



Stochastic modeling for bandwidth part switching based DRX mechanism in 5G NR networks

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Abstract

The intensification of mobile broadband services and user experience is explicitly dependent on the increased battery life of user equipment (UE) and minimized delay in service. In 5G New Radio (NR), in addition to the Discontinuous Reception (DRX) scheme, Bandwidth Part (BWP) switching plays a significant role in reducing UE power consumption. The dynamic bandwidth operation in BWP switching is power efficient as UE can adapt its operating bandwidth based on the traffic arrival. In this work, BWP switching-based DRX mechanism is modeled as an $M^X/G/1$ queue to trade-off between quality of service and power saving in UE in 5G NR. Analytical and numerical results on the proposed model show it to be promising in minimising power consumption and reducing delay in the service of UE.

Keywords 5G New Radio · $M^X/G/1$ model · Discontinuous reception · Bandwidth part · Power consumption

1 Introduction

Over the past decade, an exponential increase in the usage of smart gadgets and devices has been noticed in wireless communication networks. In the current scenario, there are approximately 8.1 billion mobile subscriptions which are expected to reach up to 8.9 billion by the end of 2027 [1]. To address the growing demand for mobile subscriptions and data rates while ensuring high quality of service (QoS), the fifth generation (5G) is introduced by the 3rd Generation Partnership Project (3GPP) telecommunications standards group. It features different new radio access technologies, i.e., 5G New Radio (NR), Low Power Wide Area (LPWA), Narrowband Internet of Things (NB-IoT) and Long Term

Evolution for Machines (LTE-M) with the three defining characteristics as enhanced mobile broadband (eMBB), massive capacity and ultra-low latency.

This evolution comes with a wider spectrum allowance having frequency band (up to 100 GHz) with approximately 20 times (up to 400 MHz) the bandwidth of fourth generation (4G) Long Term Evolution (LTE), as well as a large number of receiver antennas [2]. The high bandwidth in 5G networks, while facilitating the high data rates, results in increased power consumption of user equipment (UE). The fact behind such high power consumption is that UE has to actively monitor a wideband control channel across the entire bandwidth, even if no data or resources are present. Therefore, it is required to investigate power-saving mechanism-based models.

The layout of this work is organised as follows. State of the art related to the proposed model is elaborated in Sect. 2. The $M^X/G/1$ queue model based on BWP switching and the Discontinuous Reception (DRX) mechanism is discussed in Sect. 3. Section 4 details the mathematical analysis of the proposed model under the bivariate Markov process using the supplementary variable technique. Various performance measures, including expected delay and power consumption, are formulated in Sect. 5. Numerical illustrations to point out the impact of various intensities over these performance measures are presented in Sect. 6. Furthermore, Sect. 7 analyses this work theoretically with the existing works and proposes

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the advantages of developing this model. Finally, the underlying model is concluded with insights for future works in Sect. 8.

2 Related work

The DRX mechanism is one of the most widely adopted mechanisms in LTE-Advanced (LTE-A) [3–5] and is supported by 3GPP to implement in 5G networks as well [6, 7]. It allows UE to reduce power consumption by switching to sleep mode over short time intervals when no data is scheduled. The DRX mechanism works on two types of Radio Resource Control (RRC) modes, i.e., RRC connected and RRC idle, which are classified on the basis of the establishment of the RRC connection and release of the RRC connection, respectively. Thus, the DRX helps in saving UE power during both modes.

Besides the DRX mechanism efficient spectrum usage is considered to be a very important tool to save UE power. In this regard, UE utilises part of full bandwidth, referred to as Bandwidth Part (BWP), to transmit data or monitor data arrival. There are two types of BWP, named as narrowband and wideband, considered in this work. After serving all the available packet(s) in wideband mode, UE switches to narrowband mode rather than being available in wideband mode. This switching is termed Bandwidth Part (BWP) switching. To develop a power-efficient mathematical model, BWP switching-based DRX mechanism is proposed. In this model, the Physical Downlink Control Channel (PDCCH) for the data packets is monitored in narrowband rather than in wideband. Therefore, the power consumption of UE can be minimised to a greater extent.

Massive efforts have been undertaken over the last decade to improve UE power efficiency in wireless communication [6–9]. In this context, Li et al. [10] provided an overview on various power-saving techniques, including the DRX mechanism, bandwidth adaptation, cross-slot scheduling, etc. Kim et al. [11] analysed the key features and benefits of power-saving techniques such as the DRX with three sleep states and bandwidth adaptation through simulation results under various traffic scenarios. In 5G networks, the DRX mechanism, the legacy power-saving technique, can be facilitated in both RRC connected mode and RRC idle mode [3]. Wu et al. [8] presented the DRX with a real power-saving (RPS) factor by taking into consideration state transition to improve power efficiency.

The impact of BWP switching on UE performance in the 5G NR was studied in [12]. For more information on BWP system characteristics, it is suggested to refer to Abinader et al. [12]. BWP adaptation whitepaper [13] suggested that the power-saving gains introduced by BWP switching can be significant and applicable to different types of traffic profiles

as per the various data rate requirements. Rostami et al. [14] presented a semi-Markov model for the Wake-up scheme (WuS) to enhance power efficiency and delay trade-offs in the 5G NR networks. In the WuS, UE monitors a narrow-band channel for wake-up signalling to get the message for the arrival of a packet(s). Recently, wake-up scheduling was proposed to improve the power saving of WuS-enabled UE under predefined delay [15].

Numerous mathematical analyses have been proposed to model the DRX mechanism by several researchers. Most of these studies considered that the data arrival follows the Poisson process [7, 14–16]. To match with the realistic scenario, where data packets arrive in bulk, the compound Poisson process is the best suited to model the arrival process [9]. In this regard, Ke et al. [17] proposed an $M^X/G/1$ queueing system under a multiple vacation queue model. However, in general, the service time of the packet(s) and sleep duration of UE not only depends on the present but also depends on the elapsed service time and elapsed sleep time. Such a real-time scenario is not Markovian in continuous time, Chaudhry and Templeton [18] discussed the supplementary variable technique to model it. Ke [19] derived system size distribution for an $M^X/G/1$ system with a variant vacation policy by using supplementary variables for the elapsed time. Further, Ke et al. [20] studied the supplementary variable technique in an $M^X/G/1$ modeled system with N -policy and maximum J vacations. On a similar track, Gautam et al. [9] mathematically analysed the DRX mechanism through $M^X/G/1$ model using N -policy in LTE-A networks.

In this work, the DRX mechanism integrated with BWP switching is modeled as an $M^X/G/1$ queueing model. In the DRX mechanism, sleep states are categorised as a micro sleep state, light sleep state and deep sleep state, with the increasing duration of sleep time. Due to BWP switching, UE moves to narrowband mode after serving all the packet(s) in wideband mode and stays in this mode until either the inactivity timer expires or packet arrival happens [12]. Since the downlink traffic dominates during the uneven data traffic [21], this work focuses only on UE receiver mode.

3 Model description

In this work, RRC connected mode is categorised into a wideband mode, narrowband mode, micro sleep period, light sleep period, deep sleep period and on duration timer. All these terminologies are discussed in brief as follows:

- Wideband mode (BWP(1)): This is the wider part of the bandwidth supported by 5G networks. In this mode, UE is configured to serve the packet(s).

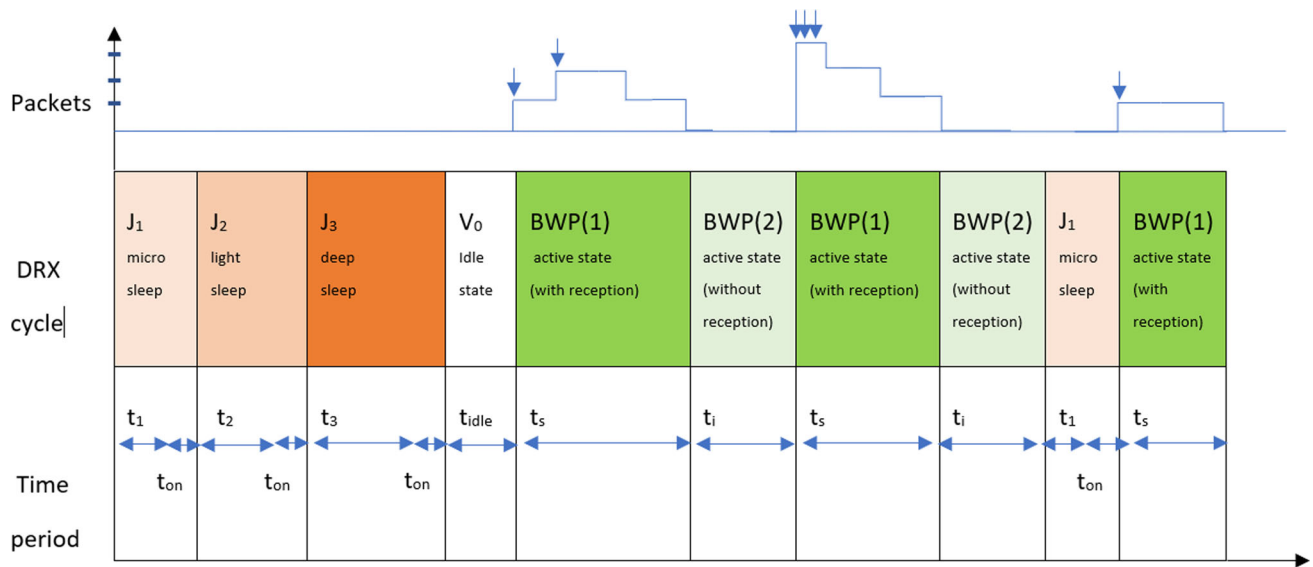


Fig. 1 Proposed DRX mechanism and BWP in 5G NR networks

- **Narrowband mode (BWP(2)):** Once the packet(s) are served in wideband mode, UE switches to a relatively narrow bandwidth part, referred to as a narrowband mode. In this mode, UE waits for the packet(s) until the packet(s) arrive or the timer, named the inactivity timer, expires, whichever happens first.

Note that wideband mode and narrowband mode collectively correspond to the active state of UE.

- **Sleep Period:** During this period, UE is configured in such a way that most of its components are turned off in order to save power. This is classified as micro sleep period (t_1), light sleep period (t_2) and deep sleep period (t_3). Note that $t_1 \leq t_2 \leq t_3$.
- **On duration timer (t_{on}):** After each sleep period, this timer starts. During this period, PDCCH is monitored by UE for packet(s) arrival.
- **Sleep state (Micro, Light and Deep):** Each sleep period with the t_{on} period constitutes a respective sleep state, described as follows:
 - **Micro sleep state:** UE switches to this state after the expiry of the inactivity timer in narrowband mode.
 - **Light sleep state:** UE switches to this state when there is no arrival of a packet(s) during the micro sleep state.
 - **Deep sleep state:** UE switches to the deep sleep state when there is no arrival of a packet(s) during the light sleep state.

The proposed model representing the transitions in the RRC connected mode and RRC idle mode with the dependence on the arrival of a packet(s) is depicted in Fig. 1.

In this work, an $M^X/G/1$ model is considered to study the DRX mechanism with BWP switching in RRC connected mode and RRC idle mode in 5G NR. The arrival follows the compound Poisson process, and the service follows the general distribution. It is noteworthy that only the downlink packet(s) transmission at UE is addressed here, and the packets are served one by one by a single server, i.e., UE. The buffer capacity of Next Generation NodeB (gNB) is assumed as infinite. Further, this work examines UE under the bivariate Markov process using the elapsed time as a supplementary variable. For the proposed system, i.e., the proposed DRX mechanism with BWP switching for UE, the state transition diagram (STD) is depicted in Fig. 2. Let the states (n, m) be described as:

- $n \in N$, where $N = \{0, 1, \dots\}$, represents the number of packet(s) in the system at time t .
- $m \in M$, where $M = \{1, 2, 3, 4, 5, 6\}$, represents the states of UE at time t , defined as:
 - **Idle state ($m = 1$):** This is RRC idle state. During this state, UE gets a paging message for the arrival of a packet(s). The arrival of batch size k to $(0, 1)$ state switches it to $(k, 2)$ state to provide the service to the packets.
 - **Wideband active state ($m = 2$):** This is RRC connected state in which packets will be served. Packet(s) arrival of size k in $(n, 2)$ state switches UE to $(n+k, 2)$ state. After providing service to all the available packet(s), UE transits to $(0, 3)$ state.
 - **Narrowband active state ($m = 3$):** Batch arrival of size k during the inactivity timer of this state shift

2. If UE is in $(n, 5)$ state where $n \neq 0$, it will move to $(n, 2)$ state, i.e., wideband active state, to serve the packets at the expiry of t_{on} .
- Deep sleep state ($m = 6$): During t_3 , if there is a batch arrival (size k) of packet(s), UE switches from state $(0, 6)$ to $(k, 6)$ state, else UE remains in the $(0, 6)$ state until t_3 expires. Depending on UE state at the end of t_3 , in the respective t_{on} period of deep sleep state, there are two scenarios:
 1. If UE is at $(0, 6)$ state, it will switch to $(0, 1)$ state at the expiry of t_{on} .
 2. If UE is in $(n, 6)$ state where $n \neq 0$, it will move to $(n, 2)$ state, i.e., wideband active state, to serve the packets at the expiry of t_{on} .

Further, all the required notations are described in Table 1.

4 Mathematical analysis

The underlying bivariate Markov process $\{(L_s(t), U^e(t)) : t \geq 0\}$ is defined by the following state space:

$$U(t) := \begin{cases} (0, 1), & \text{if UE is in idle state,} \\ (n, 2); n \in N \setminus \{0\}, & \text{if UE is in wideband active state,} \\ (n, 3); n \in N, & \text{if UE is in narrowband active state,} \\ (n, 4); n \in N, & \text{if UE is in micro sleep state,} \\ (n, 5); n \in N, & \text{if UE is in light sleep state,} \\ (n, 6); n \in N, & \text{if UE is in deep sleep state.} \end{cases},$$

Note that:

- $L_s(t)$ represents the number of packets in the system at time t .
- $U^e(t)$ represents the elapsed time in a particular state of state space $U(t)$ at time t and is defined as follows:

$$U^e(t) := \begin{cases} 0, & \text{if } U(t) = (0, 1), \\ S^e(t), & \text{if } U(t) = (n, 2), \\ I^e(t), & \text{if } U(t) = (n, 3), \\ D_i^e(t), & \text{if } U(t) = (n, 4) \text{ or } (n, 5) \text{ or } (n, 6), \end{cases}$$

where

- $S^e(t)$ is the elapsed service time of the packet under service at time t ,
- $D_i^e(t)$ is the elapsed sleep time of i^{th} sleep state at time t ; ($i = 1, 2, 3$), where $i = 1, 2, 3$ stands for micro sleep state, light sleep state, and deep sleep state, respectively,

- $I^e(t)$ is the elapsed inactivity timer at time t .

Assume that:

- $V_0(t)$ is the probability of UE being in the idle state at time t .
- $S_n(x, t)$ is the probability of UE in a wideband active state with an elapsed service time of the n^{th} packet between x and $x + dx$ at time t .
- $Z_n(x, t)$ is the probability of UE in a narrowband active state having n packets at time t , with elapsed time in a narrowband active state lying between x and $x + dx$.
- $D_{i,n}(x, t)$ is the probability of n packets in the buffer in i^{th} sleep state at time t , where the elapsed sleep time of the i^{th} sleep state lies between x and $x + dx$.

Then

$$\begin{aligned} V_0(t) &:= P\{L_s(t) = 0, U^e(t) = 0\}, \\ S_n(x, t)dx &:= P\{L_s(t) = n, U^e(t) = S^e(t); \\ &\quad x < S^e(t) \leq x + dx\}, \\ &\quad x > 0, \quad n = 1, 2, \dots, \\ Z_n(x, t)dx &:= P\{L_s(t) = n, U^e(t) = I^e(t); \\ &\quad x < I^e(t) \leq x + dx\}, \\ &\quad x > 0, \quad n = 0, 1, 2, \dots, \\ D_{i,n}(x, t)dx &:= P\{L_s(t) = n, U^e(t) = D_i^e(t); \\ &\quad x < D_i^e(t) \leq x + dx\}, \\ &\quad i = 1, 2, 3, \quad x > 0, \quad n = 0, 1, 2, \dots \end{aligned}$$

For the steady-state probabilities of the states of UE,

$$V_0 = \lim_{t \rightarrow \infty} V_0(t),$$

and limiting densities for $x > 0$,

$$S_n(x) = \lim_{t \rightarrow \infty} S_n(x, t),$$

$$Z_n(x) = \lim_{t \rightarrow \infty} Z_n(x, t),$$

$$D_{i,n}(x) = \lim_{t \rightarrow \infty} D_{i,n}(x, t).$$

4.1 Steady state equations

For the proposed model, the Kolmogorov forward equations of the system under steady-state conditions can be written using the arguments given by Cox [22] and Cox and Miller

Table 1 Notations

λ	Composite arrival rate of all the batches.
X	Random variable (RV) denoting the number of packets in a batch.
$G(z)$	PGF of X .
$P(X = k) = \epsilon_k$	PMF of X .
S	RV denoting service time of packet(s).
$F_S(x)$	CDF of S .
$S(z)$	PGF of S .
$\bar{S}(\theta)$	Laplace-Stieltjes transform (LST) of S and defined as $E(\exp^{-\theta S})$.
I	RV denoting inactivity timer.
$F_I(x)$	CDF of I .
$Z(z)$	PGF of I .
$\bar{I}(\theta)$	LST of I .
J_i	RV denoting i th sleep time, where $i = 1, 2, 3$ stands for micro sleep state, light sleep state, and deep sleep state, respectively.
F_{J_i}	CDF of J_i .
$\bar{J}_i(\theta)$	LST of J_i .
ρ	Traffic intensity of the system.
$v(x)$	Hazard rate function of RV S .
$h_i(x)$	Hazard rate function of RV J_i .
$g(x)$	Hazard rate function of RV I .

[23] as follows:

$$\lambda V_0 = \int_0^\infty D_{3,0}(x) h_3(x) dx, \quad (1)$$

$$\begin{aligned} & \frac{d}{dx} S_n(x) + (\lambda + v(x)) S_n(x) \\ &= \lambda \sum_{k=1}^{n-1} \epsilon_k S_{n-k}(x), \quad n = 1, 2, \dots, \end{aligned} \quad (2)$$

$$\frac{d}{dx} D_{i,0}(x) + (\lambda + h_i(x)) D_{i,0}(x) = 0, \quad i = 1, 2, 3, \quad (3)$$

$$\begin{aligned} & \frac{d}{dx} D_{i,n}(x) + (\lambda + h_i(x)) D_{i,n}(x) \\ &= \lambda \sum_{k=1}^n \epsilon_k D_{i,n-k}(x), \quad n = 1, 2, \dots, \end{aligned} \quad (4)$$

$$\frac{d}{dx} Z_0(x) + (\lambda + g(x)) Z_0(x) = 0, \quad (5)$$

$$\begin{aligned} & \frac{d}{dx} Z_n(x) + (\lambda + g(x)) Z_n(x) \\ &= \lambda \sum_{k=1}^n \epsilon_k Z_{n-k}(x), \quad n = 1, 2, \dots \end{aligned} \quad (6)$$

The solution of these equations is obtained under the boundary conditions at $x = 0$ and normalizing condition. The boundary conditions of the system are as follows:

$$S_n(0) = \int_0^\infty D_{1,n}(x) h_1(x) dx + \int_0^\infty D_{2,n}(x) h_2(x) dx +$$

$$\begin{aligned} & \int_0^\infty D_{3,n}(x) h_3(x) dx + \int_0^\infty S_{n+1}(x) v(x) dx + \lambda \epsilon_n V_0 \\ &+ \int_0^\infty Z_n(x) g(x) dx, \quad n = 1, 2, \dots, \end{aligned} \quad (7)$$

$$D_{1,n}(0) = \begin{cases} \int_0^\infty Z_0(x) g(x) dx, & n = 0, \\ 0, & n = 1, 2, \dots, \end{cases} \quad (8)$$

$$D_{i,n}(0) = \begin{cases} \int_0^\infty D_{i-1,0}(x) h_{i-1}(x) dx, & n = 0, \\ 0, & n = 1, 2, \dots, \end{cases} \quad \begin{matrix} i = 2, 3, \\ i = 2, 3, \end{matrix} \quad (9)$$

$$Z_n(0) = \begin{cases} \int_0^\infty S_1(x) v(x) dx, & n = 0, \\ 0, & n = 1, 2, \dots \end{cases} \quad (10)$$

The normalizing condition is as follows:

$$\begin{aligned} & V_0 + \sum_{n=1}^\infty \int_0^\infty S_n(x) dx + \sum_{i=1}^3 \sum_{n=0}^\infty \int_0^\infty D_{i,n}(x) dx \\ &+ \sum_{n=0}^\infty \int_0^\infty Z_n(x) dx = 1. \end{aligned} \quad (11)$$

Using Eqs. (1)–(11), probabilities of various states and probability generating functions (PGFs) of the proposed system are reported in the next sections.

4.2 Steady state probability that UE is in sleep state and no packet arrives

Let the steady state probability of no packet arrival while UE is in one of the sleep states be denoted by D_0 and is calculated as:

$$D_0 = \sum_{i=1}^3 D_{i,0},$$

where $D_{i,0}$ is the corresponding probability of the i^{th} sleep state. In order to obtain D_0 , the following procedure is considered.

- $D_{i,0}(x)$, obtained by integrating Eq. (3) and is given as:

$$D_{i,0}(x) = D_{i,0}(0)(1 - F_{J_i}(x))e^{-\lambda x}, \quad i = 1, 2, 3. \quad (12)$$

- For each $i = 1, 2, 3$; multiply the Eq. (12) with the corresponding $h_i(x)$ and integrate it. Further, by using Eqs. (1) and (9), we obtain:

$$D_{3,0}(0) = \frac{\lambda V_0}{\xi_3}, \quad D_{i,0}(0) = \frac{D_{i+1,0}(0)}{\xi_i}, \quad i = 1, 2, \quad (13)$$

where $\xi_i = \bar{J}_i(\lambda)$, for $i = 1, 2, 3$.

Further, the Eq. (13) provides:

$$D_{i,0}(0) = \frac{\lambda V_0}{3} \prod_{l=i}^3 \xi_l, \quad i = 1, 2, 3. \quad (14)$$

- On integrating the Eq. (12) and using Eq. (14) in the obtained integral solution, the $D_{i,0}$ is expressed as follows:

$$D_{i,0} = \frac{V_0}{3} (1 - \xi_i) \prod_{l=i}^3 \xi_l, \quad i = 1, 2, 3. \quad (15)$$

- By using Eq. (15), we obtain D_0 as follows:

$$\begin{aligned} D_0 &= \sum_{i=1}^3 D_{i,0} \\ &= \frac{V_0(1 - \xi_1)}{\xi_1 \xi_2 \xi_3} + \frac{V_0(1 - \xi_2)}{\xi_2 \xi_3} + \frac{V_0(1 - \xi_3)}{\xi_3}. \end{aligned} \quad (16)$$

4.3 Probability generating function of the system

To obtain the system characteristics, the PGF of the proposed system is calculated as follows:

- To calculate the PGF of UE in sleep state, i.e., $D(z)$:

– Define

$$\begin{aligned} D_i(x, z) &:= \sum_{n=0}^{\infty} D_{i,n}(x) z^n, \\ D_i(0, z) &:= \sum_{n=0}^{\infty} D_{i,n}(0) z^n. \end{aligned} \quad (17)$$

– Using Eqs. (9), (14) and (17), we get:

$$D_i(0, z) = \frac{\lambda V_0}{3} \prod_{l=i}^3 \xi_l, \quad i = 1, 2, 3. \quad (18)$$

– On multiplying Eqs. (3) and (4) by z^n , and adding over n from 0 to ∞ , we have $D_i(x, z)$ as:

$$D_i(x, z) = D_i(0, z)(1 - F_{J_i}(x))e^{-\lambda x(1-G(z))}, \quad i = 1, 2, 3. \quad (19)$$

– Using Eqs. (18) and (19), $D_i(x, z)$ is obtained as:

$$\begin{aligned} D_i(x, z) &= \frac{\lambda V_0}{3} (1 - F_{J_i}(x))e^{-\lambda x(1-G(z))} \prod_{l=i}^3 \xi_l, \\ i &= 1, 2, 3. \end{aligned} \quad (20)$$

– Therefore

$$\begin{aligned} D_i(z) &:= \int_0^{\infty} D_i(x, z) dx = \frac{V_0(1 - \xi_i(z))}{3 \prod_{l=i}^3 \xi_l(1 - G(z))}, \\ i &= 1, 2, 3, \end{aligned}$$

where $\xi_i(z) = \bar{J}_i(\lambda(1 - G(z)))$.

- The PGF that UE is in one of the sleep states is given by:

$$\begin{aligned} D(z) &:= \sum_{i=1}^3 D_i(z) = \frac{V_0}{G(z) - 1} \\ &\times \left(\frac{\xi_1(z) - 1}{\xi_1 \xi_2 \xi_3} + \frac{\xi_2(z) - 1}{\xi_2 \xi_3} + \frac{\xi_3(z) - 1}{\xi_3} \right). \end{aligned} \quad (21)$$

- The PGF for a narrowband active state, i.e., $Z(z)$, is determined through the below mentioned steps:

– By integrating Eq. (5), we get

$$Z_0(x) = Z_0(0)(1 - F_I(x))e^{-\lambda x}. \quad (22)$$

– Multiply Eq. (22) with $g(x)$ and integrate it. Afterwards, using Eqs. (8) and (14), we determine:

$$Z_0(0) = \frac{\lambda V_0}{\xi_1 \xi_2 \xi_3 \chi_0}, \quad (23)$$

where $\chi_0 = \bar{I}(\lambda)$.

– Define

$$\begin{aligned} Z(x, z) &:= \sum_{n=0}^{\infty} Z_n(x) z^n, \\ Z(0, z) &:= \sum_{n=0}^{\infty} Z_n(0) z^n. \end{aligned} \quad (24)$$

– It follows from Eqs. (8), (10), (14) and (24) that:

$$Z(0, z) = Z_0(0) = \frac{\lambda V_0}{\xi_1 \xi_2 \xi_3 \chi_0}. \quad (25)$$

– Multiplying Eqs. (5) and (6) by z^n , and adding over n from 1 to ∞ , $Z(x, z)$ is as follows:

$$Z(x, z) = Z(0, z)(1 - F_I(x))e^{-\lambda x(1-G(z))}. \quad (26)$$

– From Eqs. (25) and (26), $Z(x, z)$ is obtained as:

$$Z(x, z) = \frac{\lambda V_0}{\xi_1 \xi_2 \xi_3 \chi_0} (1 - F_I(x))e^{-\lambda x(1-G(z))}. \quad (27)$$

– Integrating Eq. (27), we get:

$$Z(z) = \frac{V_0}{\xi_1 \xi_2 \xi_3 \chi_0} \left(\frac{\chi(z) - 1}{G(z) - 1} \right), \quad (28)$$

where $\chi(z) = \bar{I}(\lambda(1 - G(z)))$.

- Further, to calculate the PGF for a wideband active state, i.e., $S(z)$, the following approach is adopted:

– Define

$$\begin{aligned} S(x, z) &:= \sum_{n=1}^{\infty} S_n(x) z^n, \\ S(0, z) &:= \sum_{n=1}^{\infty} S_n(0) z^n. \end{aligned} \quad (29)$$

– Multiplying Eq. (2) by z^n and adding over n , from 1 to ∞ , the $S(x, z)$ is as follows:

$$S(x, z) = S(0, z)(1 - F_S(x))e^{-\lambda x(1-G(z))}. \quad (30)$$

– On multiplying Eq. (7) by z^n and adding over n , from 1 to ∞ , the resultant $S(0, z)$ is:

$$\begin{aligned} S(0, z) &= \frac{\lambda V_0 z}{z - \eta(z)} \left(\frac{\xi_1(z) - 1}{\xi_1 \xi_2 \xi_3} \right. \\ &\quad + \frac{\xi_2(z) - 1}{\xi_2 \xi_3} + \frac{\xi_3(z) - 1}{\xi_3} \\ &\quad \left. + \frac{\chi(z) - 1}{\xi_1 \xi_2 \xi_3 \chi_0} + G(z) - 1 \right), \end{aligned} \quad (31)$$

where $\eta(z) = \bar{S}(\lambda(1 - G(z)))$.

– Using Eqs. (30) and (31), and integrating it, the $S(z)$ is obtained as follows:

$$\begin{aligned} S(z) &= \frac{V_0 z(\eta(z) - 1)}{(z - \eta(z))(G(z) - 1)} \left(\frac{\xi_1(z) - 1}{\xi_1 \xi_2 \xi_3} \right. \\ &\quad + \frac{\xi_2(z) - 1}{\xi_2 \xi_3} + \frac{\xi_3(z) - 1}{\xi_3} \\ &\quad \left. + \frac{\chi(z) - 1}{\xi_1 \xi_2 \xi_3 \chi_0} + G(z) - 1 \right). \end{aligned} \quad (32)$$

- Rewrite the Eq. (11) as $V_0 + S(1) + \sum_{i=1}^3 D_i(1) + Z(1) = 1$, and using Eqs. (21), (28) and (32), V_0 can be obtained as:

$$V_0 = \frac{1 - \rho}{1 + \frac{\lambda E(J_1)}{\xi_1 \xi_2 \xi_3} + \frac{\lambda E(J_2)}{\xi_2 \xi_3} + \frac{\lambda E(J_3)}{\xi_3} + \frac{\lambda E(I)}{\xi_1 \xi_2 \xi_3 \chi_0}}, \quad (33)$$

where $\rho = \lambda E(X)E(S)$ ([24]).

- PGF of the proposed system is:

$$\bar{\Phi}(z) := V_0 + S(z) + D(z) + Z(z),$$

which can be calculated using Eqs. (21), (28), (32), and (33).

Therefore, PGF is obtained as:

$$\begin{aligned} \bar{\Phi}(z) &= \frac{V_0(z - \eta(z) + z(\eta(z) - 1))}{(z - \eta(z))(G(z) - 1)} \\ &\quad \left(\frac{\xi_1(z) - 1}{\xi_1 \xi_2 \xi_3} + \frac{\xi_2(z) - 1}{\xi_2 \xi_3} + \frac{\xi_3(z) - 1}{\xi_3} \right. \\ &\quad \left. + \frac{\chi(z) - 1}{\xi_1 \xi_2 \xi_3 \chi_0} + G(z) - 1 \right). \end{aligned}$$

It is important to note that the integration mentioned during this section is performed with respect to x from 0 to ∞ .

5 Performance measures

The performance measures for the proposed system are calculated using the system characteristics. Following sub-sections describe the performance of the system.

5.1 Expected number of packets in the system

The expected number of packets in the system during the steady state is defined as:

$$E[L_s] := \lim_{z \rightarrow 1} \frac{d}{dz} \bar{\Phi}(z) = \frac{3V_0L}{4(1-\rho)^2(E[X])^2},$$

where

$$\begin{aligned} L = & \left((1-\rho)\lambda^2(E[X])^3 \left[\frac{E[J_1^2]}{\xi_1\xi_2\xi_3} \right. \right. \\ & + \frac{E[J_2^2]}{\xi_2\xi_3} + \frac{E[J_3^2]}{\xi_3} + \left. \frac{E[I^2]}{\xi_1\xi_2\xi_3\chi_0} \right] \\ & + 2(1-\rho)\lambda^2(E[X])^3 E[S] \\ & \left[\frac{E[J_1]}{\xi_1\xi_2\xi_3} + \frac{E[J_2]}{\xi_2\xi_3} + \frac{E[J_3]}{\xi_3} + \frac{E[I]}{\xi_1\xi_2\xi_3\chi_0} \right] \\ & + 2(1-\rho)\lambda(E[X])^3 E[S] \\ & + \left(\left[\lambda E[X(X-1)]E[S]E[X] + (\lambda)^2(E[X])^3 E[S^2] \right] \right. \\ & \times E[X] \times \left[\frac{\lambda E[J_1]}{\xi_1\xi_2\xi_3} + \frac{\lambda E[J_2]}{\xi_2\xi_3} \right. \\ & \left. \left. + \frac{\lambda E[J_3]}{\xi_3} + \frac{\lambda E[I]}{\xi_1\xi_2\xi_3\chi_0} + 1 \right] \right) \Bigg). \end{aligned}$$

5.2 Expected packet delay

Let Ω be the delay in serving the marked packet. By employing Little's formula, the expected delay of the marked packet $E[\Omega]$ is calculated as follows:

$$E[\Omega] := \frac{E[L_s]}{\lambda E[X]}.$$

5.3 Expected sleep state length

Let D denote the total number of packets during micro sleep state, light sleep state or deep sleep state of the system in the steady state. Assume D_1 , D_2 , and D_3 are the number of packets during micro sleep state, light sleep state and deep sleep state, respectively. Therefore, the expected value of D , $E[D]$ is given as:

$$E[D] := \lim_{z \rightarrow 1} \frac{d}{dz} D(z) = E[D_1] + E[D_2] + E[D_3],$$

$$E[D_1] := \frac{V_0\lambda^2 E[X]E[J_1^2]}{2\xi_1\xi_2\xi_3}, \quad E[D_2] := \frac{V_0\lambda^2 E[X]E[J_2^2]}{2\xi_2\xi_3},$$

$$E[D_3] := \frac{V_0\lambda^2 E[X]E[J_3^2]}{2\xi_3}.$$

Let $E[T_D]$ be the expected time spent in the sleep state by UE, which is expressed as the sum of the expected time spent in micro sleep state, light sleep state and deep sleep state. It is given as:

$$E[T_D] := E[T_{D_1}] + E[T_{D_2}] + E[T_{D_3}],$$

where

$$E[T_{D_1}] = \frac{E[D_1]}{\lambda E[X]}, \quad E[T_{D_2}] = \frac{E[D_2]}{\lambda E[X]}, \quad E[T_{D_3}] = \frac{E[D_3]}{\lambda E[X]}.$$

Therefore

$$\begin{aligned} E[T_{D_1}] &= \frac{\lambda V_0 E[J_1^2]}{2\xi_1\xi_2\xi_3}, \quad E[T_{D_2}] = \frac{\lambda V_0 E[J_2^2]}{2\xi_2\xi_3}, \\ E[T_{D_3}] &= \frac{\lambda V_0 E[J_3^2]}{2\xi_3}, \\ E[T_D] &:= \frac{\lambda V_0}{2} \left(\frac{E[J_1^2]}{\xi_1\xi_2\xi_3} + \frac{E[J_2^2]}{\xi_2\xi_3} + \frac{E[J_3^2]}{\xi_3} \right). \end{aligned} \quad (34)$$

5.4 Expected narrowband active state length

Let Z denote the number of packets during a narrowband active state of the system. The expected value of Z is given as:

$$E[Z] := \lim_{z \rightarrow 1} \frac{d}{dz} Z(z) = \frac{\lambda^2 V_0}{2} \left(\frac{E[X]E[I^2]}{\xi_1\xi_2\xi_3\chi_0} \right).$$

Let T_Z denote the time spent in a narrowband active state by UE. Then expected value of T_Z is given as:

$$E[T_Z] := \frac{E[Z]}{\lambda E[X]} = \frac{\lambda V_0}{2} \left(\frac{E[I^2]}{\xi_1\xi_2\xi_3\chi_0} \right).$$

5.5 Expected wideband active state length

Let T_S denote the time spent in the wideband active state by UE. Then, we have:

$$\begin{aligned} E[T_S] &= \left(\frac{\rho}{1-\rho} \right) E[T_z] \\ &= \left(\frac{\rho}{1-\rho} \right) \frac{V_0\lambda E[I^2]}{2\xi_1\xi_2\xi_3\chi_0}. \end{aligned}$$

5.6 Expected busy cycle length

Let T_C , the time spent in the busy cycle, be referred to as the time duration between two consecutive wideband active states. Hence, this is the sum of the duration spent in all the states starting from a wideband active state until UE enters back. The expected time spent in the busy cycle is given by:

$$E[T_C] = E[T_D] + E[T_Z] + E[T_S] + E[T_{V_0}],$$

where $E[T_{V_0}]$, expected time spent in idle state, is given as $V_0 E[T_C]$. Therefore

$$E[T_C] = \frac{E[T_D] + E[T_Z] + E[T_S]}{1 - V_0}.$$

5.7 Expected number of sleep state

Let $E[N_D]$ be the expected number of sleep states in a busy cycle. This corresponds to the expected number of switches from sleep period to on duration timer, hence the expected number of transitions from power off to power on. Here, the power off refers to the sleep periods, and the power on is the on duration timer and active state. Thus $E[N_D]$ can be calculated as:

$$E[N_D] = \frac{3E[T_D]}{E[J_1] + E[J_2] + E[J_3]},$$

where $E[T_D]$, $E[J_1]$, $E[J_2]$ and $E[J_3]$ are the expected time spent in the sleep state by the UE given in Eq. (34), the expectation of random variable denoting sleep time J_1 , J_2 and J_3 , respectively.

The expected time spent in switching from power off to power on state in a busy cycle is given by

$$E[T_{SW}] = \tau \cdot E[N_D],$$

where τ represents the mean time to switch from one power off to one power on state.

Define $\pi_s :=$ steady-state probability of UE belonging to s at time t ,

$$\pi_s = \frac{E[T_s]}{E[T_C]}, \quad s \in \{Z, S, D_1, D_2, D_3, SW\}.$$

5.8 Throughput of the system

Let T be the system's throughput and is defined as the probability of being in the wideband active state multiplied by the service rate of the packet. Hence, the throughput is given as follows:

$$T = \frac{\mu(E[T_S])}{E[T_C]}.$$

Table 2 List of various parameters and their respective rates

Time	Distribution	Parameter (ms^{-1})
Service (S)	Exponential	$\mu = 2$
Micro sleep (J_1)	Exponential	$\phi_1 = 1/4$
Light sleep (J_2)	Exponential	$\phi_2 = 1/5$
Deep sleep (J_3)	Exponential	$\phi_3 = 1/6$
Inactivity (I)	Exponential	$\sigma = 1/2$

5.9 Power saving factor

The power saving factor (PS) is the ratio of time spent in the sleep state by UE to the total time spent by UE in all the states.

$$PS = \frac{\pi_{D_1} E[T_{D_1}] + \pi_{D_2} E[T_{D_2}] + \pi_{D_3} E[T_{D_3}]}{\pi_S E[T_S] + \pi_Z E[T_Z] + \pi_{D_1} E[T_{D_1}] + \pi_{D_2} E[T_{D_2}] + \pi_{D_3} E[T_{D_3}]}$$

5.10 Utility of the system

Utility (U_s) is the ratio of time spent in a wideband active state to the sum of the time spent in a wideband and narrowband active state of the system and is given by:

$$U_s := \frac{E[T_S]}{E[T_S] + E[T_Z]}.$$

5.11 Power consumption of the system

Power consumption of the system is defined as the power consumed in all the states during a busy cycle. Consider P_{idle} , P_{ser} , P_{listen} , P_{micro} , P_{light} , P_{deep} and P_{sw} are the power consumption per unit time (in mW) of a UE during the idle state, wideband active state, narrowband active state, micro sleep state, light sleep state, deep sleep state, and switching from power off to power on state, respectively. Hence, the power consumption of the system is defined as

$$P_{DRX} := V_0 P_{idle} + \pi_S P_{ser} + \pi_Z P_{listen} + \pi_{D_1} P_{micro} + \pi_{D_2} P_{light} + \pi_{D_3} P_{deep} + \pi_{SW} P_{sw}.$$

6 Numerical illustration

The theoretical results obtained here are justified numerically with the help of a few experiments. Let us consider the arrival of a packet(s) in a batch following the Poisson distribution with the rate λ .

Experiments 1–4 are aimed to examine the influence of the arrival rate (λ), service rate (μ), inactivity rate (σ), micro sleep rate (ϕ_1), light sleep rate (ϕ_2) and deep sleep rate (ϕ_3) on the performance of the system in 5G NR during RRC

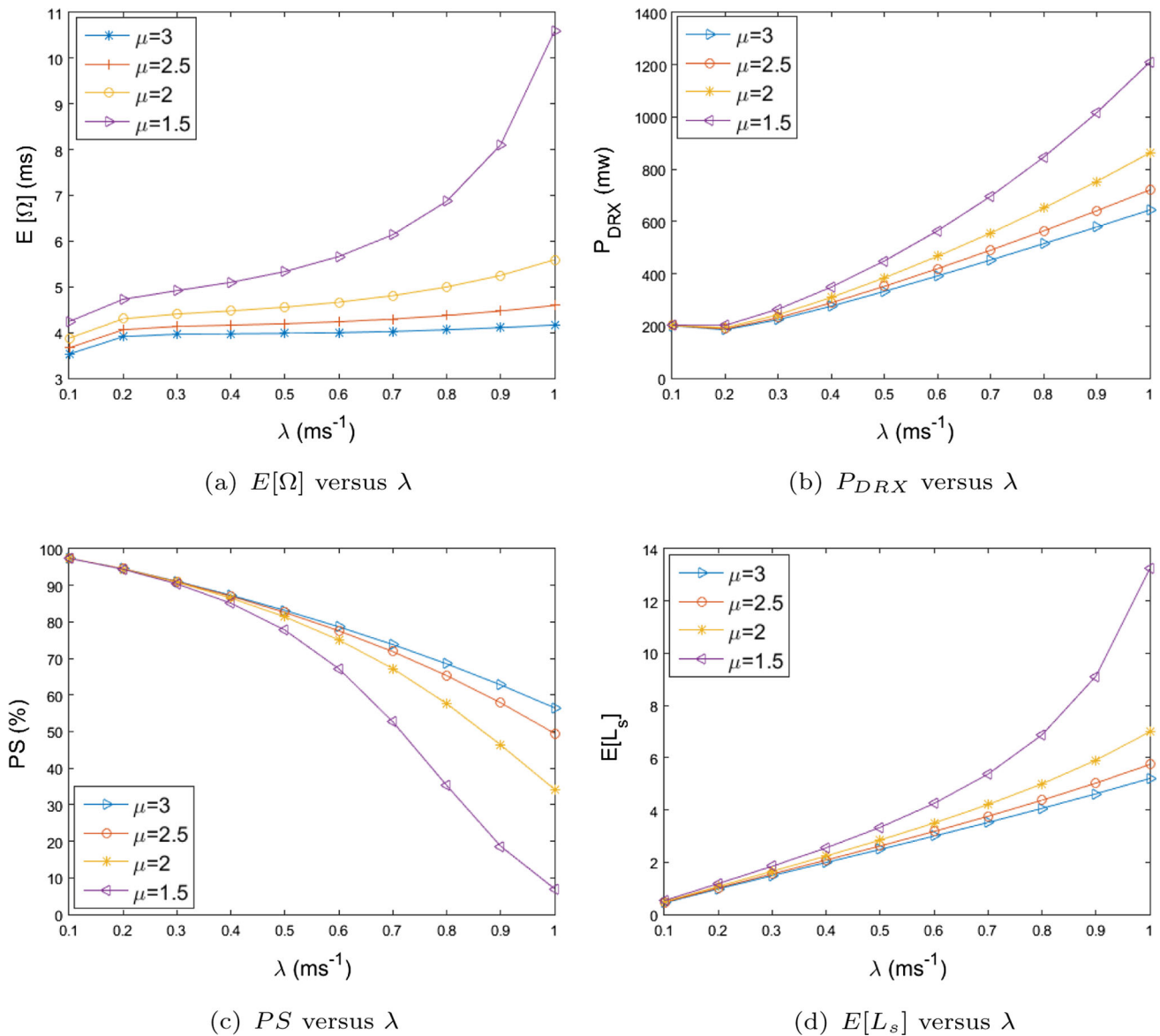


Fig. 3 Expected delay $E[\Omega]$, power consumption of the system P_{DRX} , power saving factor PS and expected number of packets in the system $E[L_s]$ versus arrival rate λ

connected and idle mode. Consider the batch size X follows a geometric distribution with parameter $p = 0.8$.

As discussed in [7], assume power consumption in the states of UE as $P_{idle} = 594.3$ mW, $P_{ser} = 1680.2$ mW, $P_{lis} = 1060$ mW, $P_{micro} = 11.4$ mW, $P_{light} = 11.4$ mW, $P_{deep} = 11.4$ mW and $P_{sw} = 39$ mW. On the similar lines of [9], the assumptions for the distribution of various random variables and notations for the corresponding rate parameters are defined in Table 2.

Experiment 1 The objective of this experiment is to analyse the effect of arrival rate (λ) on the expected delay of a packet ($E[\Omega]$), power consumption of the system (P_{DRX}), power saving factor (PS) and expected number of packets

in the system ($E[L_s]$) for the various service rates ($\mu = 3, 2.5, 2, 1.5$).

Figure 3 exhibits $E[\Omega]$, P_{DRX} , PS and $E[L_s]$ as a function of λ and μ . From Fig. 3a, it can be seen that as λ increases, the value of $E[\Omega]$ increases sharply for a fixed μ . Moreover, for any specific value of λ , an increment in μ leads to a decrease in $E[\Omega]$. This behaviour can be explained as follows. When a large number of packets arrive during sleep states, it takes longer to serve the packets after sleep ends. If the rate of the service of a packet is increased, the packet(s) in the queue need to wait for a lesser time to get the service. Therefore, the value of $E[\Omega]$ increases with the increment in λ . Figure 3b represents that P_{DRX} increases with the increasing λ value. The cause of this lies in the fact that UE goes to

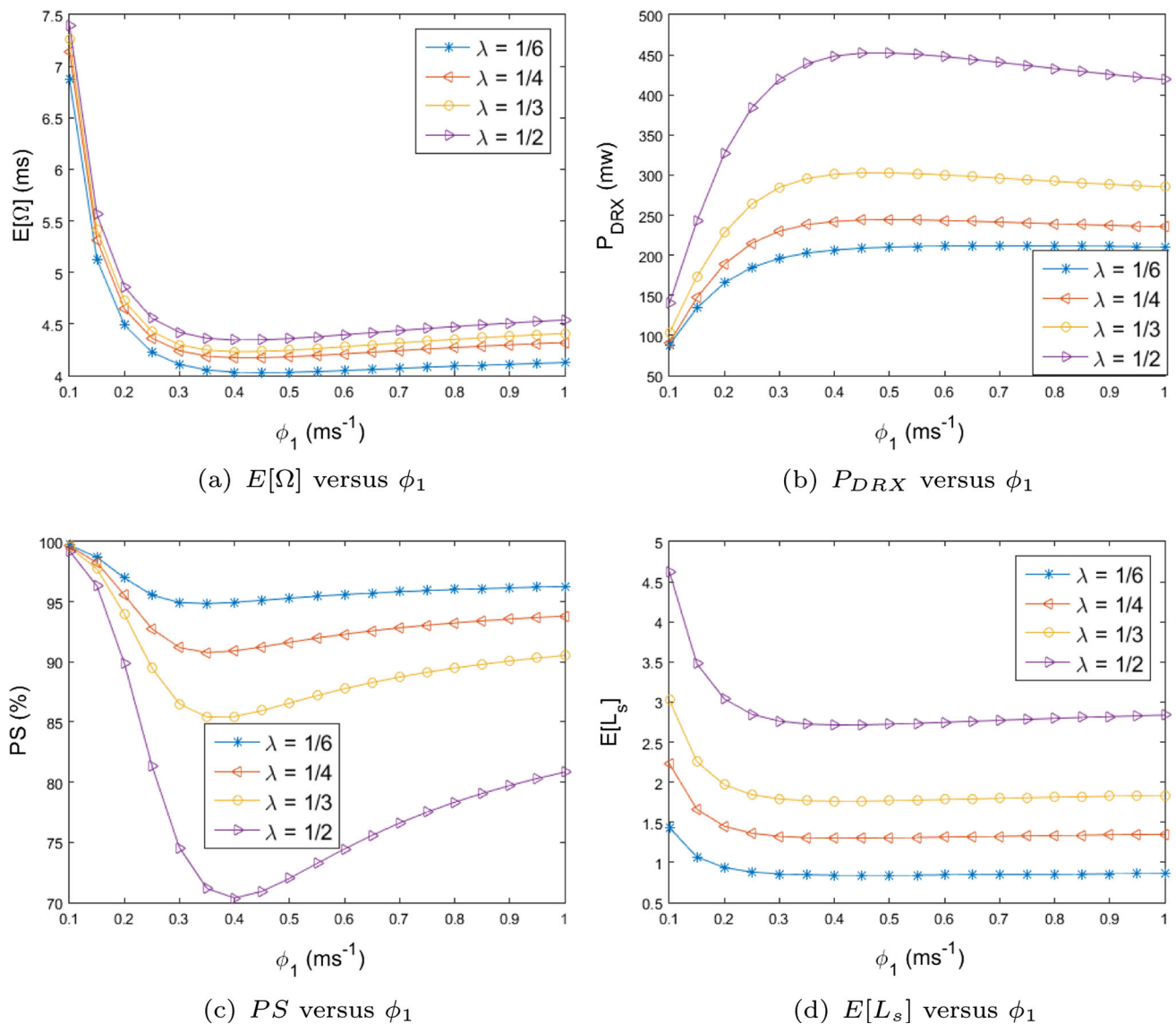


Fig. 4 Expected delay $E[\Omega]$, power consumption of the system P_{DRX} , power saving factor PS and expected number of packets in the system $E[L_s]$ versus micro sleep rate ϕ_1

sleep less often when the rate of arrival increases. Whereas P_{DRX} decreases as μ increases, which is an intuitive observation. PS decreases with the increase in value of λ for a fixed μ and it increases with the increase in value of μ along the same view point as illustrated in Fig. 3c. Additionally, with the increment in λ , $E[L_s]$ increases as depicted in Fig. 3d. Furthermore, it is obvious that with the increase in μ , $E[L_s]$ will decrease.

Experiment 2 The motivation behind this experiment is to analyse the impact of micro sleep rate (ϕ_1) on the expected delay of a packet ($E[\Omega]$), power consumption of the system (P_{DRX}), power saving factor (PS) and expected number of packets in the system ($E[L_s]$) for the various arrival rates ($\lambda = 1/6, 1/4, 1/3, 1/2$).

Figure 4 shows the variation in $E[\Omega]$, P_{DRX} , PS and $E[L_s]$ with ϕ_1 and the various arrival rates ($\lambda = 1/6, 1/4, 1/3, 1/2$). It is considered that ϕ_1 varies from 0 to 1 per millisecond. It is observed from Fig. 4a that the $E[\Omega]$ decreases abruptly with an increase in ϕ_1 value, as the decrease in micro sleep duration decreases the waiting time, and hence the $E[\Omega]$ decreases. Moreover, the $E[\Omega]$ increases as λ increases is obvious and verified here. The increase in P_{DRX} with the increase in ϕ_1 is depicted in Fig. 4b. It is based on the fact that micro sleep duration decreases with the increase in ϕ_1 giving rise to an increase in P_{DRX} value.

While the PS decreases for a certain value of ϕ_1 due to a decrease in micro sleep duration. Afterwards, it becomes constant for lower λ values. Further, as depicted in Fig. 4c,

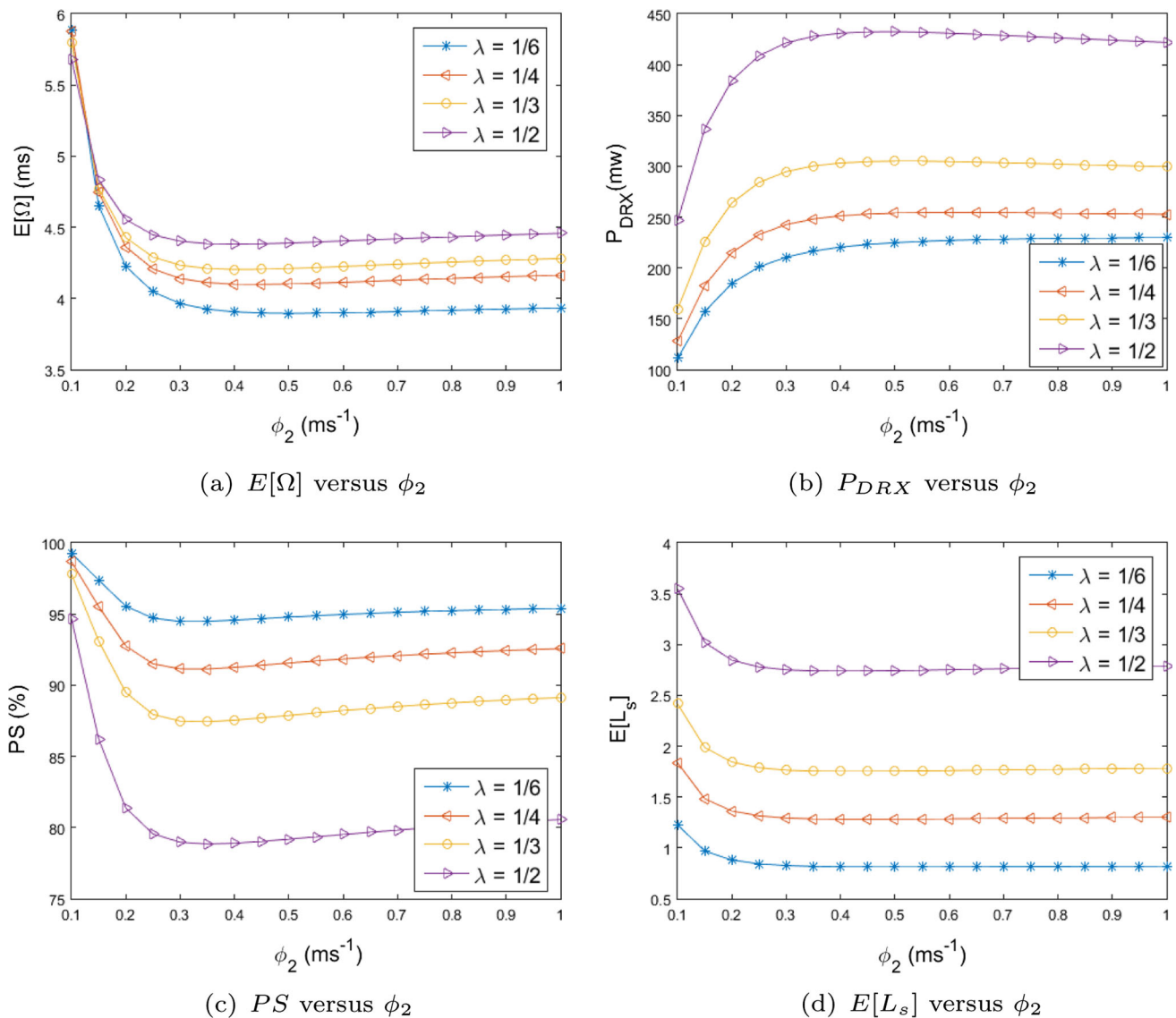


Fig. 5 Expected delay $E[\Omega]$, power consumption of the system P_{DRX} , power saving factor PS and expected number of packets in the system $E[L_s]$ versus light sleep rate ϕ_2

with the increase in λ value and after a certain value of ϕ_1 , the curve increases abruptly. This is due to the impact of light sleep, deep sleep, and idle states on increasing power saving. Figure 4d depicts that the $E[L_s]$ decreases as the ϕ_1 increases due to the decrease in micro sleep duration.

Experiment 3: The main purpose here is to observe the impact of light sleep rate (ϕ_2) on the expected delay of a packet ($E[\Omega]$), power consumption of the system (P_{DRX}), power saving factor (PS) and expected number of packets in the system ($E[L_s]$) for the various arrival rates ($\lambda = 1/6, 1/4, 1/3, 1/2$).

Figure 5a–d depict the behaviour of $E[\Omega]$, P_{DRX} , PS and $E[L_s]$, respectively for the varying ϕ_2 and the various arrival rates ($\lambda = 1/6, 1/4, 1/3, 1/2$). The curves show similar behaviour as it depicts for ϕ_1 . While the various val-

ues of performance measures have significant meaning as ϕ_2 increases and are illustrated in Fig. 5.

Moreover, the performance measures show an abrupt increase or decrease for initial values of ϕ_2 , and become constant after a fixed ϕ_2 , showing that the further decrease in light sleep duration has very less impact on the performance measures.

Experiment 4 The motivation here is to observe the impact of deep sleep rate (ϕ_3) on the expected delay of a packet ($E[\Omega]$), power consumption of the system (P_{DRX}), power saving factor (PS) and expected number of packets in the system ($E[L_s]$) for the various arrival rates ($\lambda = 1/6, 1/4, 1/3, 1/2$).

Figure 6a–d infer the functioning of $E[\Omega]$, P_{DRX} , PS and $E[L_s]$ respectively, for the varying ϕ_3 and the different

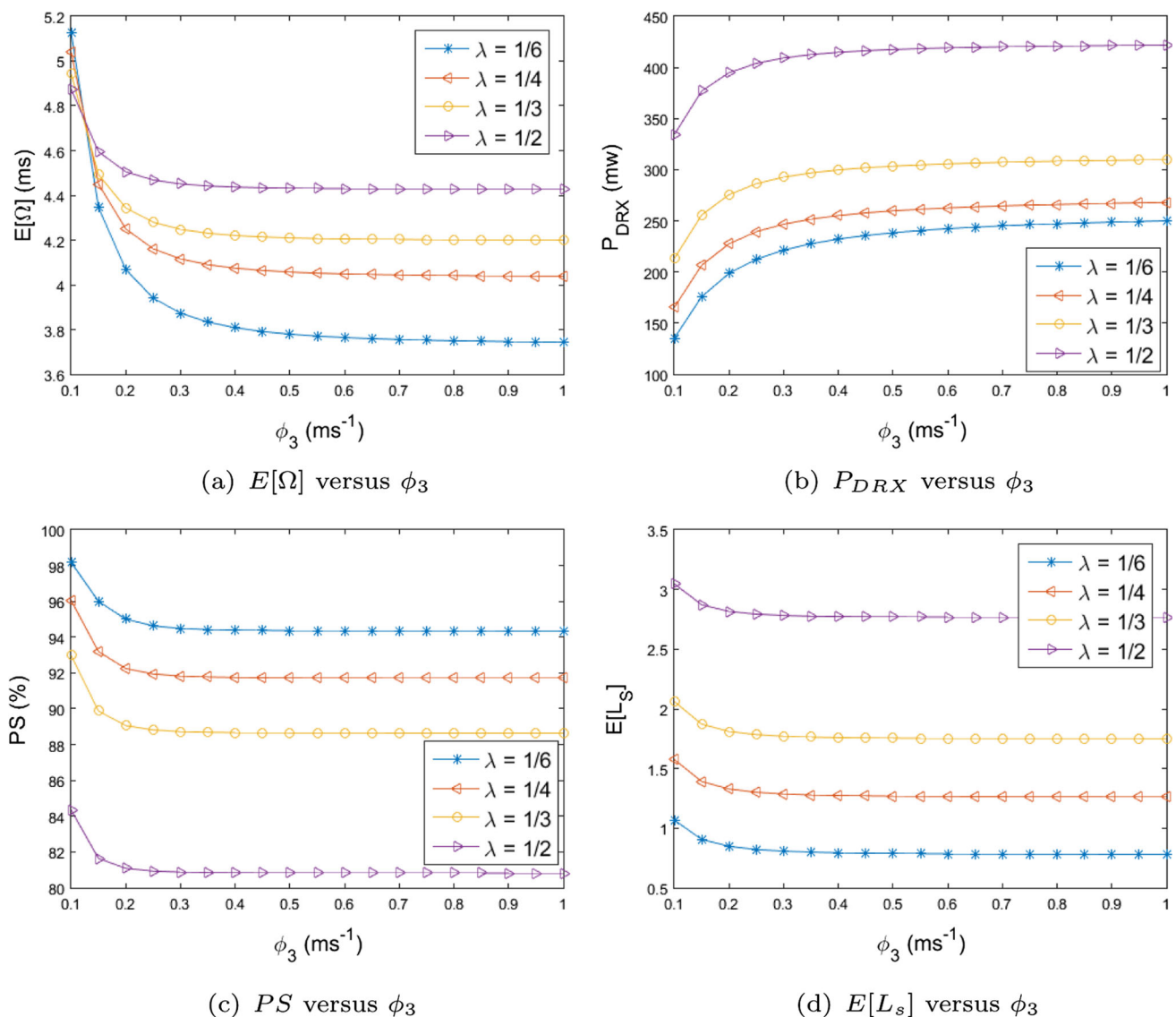


Fig. 6 Expected delay $E[\Omega]$, power consumption of the system P_{DRX} , power saving factor PS and expected number of packets in the system $E[L_s]$ versus deep sleep rate ϕ_3

Table 3 List of various parameters and their respective rates

Time	Distribution	Parameter (ms)
Service (S)	Deterministic	$t_s = 15$
Micro sleep (J_1)	Deterministic	$t_{\phi_1} = 5$
Light sleep (J_2)	Deterministic	$t_{\phi_2} = 10$
Deep sleep (J_3)	Deterministic	$t_{\phi_3} = 15$
Inactivity (I)	Deterministic	$t_\sigma = 2$

arrival rates ($\lambda = 1/6, 1/4, 1/3, 1/2$). As depicted in Fig. 6, the curves vary for very low values of ϕ_3 and become nearly constant afterwards. This behaviour can be illustrated as follows. The increase in the ϕ_3 value results in a decrease in the deep sleep duration, and decreasing the deep sleep duration

after a certain value makes it equal to the light sleep duration. Thus, the performance measures show almost no effect with the further increase in the ϕ_3 value.

As per the 3GPP standard, the sleep state time, inactivity timer and service time may follow the deterministic distribution. This scenario is discussed in Experiment 5 as described below. It is considered that the batch size X follows a geometric distribution with parameter $p = 0.5$. All other necessary assumptions are mentioned in Table 3.

Experiment 5 The purpose here is to observe the impact of arrival rate (λ) on the expected delay of a packet ($E[\Omega]$) for the various service time ($t_s = 15, 20, 25, 30$). Moreover, the motivation is to illustrate the effect of micro sleep time (t_{ϕ_1}), light sleep time (t_{ϕ_2}) and deep sleep time (t_{ϕ_3}) on the

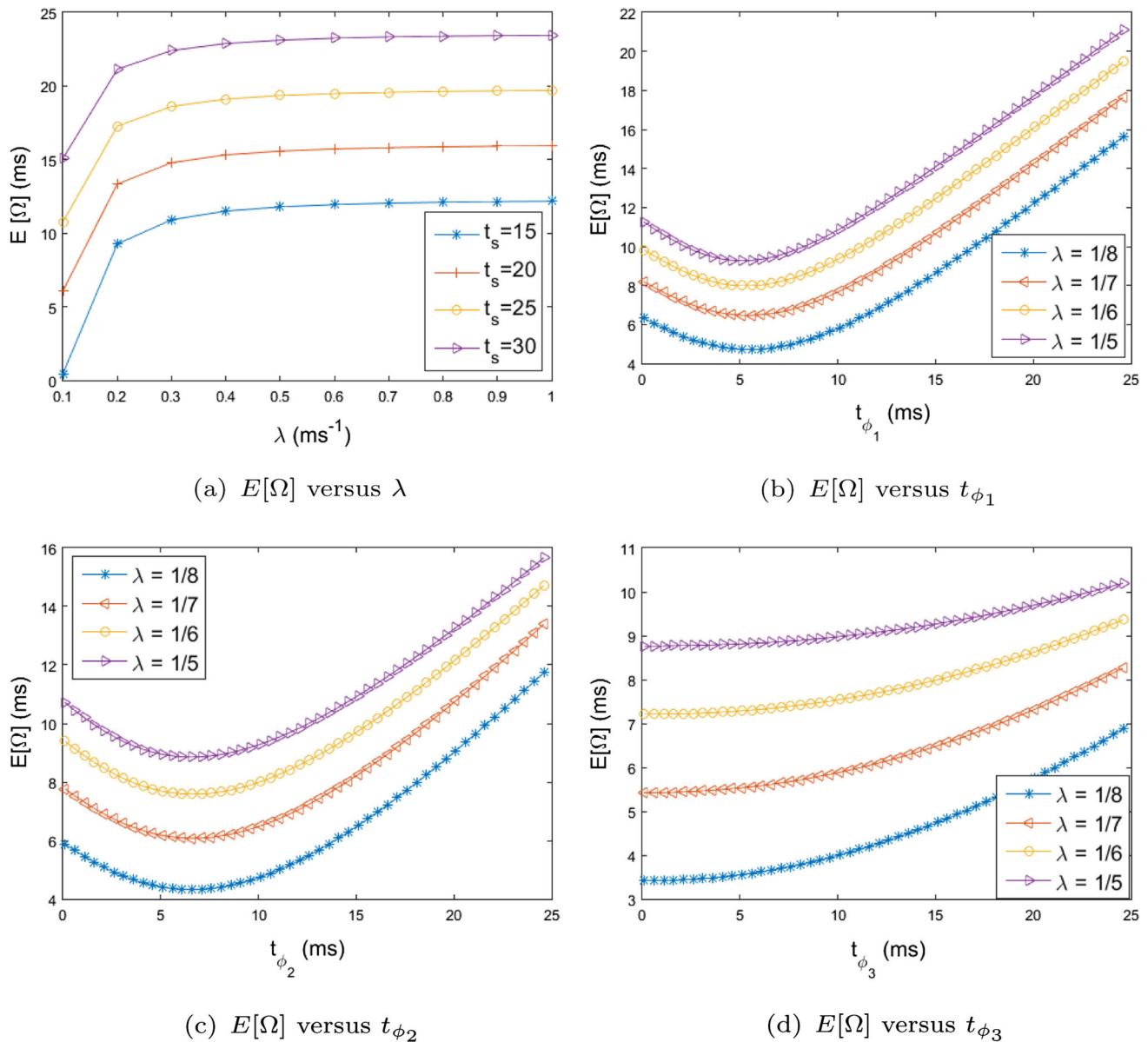


Fig. 7 Expected delay $E[\Omega]$ versus arrival rate (λ), micro sleep duration (t_{ϕ_1}), light sleep duration (t_{ϕ_2}) and deep sleep duration (t_{ϕ_3})

expected delay of a packet ($E[\Omega]$) for the various arrival rates ($\lambda = 1/8, 1/7, 1/6, 1/5$).

Figure 7a illustrates the behaviour of $E[\Omega]$ for the varying λ for the various service time ($t_s = 15, 20, 25, 30$). The represented curve shows that the $E[\Omega]$ increases with the λ up to a certain value, and eventually becomes constant. This functioning can be explained as follows. For a large amount of packet arrival during sleep states, UE takes longer to serve the packets after the sleep ends. In addition, the increase in t_s leads to an increase in $E[\Omega]$, which is intuitive. Furthermore, the rise in $t_{\phi_1}, t_{\phi_2}, t_{\phi_3}$ prompts the increase in $E[\Omega]$ with a varying extent as depicted in Fig. 7b–d. This performance can be justified based on the fact that with the rise in sleep

time, $E[\Omega]$ increases due to the enhancement in waiting time for the packets that arrive during sleep duration. Moreover, the effect enhances more promptly with the increase in λ .

7 Comparison with the existing works

In this section, the performance of the proposed system is compared with the existing & relevant works in the literature. Table 4 exhibits this comparison in brief.

It can be observed that there has been commendable work completed in the direction of saving UE power using the DRX mechanism in various wireless network generations. One

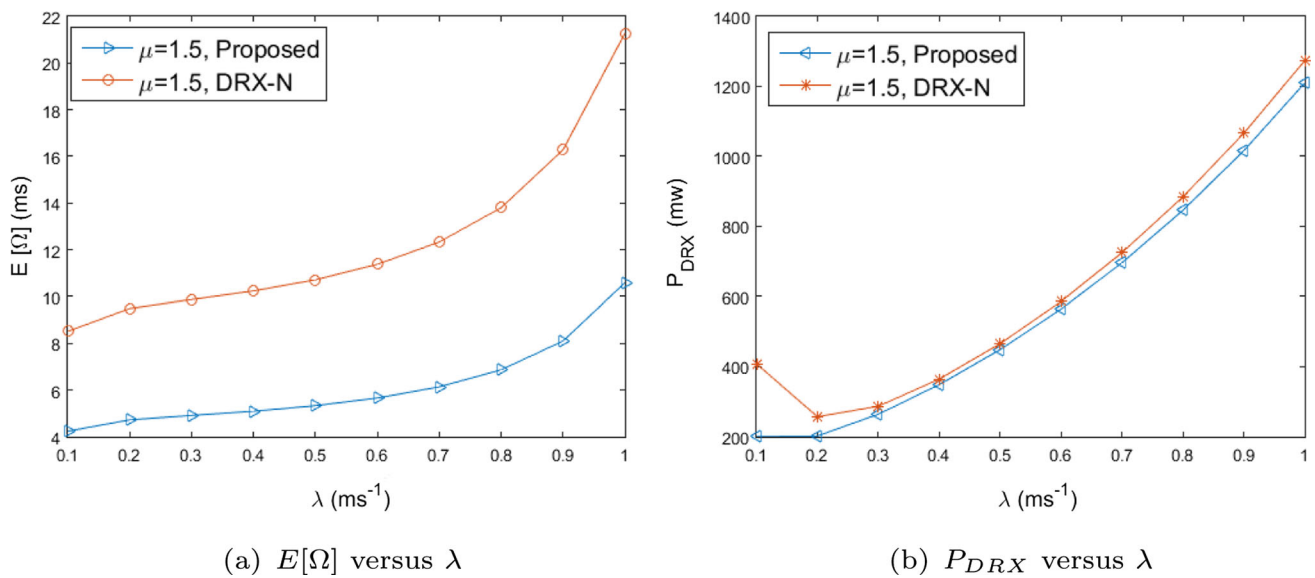


Fig. 8 Expected delay $E[\Omega]$, power consumption of the system P_{DRX} versus arrival rate λ

Table 4 Comparison with the existing and relevant work

Related work	LTE	LTE -A	5G	RRC connected and idle mode	Power consumption	Power saving factor	Delay	Mathematical model
Bontu and Nortel [3]	✓			✓		✓	✓	Not defined
Tseng et al. [4]		✓			✓	✓	✓	Markov model
Abinader et al. [12]			✓			✓	✓	Not defined
Rostami et al. [16]			✓			✓	✓	Semi-Markov
Philip and Balakrishnan [6]			✓		✓	✓	✓	Semi-Markov
Wu and Park [8]	✓				✓	✓	✓	Not defined
Gautam et al. [9]		✓		✓	✓		✓	$M^X/G/1$
Arunsun-dar et al. [5]		✓		✓	✓	✓	✓	Semi-Markov
Rostami et al. [14]			✓			✓	✓	Semi-Markov
Rostami et al. [15]			✓			✓	✓	Poisson arrival
Mahesh-wari et al. [7]			✓	✓	✓	✓	✓	Semi-Markov
The proposed model in this work			✓	✓	✓	✓	✓	$M^X/G/1$

thing that needs to be looked into is that there are restricted mathematical models exist in the literature over the DRX mechanism employed in both the RRC modes. Therefore, this observation is considered as the reference point for providing the comparison.

- Bontu and Nortel [3] discussed the power saving of UE and delay in the LTE network. As per the current scenario, the power saving of UE needs to be taken care of in 5G networks.
- Arunsundar et al. [5] worked over a semi-Markov model to trade-off between delay and power consumption in LTE-A. However, UE is configured with wide bandwidth to transmit signal leading to more power consumption in

5G networks. This has been taken care in this proposed work.

- Maheshwari et al. [7] proposed a semi-Markov modeling based DRX mechanism in 5G networks, where the arrival of packets follows the Poisson process. The proposed model in this paper represents the relevance of arrival with the realistic scenario since packet(s) may arrive in batches to UE in a realistic scenario.
- Gautam et al. [9] presented an $M^X/G/1$ queue model which dealt with the batch arrival of packet(s) for LTE-A networks. Moreover, it was shown that after providing service, UE switched directly to sleep states. In our proposed model, we improvise the model proposed by Gautam et al. [9] by considering waiting for the arrival of packets during the inactivity timer. In addition, the

DRX mechanism with BWP in 5G NR is proposed here as it is very obvious that the current scenario demands the DRX for a wide range spectrum.

To see the strength of the proposed work, the performance measures are numerically illustrated for $\mu = 1.5$, $N = 1$ and $K_2 = 3$ of Gautam et al. [9] in Fig. 8a, b. Due to the presence of the $M^X/G/1$ queue model, the comparison is made with the DRX mechanism having N-policy (DRX-N) analysed in Reference [9]. The numerical results show that the proposed work in this paper has a lesser expected delay of packet and power consumption of the system as compared to the Gautam et al. [9].

Therefore, based on the above comparison, it is clear that this model can easily capture the real-time scenario.

8 Conclusion with future directions

Queueing models are a driving force in cellular networks to minimise the power consumption of UE. Various power-saving techniques are modeled as a queue intending to reduce power consumption. A full bandwidth usage to monitor the gNB for the packet(s) arrival increases the power consumption of UE. When no packet is available to serve, the use of part of the bandwidth is an effective method for power saving. In this work, batch arrival based DRX for the RRC connected and RRC idle mode having three kinds of sleep states is explored. These sleep states with RRC idle state and narrowband active state are modeled as $M^X/G/1$ queue to reduce the power consumption in the DRX. In this single server queueing model, UE is considered as a server and only the downlink packet at UE in 5G NR is addressed. To analyse the delay and power consumption of the system, the proposed work introduces a narrowband active state before going to sleep states to reduce the delay. The analytical expression for various performance parameters such as the expected number of packets in the system, power saving factor, power consumption and delay of a packet are obtained. The numerical results show that UE can attain a significant power saving.

For future studies, the authors will investigate the findings of this proposed research work by integrating it with a power-saving mechanism (PSM) and beam-forming technique. The expected delay due to switching from switch off to switch on is very small as it has no effect on power consumption of the system. This factor could be modeled in future work. Additionally, we extend this work to study the power-saving and cooling parameters of a base station.

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Declarations

Conflict of interest The authors declare that there is no conflict of interest regarding the publication of this paper.

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