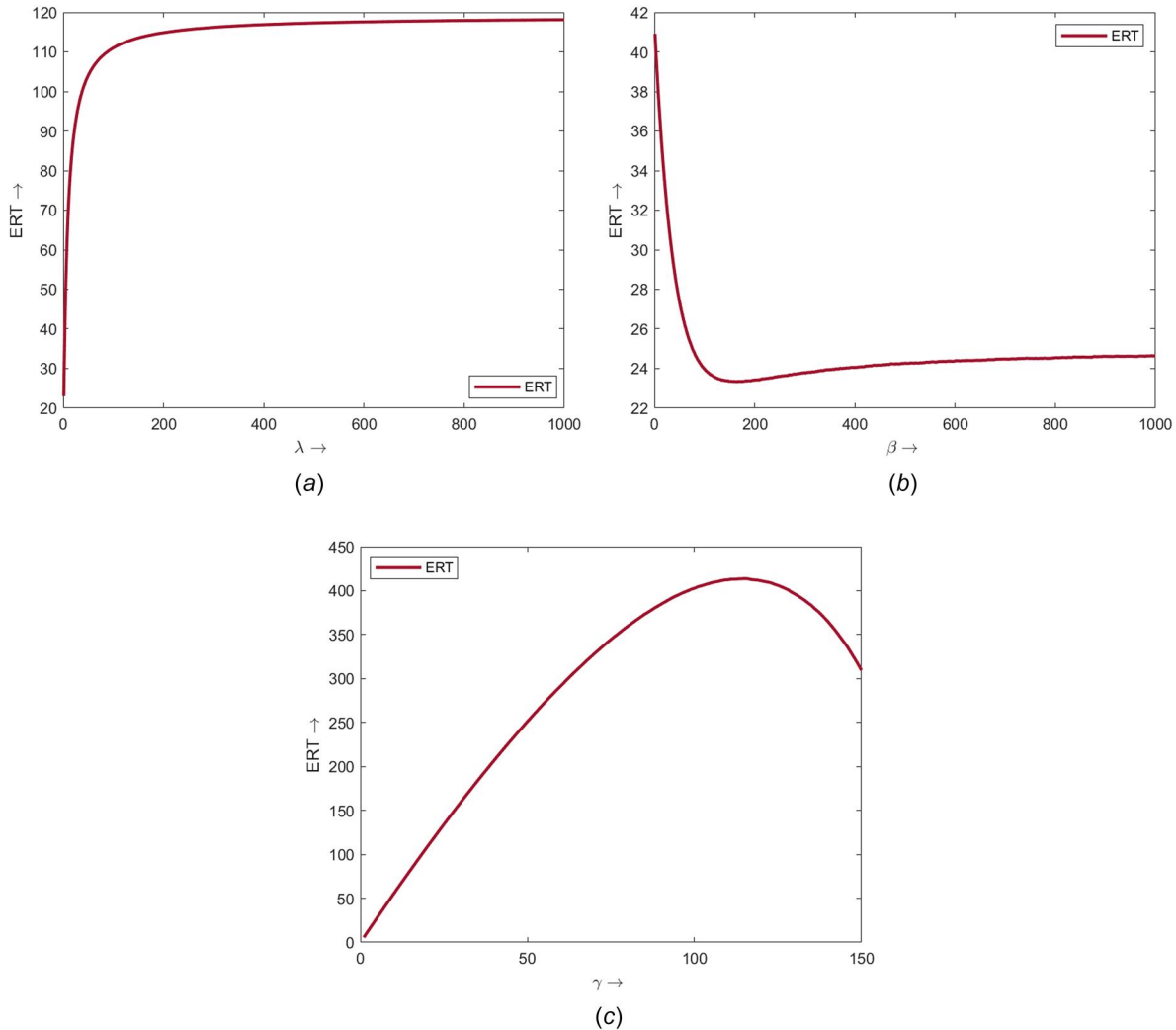


Fig. 6 Effect on ERT when λ , β , and γ varies: (a) when λ varies, (b) when β varies, and (c) when γ varies

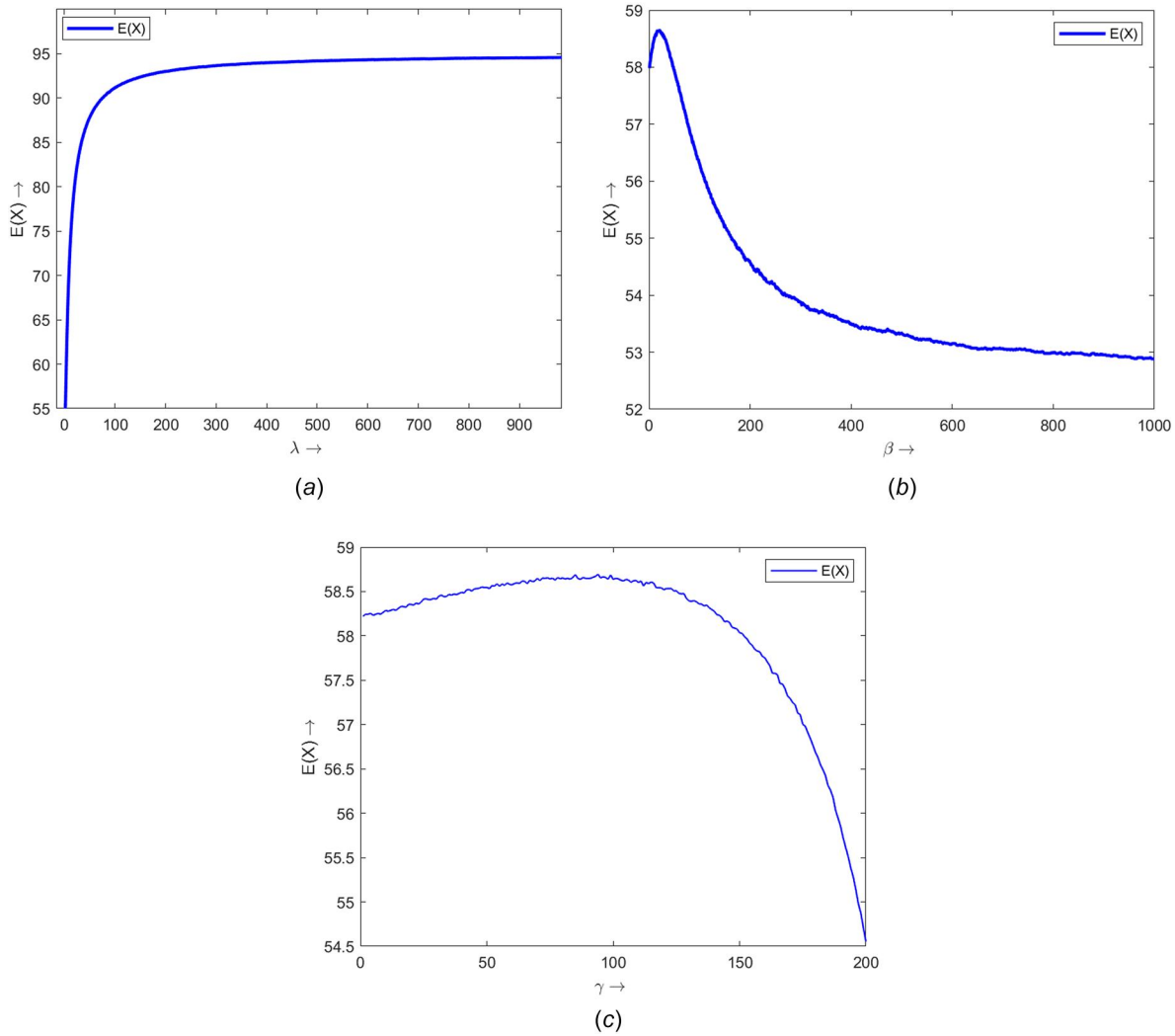


Fig. 7 Effect on $E(X)$ when λ , β , and γ varies: (a) when λ varies, (b) when β varies, and (c) when γ varies

significant variations in the cost function across different parameter values, refer to Fig. 5. The figures depict 3D surface plots of a cost function with respect to N and L . The cost function is represented on the z-axis, while N and L are on the x and y axes, respectively. Each figure contains a 3D surface plot showing the relationship between these variables. The color gradient effectively represents the cost distribution across the surface from the lower to the higher costs. We employ the PSO algorithm [20] to find the optimal cost and the corresponding optimal pair (L^*, N^*) , with the pseudocode for optimization provided in Table 1 and results presented in Table 2.

The figures illustrate the behavior of the cost function in relation to variables N and L . The cost function first decreases and then increases as N decreases, with notable variations in the highlighted cost function values. Figure 5(g) shows the lowest cost at (105, 104, 710.735), while Fig. 5(c) shows the highest cost at (103, 102, 5423.74).

Our findings indicate that the cost function achieves a lower overall cost when $\lambda < \beta < \gamma$ and the highest cost when $\lambda > \beta > \gamma$. Specifically, when the repair rate of failed batteries exceeds both the replenishment rate and the failure rate, with the failure rate being lower than the replenishment rate, the overall cost is minimized compared to other parameter selections. Moreover, we observe that the penalty cost exerts a negligible influence on the cost function, reducing the overall cost. This suggests that the system is less likely to encounter failures. This outcome aligns with our expectations, given that the repair cost is significantly lower than the penalty and replenishment costs in the cost function. Consequently, a system

characterized by a higher repair rate will incur a lower overall cost over time, as the repair cost remains relatively low.

However, when the failure rate surpasses the rates of repair and replenishment within a system, the likelihood of encountering a breakdown increases. Consequently, in scenarios where the failure rate outweighs the other contributing factors, the imposition of penalty costs significantly impacts the cost function, escalating the overall expenses incurred. This elucidates the rationale behind observing markedly elevated optimal costs when the failure rate exceeds the repair and replenishment rates, that is, $\lambda > \beta > \gamma$.

Thus, we achieve a highly reliable and cost-effective system for the model under study when the condition $\lambda < \beta < \gamma$ is met. Specifically, an optimal cost of \$710.735 is obtained for the 96-out-of-112 : G, COLD system, corresponding to the optimal pair $(L^*, N^*) = (104, 103)$ under the given condition.

4.2 Analysis of System Characteristics. In this section, we graphically illustrate the system's performance as underlying parameters vary over time. Based on the results obtained in Sec. 4.1, we focus on a 96-out-of-112 : G, COLD system with optimal values $L^* = 105$ and $N^* = 104$. We now analyze how variations in key parameters impact the continuous power supply to the tethered HAPS, which subsequently affects the system's reliability and performance. We study these variations by employing the one-parameter variation approach while keeping the other two parameters constant. Table 3 presents the pseudocode of the sensitivity analysis.

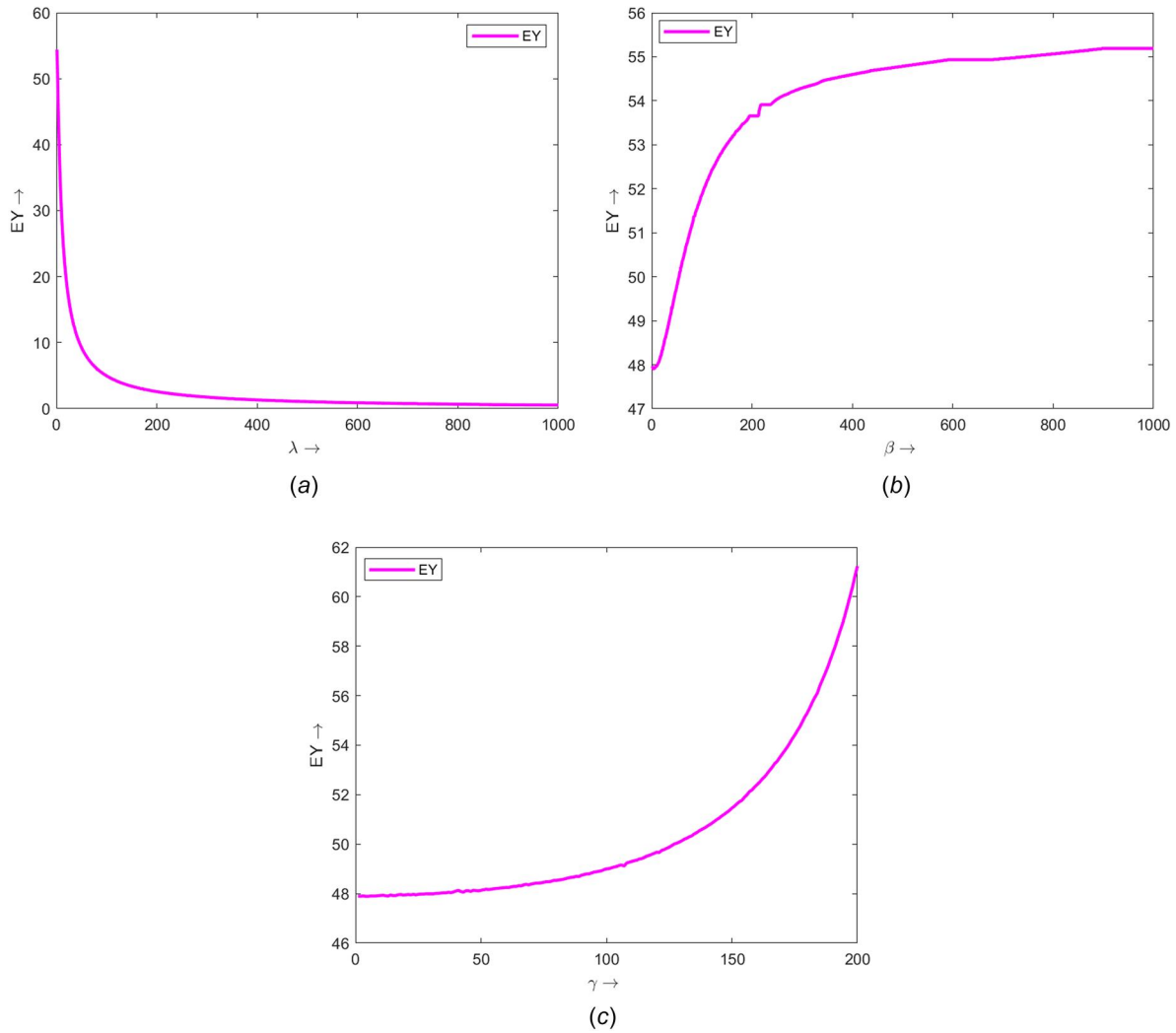


Fig. 8 Effect on $E(Y)$ when λ , β , and γ varies: (a) when λ varies, (b) when β varies, and (c) when γ varies

Specifically, when varying λ , we maintain $\gamma = 7$ and $\beta = 5$. When varying β , we fix $\lambda = 3$ and $\gamma = 7$. Finally, while varying the γ , we keep $\lambda = 3$ and $\beta = 5$. This methodical analysis allows us to understand the impact of each parameter on system performance, providing a comprehensive understanding of the system's behavior under different conditions.

Figure 6(a) illustrates the relationship between the λ and ERT. As λ increases, ERT correspondingly rises due to the higher accumulation of failed batteries awaiting repair. This increase is attributable to the augmented number of failures necessitating attention and leading to a greater accumulation of failed batteries awaiting repair, thereby increasing the system's downtime and decreasing reliability. Thus, the system becomes less reliable when battery failures occur more frequently. Additionally, the positive correlation between the repair rate and ERT is evident, as an elevated repair rate naturally enhances the efficiency of the repair process, implying that systems with higher repair rates can quickly address failures and reduce downtime, thus improving reliability. Conversely, an increase in the replenishment rate reduces ERT based on the assumption that repair activities cease upon the arrival of new orders. This indicates that the system becomes more reliable as the replenishment rate increases. Specifically, higher replenishment rates minimize the time batteries remain in a failed state, thereby enhancing the system's overall reliability. The impacts of varying β and γ on ERT are depicted in Figs. 6(b) and 6(c), respectively.

Figure 7 depicts $E(X)$ as a function of three different parameters: λ , β , and γ , respectively. Each figure shows how $E(X)$ changes as one of these parameters varies while holding the others constant. In Fig. 7(c), as γ increases, the system initially benefits from more efficient repairs, leading to a rise in $E(X)$. However, the effective repair rate decreases after a certain point, likely due to diminishing returns or over-repair scenarios where the system is overloaded with repairs, reducing the overall efficiency and thus decreasing $E(X)$. In Fig. 7(b), we observe that $E(X)$ decreases as the replenishment rate increases, which is an unexpected outcome. In theory, a higher replenishment rate should increase $E(X)$. However, the effective repair rate decreases with increasing β , suggesting that the system might not effectively utilize the replenished components due to bottlenecks or inefficiencies in the repair process. This could lead to a net decrease in $E(X)$ as β increases. Counterintuitively, the increasing λ leads to an increase in the expected number of operational components $E(X)$, see Fig. 7(a). This can be explained by the fact that the ERT also increases with λ . A higher failure rate likely prompts the system to improve its repair mechanisms or prioritize repairs more efficiently, leading to a higher $E(X)$ as more batteries are kept in working condition. In summary, λ , β , and γ influence the expected number of working batteries $E(X)$ in complex ways. Increasing β and γ beyond certain points can decrease $E(X)$ due to reduced repair efficiency while increasing λ improves repair efficiency, leading to higher $E(X)$. Understanding these dynamics is crucial for optimizing maintenance and repair strategies to maintain

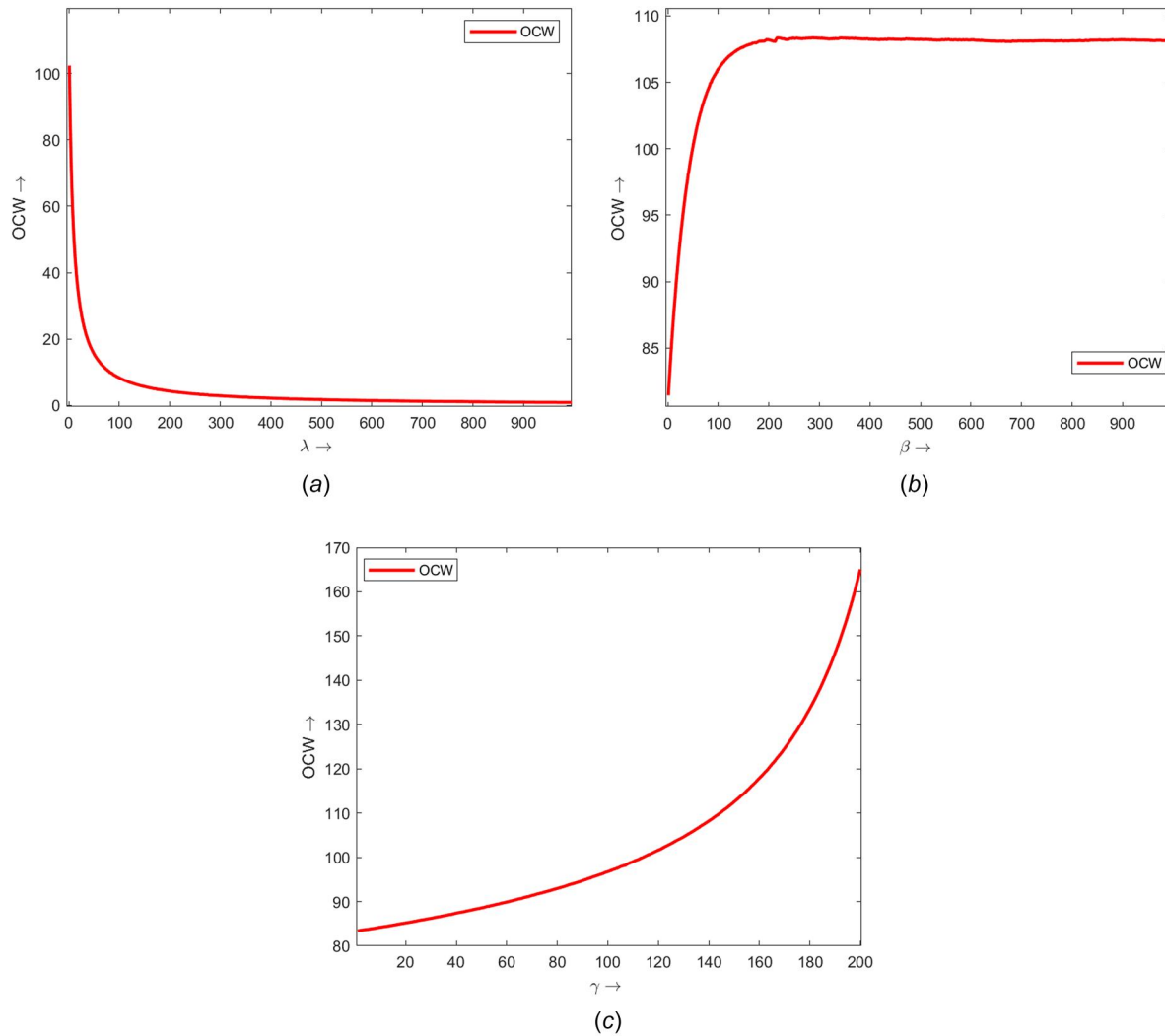


Fig. 9 Effect on OCW when λ , β , and γ varies: (a) when λ varies, (b) when β varies, and (c) when γ varies

a high number of operational components in the system, which, in effect, increases the system's reliability.

When λ increases, batteries fail more frequently, thus reducing $E(Y)$. This suggests that higher failure rates strongly negatively impact the system's reliability, as depicted by the rapid decline in Fig. 8(a). This effect is even more pronounced when the repair is OFF, highlighting the critical need for effective repair mechanisms.

Increasing the repair rate will, in turn, influence the ability to bring failed batteries back into working condition, thus increasing $E(Y)$, as shown in Fig. 8(c). The sharp increase at higher γ values suggests a critical threshold where the repair rate becomes very effective in maintaining the working batteries, implying that a robust repair mechanism is essential for enhancing system reliability, especially when the failure rate is high.

Higher replenishment rates mean more batteries are added to the system, thus increasing $E(Y)$. As β continues to increase, the system reaches a saturation point where further increases in β do not significantly impact $E(Y)$, as indicated in Fig. 8(b). This suggests that while replenishment is crucial, there is a limit to its effectiveness in enhancing system reliability beyond a certain point.

The system's mean number of operational batteries rises with increasing repair and replenishment rates, as these factors contribute to quicker restoration and replacement of failed batteries. Conversely, the mean number decreases as λ increases since more frequent failures lead to more batteries being out of operation at any given time. Altogether, higher repair and replenishment rates enhance the system's reliability, which facilitates quicker restoration and replacement of failed batteries. Conversely, an increased

failure rate diminishes reliability by causing more batteries to be out of operation. The interplay of these factors, as illustrated in Fig. 9, highlights the importance of efficient repair and replenishment mechanisms in maintaining a reliable system.

As the failure rate increases while the replenishment and repair rates remain constant, the number of spare batteries in the system decreases over time. This decline occurs because spare batteries are utilized to replace the failed ones, leading to a corresponding reduction in the MSC available in the system. Thus, increasing λ decreases the availability of spare batteries, thus reducing the system reliability. Similarly, an increase in the repair rate results in a decrease in MSC. This is because a significantly higher repair rate can lead to the cancellation of existing orders once all failed batteries are repaired, leaving no spare batteries available. Thus, increased repair rates, while beneficial for restoring failed batteries, can also lead to lower MSC if not managed properly. Conversely, as the replenishment rate increases, the number of spare batteries in the system rises, thereby increasing the MSC. Thus, higher replenishment rates improve system reliability by maintaining a sufficient stock of spare batteries. For graphical illustration, see Fig. 10.

The system's reliability exhibits a steady increase with variations in the replenishment rate. As β rises, the number of operational batteries within the system increases, thereby reducing the likelihood of system failure. Similarly, variations in the repair rate initially cause an increase in FTU due to enhanced efficiency in managing failures. However, after reaching a certain threshold, the FTU experiences a slight decline. This phenomenon occurs despite maintaining $\beta > \lambda$; an excessively high repair rate can disrupt the

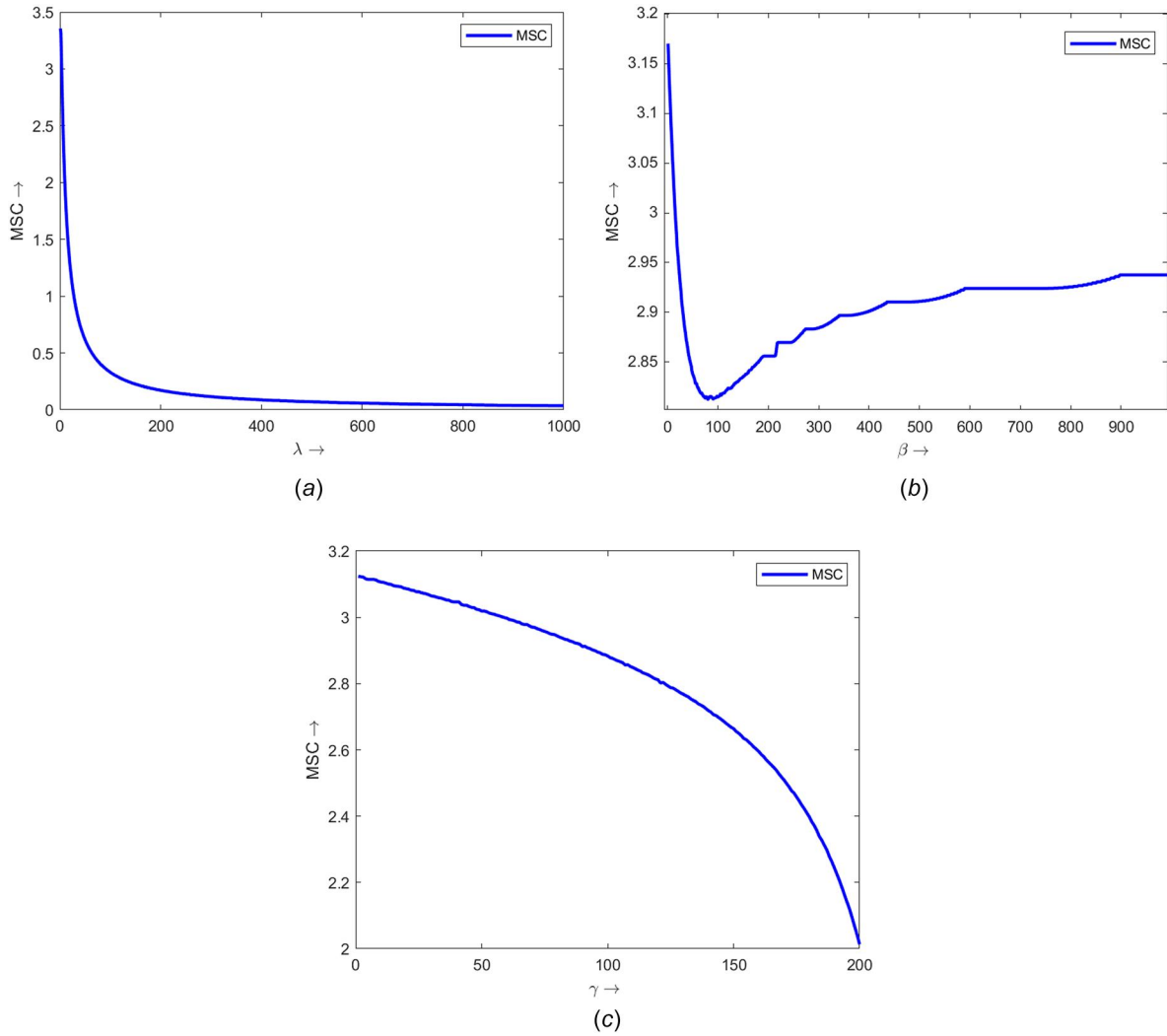


Fig. 10 Effect on MSC when λ , β , and γ varies: (a) when λ varies, (b) when β varies, and (c) when γ varies

equilibrium the replenishment process establishes. In such cases, overly frequent repairs may prevent the system from fully benefiting from the replenishment process, introducing inefficiencies and resulting in a minor decrease in FTU. The decline in FTU with an increasing failure rate is self-evident. The patterns observed in the fraction of time the system is down, FTD, are consistent with those seen in FTU. Figures 11 and 12 illustrate the impacts of varying λ , β , and γ on FTU and FTD, respectively.

Altogether, the uninterrupted power supply and the system's reliability are highly dependent on the balance between λ , β , and γ . Increasing failure rates of batteries cause a break in power supply, which in effect degrades the system's reliability, while high repair and replenishment rates improve it by reducing the time failed batteries remain unaddressed. Thus, the sensitivity analysis emphasizes these dynamics and highlights the need to maintain an optimal balance among the rates to achieve a stable power supply, ensuring overall system reliability.

4.3 Sensitivity Analysis Using Morris Method. In this section, we perform sensitivity analysis using the Morris method to identify the key parameters affecting system performance, complementing the numerical analysis conducted in Sec. 4.2, which utilized a one-parameter variation approach. While Sec. 4.2 examined how individual parameters influence performance metrics independently, the Morris method provides a broader perspective by

quantifying the relative importance, μ^* , and nonlinear effects, σ , of all parameters across specified bounds.

For the Morris sensitivity analysis, we set the bounds for each parameter (λ, β, γ) as $[1, 10]$, with `num_levels`=10 and `N`=10,000. The pseudocode for sensitivity analysis using the Morris method is provided in Table 4, and the results obtained are provided in Table 5.

Results indicate that the failure rate (λ) significantly impacts $E(X)$, with a high μ^* (7.0027) and moderate σ (11.10), suggesting nonlinear effects. Conversely, replenishment and repair rates have no measurable influence, as μ^* and σ are zero. The failure rate is the dominant factor for $E(Y)$, with nonzero sensitivity indices (μ^* and σ), highlighting its critical role in system performance during nonrepair scenarios. Replenishment and repair rates have zero sensitivity, indicating they do not affect $E(Y)$ when repairs are inactive. These findings underscore the need to minimize failure rates to enhance reliability. The repair rate is the most influential parameter for ERT, with a high μ^* (5210.37) and low σ , indicating a dominant and stable effect. The failure rate also contributes significantly to ERT, albeit with high variability ($\sigma = 377.23$), reflecting nonlinear effects. The replenishment rate shows no influence, with zero sensitivity indices.

For OCW, the failure rate strongly impacts outcomes, as evidenced by nonzero μ^* and σ , emphasizing its role in determining system performance. Replenishment and repair rates have no impact, indicating OCW depends primarily on failure dynamics, reinforcing the importance of minimizing failure rates during operation. The failure rate also drives MSC, as its nonzero

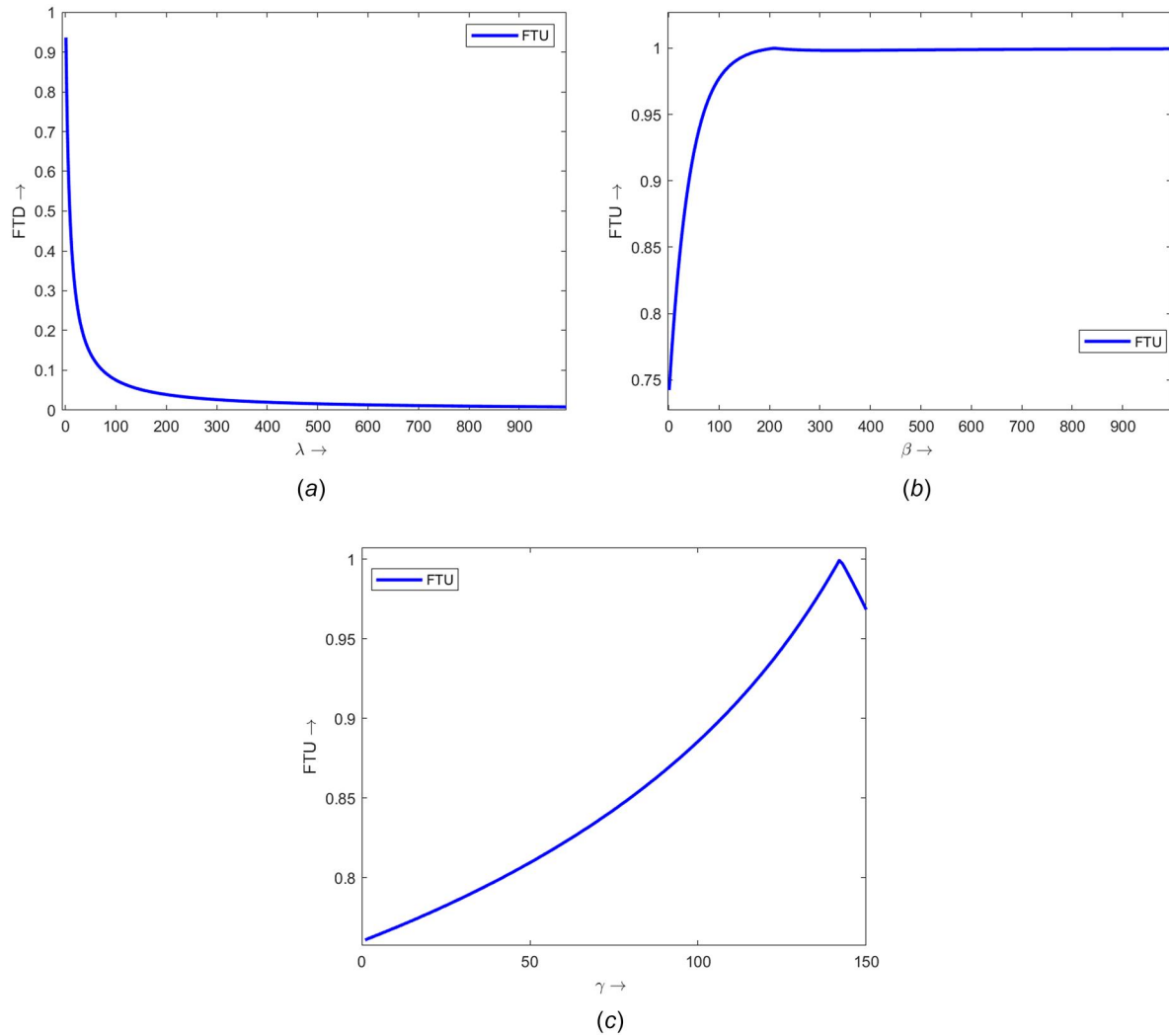


Fig. 11 Effect on FTU when λ , β , and γ varies: (a) when λ varies, (b) when β varies, and (c) when γ varies

sensitivity indices show. In contrast, replenishment and repair rates show no impact, which indicates spare battery availability is governed by failure dynamics, with β having negligible influence. Effective failure rate management is essential for optimizing spare utilization.

Finally, for FTD, the failure rate significantly affects the fraction of time the system is in a failed state, with $\mu^* = 0.075075$. In contrast, replenishment and repair rates exhibit no sensitivity, highlighting failure dynamics' dominant role in determining system downtime. Similarly, for FTU, the failure rate is the crucial parameter influencing reliability, with replenishment and repair rates having no notable impact.

Thus, the Morris sensitivity analysis highlights that λ is the most critical parameter influencing system performance across the metrics, including $E(X)$, $E(Y)$, OCW, MSC, FTD, and FTU, while γ is the critical parameter influencing ERT. Replenishment and repair rates have a negligible impact within the specified bounds, emphasizing the need to prioritize failure rate reduction to enhance system reliability and operational efficiency. These findings underscore the dominant role of failure dynamics in system performance.

5 Conclusion and Future Work

This paper presents an integrated system aimed at ensuring an efficient and uninterrupted power supply to the flight module of tethered HAPS through the strategic management of batteries in the GCS. The batteries are modeled as components of a k -out-of- n : G

COLD system, incorporating both repair and replenishment mechanisms for effective maintenance. We derived explicit expressions for system-state probabilities and computed several critical performance measures, providing a detailed sensitivity analysis. Using the PSO algorithm, we identified the optimal pair of N and L that minimize system costs, and these results were illustrated graphically. Our analysis shows that achieving an efficient power supply and system reliability can be optimized by ensuring the condition: $\lambda < \beta < \gamma$. The analysis of system characteristics further demonstrates that increasing repair and replenishment rates improves system reliability by reducing downtime due to failed batteries. However, further increases in these rates no longer significantly improve reliability beyond a certain threshold. Therefore, it is essential to maintain an optimal balance between repair and replenishment rates to sustain both power supply efficiency and overall system reliability. Further, the Morris sensitivity analysis identifies the λ as the most critical factor for most performance metrics and the repair rate as key for ERT, highlighting the dominant role of failure dynamics and the need to prioritize failure rate reduction to improve system reliability and efficiency.

Our findings indicate that a critical extension to existing work lies in setting an upper bound to cease the repairs and replenishment. This would introduce the hysteresis policy, allowing the system to maintain efficiency by stopping repair and replenishment when no longer necessary. This direction offers a valuable avenue for future research. Applying the proposed model to WARM or HOT systems with a hysteresis policy can be practically applicable to enhance

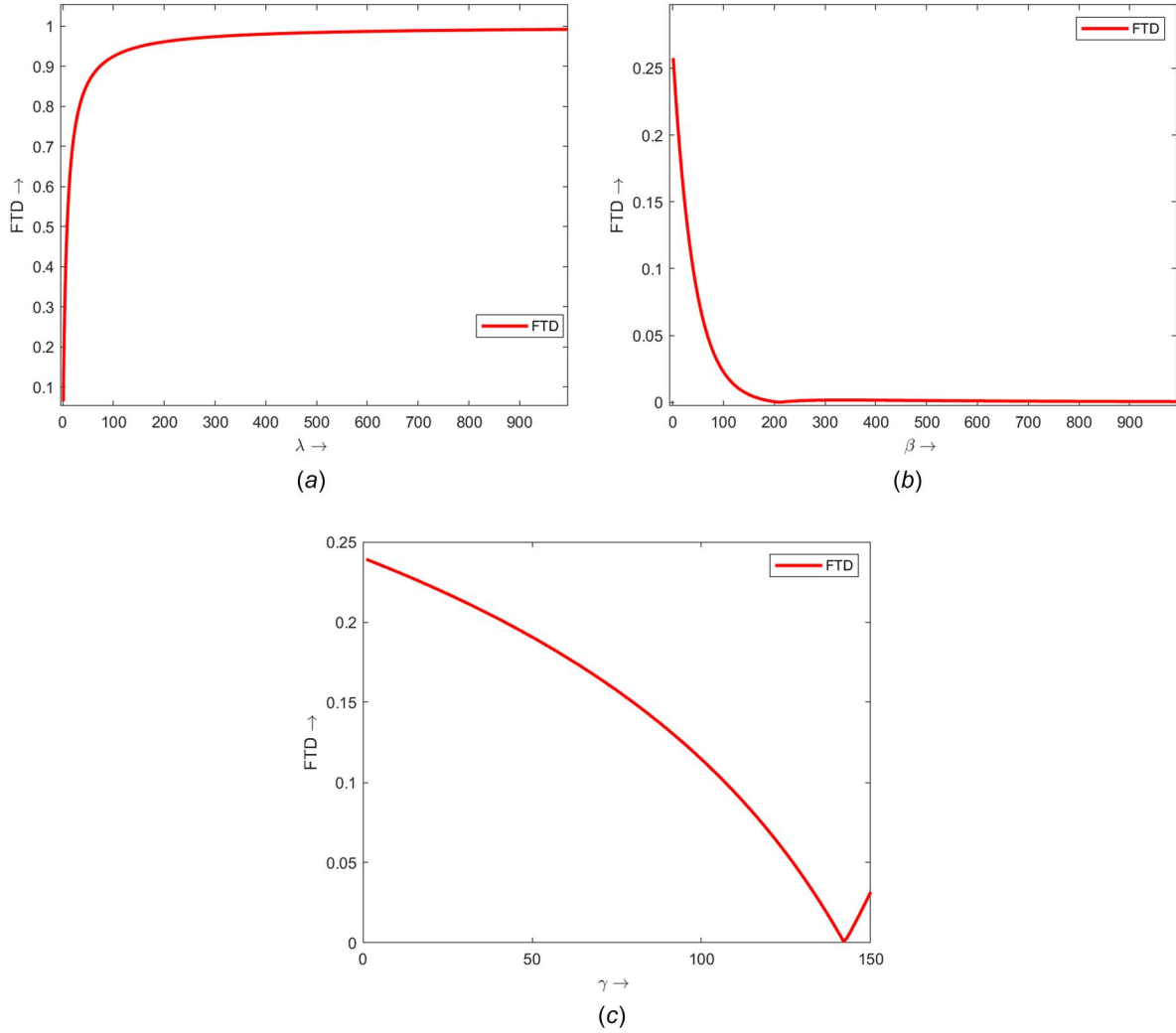


Fig. 12 Effect on FTD when λ , β , and γ varies: (a) when λ varies, (b) when β varies, and (c) when γ varies

efficiency, reliability, and cost-effectiveness, providing a robust framework for future research and practical applications. In this paper, we consider the lifetime of batteries as independent of each

other; hence, the case of dependent failures is another possible extension. In the present work, we consider the k -out-of- n : G system; the case of consecutive k -out-of- n systems can be

Table 4 Pseudocode for sensitivity analysis using Morris method

Pseudocode for sensitivity analysis using Morris method

- **Step 1.** Import libraries:
 - Import tools for sampling, analysis, and numerical operations: NumPy, SALib
- **Step 2.** Define model function (`callexe(La, B, G)`):
 - Initialize constants and state space (`r1` and `c1`).
 - Compute transition matrix (`D1`) based on specific conditions.
 - Create a NumPy array of the transition matrix (`matrixtrmarray`).
 - Solve for steady-state probabilities (`pi`) and map results to states.
 - Calculate performance metrics.
 - Output: A list of metrics [`EX`, `EY`, `OCW`, `MSC`, `ERT`, `FTD`, `FTU`].
- **Step 3.** Define problem:
 - Specify the number of parameters: `num_vars`, names: [`'La'`, `'B'`, `'G'`], and bounds: ranges for each parameter.
- **Step 4.** Perform sampling:
 - Use Morris method to generate `param_values` based on the number of samples (`N`) and levels (`num_levels`).
- **Step 5.** Evaluate model:
 - Loop through `param_values` to compute outputs using `callexe`.
 - Store results as a NumPy array `Y`.
- **Step 6.** Perform sensitivity analysis:
 - Analyze outputs using Morris sensitivity analysis.
 - Results include `mu_star` (importance) and `sigma` (nonlinearity).
- **Step 7.** Display outputs:
 - Show computed metrics, and sensitivity indices.

Table 5 Sensitivity analysis

Performance measure	Parameter	μ^*	σ
$E(X)$	λ	7.002687	11.100149
	β	0.000000	0.000000
	γ	0.000000	0.000000
$E(Y)$	λ	8.556843	13.306531
	β	0.000000	0.000000
	γ	0.000000	0.000000
OCW	λ	1.263977	1.988599
	β	0.000000	0.000000
	γ	0.000000	0.000000
ERT	λ	190.401797	377.227127
	β	0.000000	0.000000
	γ	5210.373844	261.447307
MSC	λ	0.539422	0.838375
	β	0.000000	0.000000
	γ	0.000000	0.000000
FTD	λ	0.075075	0.117707
	β	0.000000	0.000000
	γ	0.000000	0.000000
FTU	λ	0.072265	0.124663
	β	0.000000	0.000000
	γ	0.000000	0.000000

considered for future work. The presented model can also be explored through various policies like T -policy and (N, T) policy. The proposed system can be thoroughly examined for the situation where we consider replenishment order and repair of failed batteries occurring simultaneously.

Acknowledgment

We thank the anonymous referees for their meticulous review and valuable suggestions, which have greatly improved the quality and presentation of our paper. Manikandan Rangaswamy expresses gratitude to the Science and Engineering Research Board (SERB), Government of India, for providing full financial support for the research and to the Department of Science and Technology (DST), Government of India, for partial support. S. Dharmaraja thanks DST, the Government of India, for its full financial support. P. S. Arya acknowledges the Department of Mathematics, Central University of Kerala, for access to facilities and financial assistance.

Funding Data

- SERB, Government of India (Grant No. EEQ/2022/000229; Funder ID: 10.13039/501100001843).
- DST, Government of India (Research Project No. 64800; Funder ID: 10.13039/501100001409).

Data Availability Statement

The authors attest that all data for this study are included in the paper.

References

- [1] D'Oliveira, F. A., Melo, F. C. L. D., and Devezas, T. C., 2016, "High-Altitude Platforms—Present Situation and Technology Trends," *J. Aerosp. Technol. Manage.*, **8**(3), pp. 249–262.
- [2] Jain, V., Vishnevsky, V., Selvamuthu, D., and Raj, R., 2022, "Analysis of Power Management in a Tethered High Altitude Platform Using MAP/PH [3]/1 Retrial Queueing Model," *International Conference on Distributed Computer and Communication Networks*, Moscow, Russia, Sept. 26–29, pp. 218–230.
- [3] Aragon-Zavala, A., Cuevas-Ruiz, J. L., and Delgado-Penín, J. A., 2008, *High-Altitude Platforms for Wireless Communications*, Wiley, Chichester, West Sussex, UK.
- [4] Belmekki, B. E. Y., and Alouini, M.-S., 2022, "Unleashing the Potential of Networked Tethered Flying Platforms: Prospects, Challenges, and Applications," *IEEE Open J. Veh. Technol.*, **3**, pp. 278–320.
- [5] Vishnevsky, V., and Meshcheryakov, R., 2019, "Experience of Developing a Multifunctional Tethered High-Altitude Unmanned Platform of Long-Term Operation," *Proceedings of the Fourth International Conference on Interactive Collaborative Robotics (ICR 2019)*, Istanbul, Turkey, Aug. 20–25, pp. 236–244.
- [6] Arum, S. C., Grace, D., Mitchell, P. D., Zakaria, M. D., and Morozs, N., 2020, "Energy Management of Solar-Powered Aircraft-Based High Altitude Platform for Wireless Communications," *Electronics*, **9**(1), p. 179.
- [7] Alam, M. S., Kurt, G. K., Yanikomeroglu, H., Zhu, P., and Đào, N. D., 2021, "High Altitude Platform Station Based Super Macro Base Station Constellations," *IEEE Commun. Mag.*, **59**(1), pp. 103–109.
- [8] Choi, S., and Wang, G., 2018, "Advanced Lithium-Ion Batteries for Practical Applications: Technology, Development, and Future Perspectives," *Adv. Mater. Technol.*, **3**(9), p. 1700376.
- [9] Perelomov, V., Myrova, L., Aminev, D., and Kozyrev, D., 2018, "Efficiency Enhancement of Tethered High Altitude Communication Platforms Based on Their Hardware-Software Unification," *Proceedings of the 21st International Conference on Distributed Computer and Communication Networks (DCCN 2018)*, Moscow, Russia, Sept. 17–21, pp. 184–200.
- [10] Vishnevsky, V., Selvamuthu, D., Rykov, V., Kozyrev, D., and Ivanova, N., 2022, "Reliability Modeling of a Flight Module of a Tethered High-Altitude Telecommunication Platform," *2022 International Conference on Information, Control, and Communication Technologies (ICCT)*, Astrakhan, Russian Federation, Oct. 3–7, pp. 1–6.
- [11] Li, M., Hu, L., Peng, R., and Bai, Z., 2021, "Reliability Modeling for Repairable Circular Consecutive-k-Out-of-n: F Systems With Retrial Feature," *Reliab. Eng. Syst. Saf.*, **216**, p. 107957.
- [12] Yam, R. C., Zuo, M. J., and Zhang, Y. L., 2003, "A Method for Evaluation of Reliability Indices for Repairable Circular Consecutive-k-Out-of-n: F Systems," *Reliab. Eng. Syst. Saf.*, **79**(1), pp. 1–9.
- [13] Li, X., Zuo, M. J., and Yam, R. C., 2006, "Reliability Analysis of a Repairable k-Out-of-n System With Some Components Being Suspended When the System Is Down," *Reliab. Eng. Syst. Saf.*, **91**(3), pp. 305–310.
- [14] Kozyrev, D., Phuong, N. D., Houankpo, H., and Sokolov, A., 2019, "Reliability Evaluation of a Hexacopter-Based Flight Module of a Tethered Unmanned High-Altitude Platform," *Revised Selected Papers of the 22nd International Conference on Distributed Computer and Communication Networks (DCCN 2019)*, Moscow, Russia, Sept. 23–27, pp. 646–656.
- [15] Nguyen, D. P., and Kozyrev, D. V., 2020, "Reliability Analysis of a Multirotor Flight Module of a High-Altitude Telecommunications Platform Operating in a Random Environment," *2020 International Conference Engineering and Telecommunication (En&T)*, Dolgoprudny, Russia, Nov. 25–26, pp. 1–5.
- [16] Mittal, N., Ivanova, N., Jain, V., and Vishnevsky, V., 2024, "Reliability and Availability Analysis of High-Altitude Platform Stations Through Semi-Markov Modeling," *Reliab. Eng. Syst. Saf.*, **252**, p. 110419.
- [17] Dharmaraja, S., Sivam, A. H., Raj, R., and Vishnevsky, V., 2023, "Study of Reliability of the On-Tether Subsystem of a Tethered High-Altitude Unmanned Telecommunication Platform," *Reliab.: Theory Appl.*, **18**(1 (72)), pp. 172–178.
- [18] Krishnamoorthy, A., 2021, "A Reliability-Inventory Problem Under N-Policy of Replenishment of Component," *Reliab.: Theory Appl.*, **16**(4 (65)), pp. 73–87.
- [19] SoftBank Research Institute of Advanced Technology, 2023, "Exploring HAPS' Long-Duration Flight From an Energy Perspective," SoftBank Research Institute of Advanced Technology, Japan, accessed May 27, 2024, <https://www.softbank.jp/en/corp/technology/research/story-event/028/>
- [20] Gad, A. G., 2022, "Particle Swarm Optimization Algorithm and Its Applications: A Systematic Review," *Arch. Comput. Methods Eng.*, **29**(5), pp. 2531–2561.