



Limit theorems for marked dynamic contagion processes with mean-reverting Cox–Ingersoll–Ross intensity

Shamiksha Pandey, Dharmaraja Selvamuthu^{ID*}

Department of Mathematics, Indian Institute of Technology Delhi, India

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ABSTRACT

In this paper, we introduce and study the marked dynamic contagion process, where the intensity process is governed by the Cox–Ingersoll–Ross (CIR) model. This framework extends both the Hawkes and the Cox process with Poisson shot-noise intensity by incorporating self-exciting and externally-exciting jumps, and random noise. This allows to capture the effects of events caused by internal (endogenous) and external (exogenous) factors, along with random fluctuations of the market. Furthermore, the process includes an independent and identically distributed (i.i.d.) sequence of marks, which represent distinct event characteristics and influence the timing and behavior of subsequent occurrences. For the proposed model, we examine the theoretical properties, including the Law of Large Numbers (LLN), the Central Limit Theorem (CLT), and the Large Deviation Principle (LDP).

1. Introduction

Point processes have become a crucial tool in finance, providing a systematic approach to describe various phenomena such as market arrivals, price fluctuations, defaults, and contagion effects. In today's interconnected financial markets, understanding the timing and occurrence of events is crucial for risk management, portfolio optimization, and maintaining overall market stability. In particular, the study of default events and their contagion effects emphasize the importance of point processes in finance. By contagion effects, we refer to the phenomena in which the default of a single entity triggers a series of defaults among interconnected organizations. However, most of the existing literature (e.g., [Jarow and Turnbull \(1995\)](#), and [Jang \(2007\)](#)) employs simple Poisson processes to simulate defaults. These models, however, fail to capture the contagion effects associated with sudden jumps or defaults in markets, particularly during financial crises.

Hawkes process (or self-exciting process) was introduced by [Hawkes \(1971\)](#) and [Hawkes and Oakes \(1974\)](#) for modeling the arrival of events with clustering and contagion effects. In this regard, [Errais et al. \(2010\)](#) observed using real data that the default clustering could be addressed more consistently by a self-exciting process. Moreover, it is important to take into consideration that defaults can also be influenced by some external factors. Thus, [Dassios and Zhao \(2011\)](#) introduced a new model called the dynamic contagion process, which is a generalization of the Hawkes process and Cox process with Poisson shot-noise intensity, and it captures the externally and self-exciting jumps.

In this paper, we incorporate a mean-reverting CIR diffusion process (introduced by [Cox et al. \(1985\)](#) and [Feller \(1951\)](#)) into the intensity of a point process. A wide range of studies on CIR model have been done in various areas of finance. The existence and

* Corresponding author.

E-mail addresses: shamiksha@maths.iitd.ac.in (S. Pandey), dharmar@maths.iitd.ac.in (D. Selvamuthu).

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uniqueness of the solution for the CIR process under some assumptions have been done by Akhtari and Li (2024) and others. Using these insights, we define the point process, which is excited by its own occurrences and other external jumps (which we refer to as a dynamic contagion process) with CIR intensity process. This framework generalizes the dynamic contagion process by incorporating an intensity process that exhibits exponential decay and is also influenced by an independent mean-reverting diffusion process. By incorporating a diffusion component into the intensity process, the model become useful in capturing noises that persistently and externally exist in the markets. Additionally, in the proposed model, we consider an i.i.d. sequence of marks. These marks can be considered as the characteristics of an event, where events with distinct characteristics exert different impacts on the occurrence of future events. Consequently, we study the asymptotic results of the proposed model.

The paper is organized as follows: Section 2 explores the literature on asymptotic behaviors of the Hawkes and the dynamic contagion processes. In Section 3, we introduce the proposed model, and also discuss several assumptions and preliminary results. In Section 4, we present the main results, namely, the LLN, the CLT and the LDP with their detailed proofs. Finally, Section 5 concludes the paper with some remark and future research directions.

2. Hawkes processes and dynamic contagion processes

This section discusses about the Hawkes processes and the dynamic contagion processes along with their asymptotic behaviors. Let, $(\Omega, \mathcal{F}, \{F_t\}_{t \geq 0}, P)$ be the filtered probability space where the filtration F_t satisfies the usual conditions of completeness and right continuity and on which all the stochastic processes are defined.

A Hawkes process is a point process $N = \{N_t, t \geq 0\}$ which admitting an F_t adapted intensity defined as

$$\lambda_t = \phi \left(\int_0^t h(t-s) dN_s \right),$$

where $\phi(\cdot) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a locally integrable, left continuous function known as intensity function and $h(\cdot) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is assumed to be integrable, i.e., $\|h\|_{L^1} = \int_0^\infty h(t)dt < \infty$ and it is known as the exciting function.

Whenever $\phi(\cdot)$ is linear, say $\phi(z) = v + z$, $v > 0$, the Hawkes process is said to be linear, and admits an immigration-birth representation, see Hawkes and Oakes (1974). Within this framework, and using the stability assumption, i.e., $\|h\|_{L^1} < 1$, Daley and Vere-Jones (2003), proved the LLN and Bacry et al. (2013), proved the CLT. Moreover, Bordenave and Torrisi (2007) studied the LDP for $P(\frac{N_t}{t} \in \cdot)$. For the extension of linear Hawkes process with CIR process, Zhu (2014a) studied the limit theorems.

When $\phi(\cdot)$ is nonlinear, then the Hawkes process is referred to as a nonlinear Hawkes process, introduced by Brémaud and Massoulié (1996). In nonlinear Hawkes process the usual immigration-birth representation is no longer valid. Hence, different approach has been used to study the limit theorems. Zhu (2013) proved the Functional CLT, while LDP for the nonlinear case has been studied by Zhu (2014b), Zhu (2015).

A dynamic contagion process is a point process $N = \{N_t, t \geq 0\}$ whose intensity is given by

$$\lambda_t = v + \int_0^t g(t-s) dM_s + \int_0^t h(t-s) dN_s, \quad t \geq 0$$

where $\{M_t, t \geq 0\}$ is a Poisson process with parameter $\rho > 0$. This process is designed to account for scenarios where the default intensity is influenced by many external factors, as investigated by Duffie and Gârleanu (2001) and Longstaff and Rajan (2008). The asymptotic results of the dynamic contagion process, considering a general exciting function with or without marks, have been studied by Yao and Xiao (2018). Additionally, Pandey et al. (2025) and Selvamuthu et al. (2023) explored a more general form of the intensity function in the dynamic contagion process and examined its limit theorems. Furthermore, the distributional properties of the dynamic contagion process with CIR intensity have been investigated by Dassios and Zhao (2017), Dassios et al. (2019), and Pasricha and Selvamuthu (2019).

3. Marked dynamic contagion process with CIR intensity

A marked dynamic contagion process with CIR intensity is a point process $N = \{N_t, t \geq 0\}$ with non-negative F_t intensity given as

$$\lambda_t = a + (\lambda_0 - a)e^{-\delta t} + \sigma \int_0^t \sqrt{\lambda_s} e^{-\delta(t-s)} dW_s + \sum_{0 \leq T_i^{(1)} < t} Y_i e^{-\delta(t-T_i^{(1)})} + \sum_{0 \leq T_k^{(2)} < t} Z_k e^{-\delta(t-T_k^{(2)})}, \quad (1)$$

where

- $0 < \lambda_0 < \infty$ is the initial intensity at $t = 0$, $a \geq 0$ is the mean-reversion level and $\delta > 0$ is the mean-reversion rate.
- $\{W_t, t \geq 0\}$ is a standard Brownian motion.
- $\{T_i^{(1)}\}_{i=1,2,\dots}$ are the arrival times of a Poisson process $\{M_t, t \geq 0\}$ with constant intensity $\rho > 0$.
- $\{Y_i\}_{i=1,2,\dots}$ is a sequence of i.i.d. positive random variables with cumulative distribution function (CDF) $H(y), y > 0$ at the corresponding arrival times $\{T_i^{(1)}\}_{i=1,2,\dots}$.
- $\{T_k^{(2)}\}_{k=1,2,\dots}$ are the arrival times of a point process $\{N_t, t \geq 0\}$ with intensity process $\{\lambda_t, t \geq 0\}$ as defined in Eq. (1).
- $\{Z_k\}_{k=1,2,\dots}$ is a sequence of i.i.d. positive random variables with CDF $G(z), z > 0$ at the corresponding arrival times $\{T_k^{(2)}\}_{k=1,2,\dots}$.

• $\{T_i^{(1)}\}_{i=1,2,\dots}$, $\{Y_i\}_{i=1,2,\dots}$, $\{Z_k\}_{k=1,2,\dots}$ and $\{W_t, t \geq 0\}$ all are assumed to be independent of each other.

To ensure that $\lambda_t > 0$, Feller condition is needed, i.e., $2\delta a \geq \sigma^2$. Moreover, along with the assumptions that the exciting function is exponential, the couple given by the marked dynamic contagion process, along with its intensity has the Markov property. Therefore, the joint process $\{(\lambda_t, N_t), t \geq 0\}$ is a Markov process in the state space $D = \mathbb{R}^+ \times (\mathbb{N} \cup \{0\})$. This implies that the infinitesimal generator $\mathcal{A}$ of the process (t, λ_t, N_t) defined on a function $f : \mathbb{R}^+ \times D \rightarrow \mathbb{R}$ is given by

$$\begin{aligned} \mathcal{A}f(t, \lambda, n) &= \frac{\partial f}{\partial t} - \delta(\lambda - a) \frac{\partial f}{\partial \lambda} + \frac{1}{2} \sigma^2 \lambda \frac{\partial^2 f}{\partial \lambda^2} + \rho \left(\int_0^\infty f(t, \lambda + y, n) dH(y) - f(t, \lambda, n) \right) \\ &\quad + \lambda \left(\int_0^\infty f(t, \lambda + z, n+1) dG(z) - f(t, \lambda, n) \right) \end{aligned} \quad (2)$$

where the function $f(t, \lambda, n)$ is differentiable with respect to λ and t , and satisfies the following conditions

$$\left| \int_0^\infty f(t, \lambda + y, n) dH(y) - f(t, \lambda, n) \right| < \infty, \quad \text{and} \quad \left| \int_0^\infty f(t, \lambda + z, n+1) dG(z) - f(t, \lambda, n) \right| < \infty.$$

Let us discuss some of the assumptions which will be required in the proof of the main results.

Assumption 3.1.

- (i) $\mathbb{E}^H[Y] = \int_0^\infty y dH(y) < \infty$ and $\text{Var}^H[Y] < \infty$.
- (ii) $\mathbb{E}^G[Z] = \int_0^\infty z dG(z) < \infty$ and $\text{Var}^G[Z] < \infty$.
- (iii) $\delta > \mathbb{E}^G[Z]$, which is known as the stability condition.

Now, we present one lemma which will be needed for the proof of main results. The following lemma provides us the expression for the first moment of λ_t and shows that $\sup_{t \geq 0} \mathbb{E}[\lambda_t] < \infty$.

Lemma 3.2. Consider the intensity process $\{\lambda_t, t \geq 0\}$ as defined in Eq. (1), then we have

$$\sup_{t \geq 0} \mathbb{E}[\lambda_t] = \max \left\{ \frac{a\delta + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]}, \lambda_0 \right\} < \infty. \quad (3)$$

Proof. Using Dynkin's formula, we have

$$\mathbb{E}[f(\lambda_t)] = f(\lambda_0) + \mathbb{E} \left(\int_0^t \mathcal{A}f(\lambda_s) ds \right). \quad (4)$$

In Eq. (2) consider $f(t, \lambda, n) = \lambda$, we get

$$\begin{aligned} \mathcal{A}(\lambda) &= \delta(a - \lambda) + \rho \left(\int_0^\infty y dH(y) \right) + \lambda \left(\int_0^\infty z dG(z) \right) \\ &= \delta a + \rho \mathbb{E}^H[Y] - \lambda (\delta - \mathbb{E}^G[Z]). \end{aligned}$$

Substitute this in Eq. (4), we obtain

$$\begin{aligned} \mathbb{E}[\lambda_t] &= \lambda_0 + \mathbb{E} \left(\int_0^t (\delta a + \rho \mathbb{E}^H[Y] - \lambda_s (\delta - \mathbb{E}^G[Z])) ds \right) \\ &= \lambda_0 + \int_0^t (\delta a + \rho \mathbb{E}^H[Y] - (\delta - \mathbb{E}^G[Z]) \mathbb{E}[\lambda_s]) ds. \end{aligned} \quad (5)$$

Now differentiate Eq. (5) w.r.t. t , we get $\frac{d}{dt} \mathbb{E}[\lambda_t] = \delta a + \rho \mathbb{E}^H[Y] - (\delta - \mathbb{E}^G[Z]) \mathbb{E}[\lambda_t]$. After simplification, we deduce that

$$\begin{aligned} \mathbb{E}[\lambda_t] &= \frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \left(1 - e^{-(\delta - \mathbb{E}^G[Z])t} \right) + \lambda_0 e^{-(\delta - \mathbb{E}^G[Z])t} \\ &= \frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} + \left(\lambda_0 - \frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \right) e^{-(\delta - \mathbb{E}^G[Z])t}. \end{aligned}$$

By considering Assumption 3.1 (iii), we obtain that $\sup_{t \geq 0} \mathbb{E}[\lambda_t] = \max \left\{ \frac{a\delta + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]}, \lambda_0 \right\}$, which is finite. Hence, Eq. (3) holds. $\square$

4. Main results

This section focuses on the asymptotic behavior of the marked dynamic contagion process with CIR intensity. The main results of this paper are presented in Theorem 4.1, Theorem 4.2 and Theorem 4.3.

Theorem 4.1 (Law of Large Numbers). Under the Assumption 3.1, let, $\{N_t, t \geq 0\}$ be the marked dynamic contagion process with CIR intensity process $\{\lambda_t, t \geq 0\}$ as defined in Eq. (1), then we have

$$\frac{N_t}{t} \rightarrow \frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \text{ in probability as } t \rightarrow \infty.$$

Proof. From the definition of λ_t as given in Eq. (1), we can write that

$$\int_0^t \lambda_s ds = at + \frac{(\lambda_0 - a)}{\delta} (1 - e^{-\delta t}) + \sigma \int_0^t \int_0^s e^{-\delta(s-\tau)} \sqrt{\lambda_\tau} dW_\tau ds + \int_0^t \int_0^s Y_\tau e^{-\delta(s-\tau)} dM_\tau ds + \int_0^t \int_0^s Z_\tau e^{-\delta(s-\tau)} dN_\tau ds.$$

Using the change of variables method under integration, we deduce that

$$\begin{aligned} \int_0^t \lambda_s ds &= at + \frac{(\lambda_0 - a)}{\delta} (1 - e^{-\delta t}) + \sigma \int_0^t \int_\tau^t e^{-\delta(s-\tau)} \sqrt{\lambda_\tau} ds dW_\tau + \int_0^t \int_\tau^t Y_\tau e^{-\delta(s-\tau)} ds dM_\tau + \int_0^t \int_\tau^t Z_\tau e^{-\delta(s-\tau)} ds dN_\tau \\ &= at + \frac{(\lambda_0 - a)}{\delta} (1 - e^{-\delta t}) + \frac{\sigma}{\delta} \int_0^t (1 - e^{-\delta(t-\tau)}) \sqrt{\lambda_\tau} dW_\tau + \frac{1}{\delta} \sum_{T_i^{(1)} < t} Y_i + \frac{1}{\delta} \sum_{T_k^{(2)} < t} Z_k - \epsilon_t \end{aligned} \quad (6)$$

where

$$\epsilon_t = \frac{1}{\delta} \sum_{T_i^{(1)} < t} Y_i e^{-\delta(t-T_i^{(1)})} + \frac{1}{\delta} \sum_{T_k^{(2)} < t} Z_k e^{-\delta(t-T_k^{(2)})}. \quad (7)$$

It can be observe that

$$N_t = \frac{N_t - \int_0^t \lambda_s ds}{\left(1 - \frac{\mathbb{E}^G[Z]}{\delta}\right)} - \frac{\frac{\mathbb{E}^G[Z]}{\delta}}{\left(1 - \frac{\mathbb{E}^G[Z]}{\delta}\right)} N_t + \frac{1}{\left(1 - \frac{\mathbb{E}^G[Z]}{\delta}\right)} \int_0^t \lambda_s ds. \quad (8)$$

Now substituting Eq. (6) into Eq. (8), we obtain

$$\begin{aligned} N_t &= \frac{N_t - \int_0^t \lambda_s ds}{\left(1 - \frac{\mathbb{E}^G[Z]}{\delta}\right)} - \frac{\frac{\mathbb{E}^G[Z]}{\delta}}{(\delta - \mathbb{E}^G[Z])} N_t + \frac{a}{\left(1 - \frac{\mathbb{E}^G[Z]}{\delta}\right)} t + \frac{\lambda_0 - a}{(\delta - \mathbb{E}^G[Z])} (1 - e^{-\delta t}) \\ &\quad + \frac{\sigma}{(\delta - \mathbb{E}^G[Z])} \int_0^t (1 - e^{-\delta(t-\tau)}) \sqrt{\lambda_\tau} dW_\tau + \frac{1}{(\delta - \mathbb{E}^G[Z])} \sum_{T_i^{(1)} < t} Y_i \\ &\quad + \frac{1}{(\delta - \mathbb{E}^G[Z])} \sum_{T_k^{(2)} < t} Z_k - \frac{\delta}{(\delta - \mathbb{E}^G[Z])} \epsilon_t. \end{aligned} \quad (9)$$

After some simplification, we get

$$\begin{aligned} &\frac{N_t - \left(\frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]}\right) t}{t} \\ &= \frac{\delta}{(\delta - \mathbb{E}^G[Z])} \left(\frac{N_t - \int_0^t \lambda_s ds}{t} \right) + \frac{1}{t} \frac{\mathbb{E}^H[Y]}{(\delta - \mathbb{E}^G[Z])} (M_t - \rho t) \\ &\quad + \frac{1}{t} \frac{\sigma}{(\delta - \mathbb{E}^G[Z])} \left(\int_0^t (1 - e^{-\delta(t-\tau)}) \sqrt{\lambda_\tau} dW_\tau \right) + \frac{1}{t} \frac{1}{(\delta - \mathbb{E}^G[Z])} \left(\sum_{T_i^{(1)} < t} Y_i - \mathbb{E}^H[Y] M_t \right) \\ &\quad + \frac{1}{t} \frac{1}{(\delta - \mathbb{E}^G[Z])} \left(\sum_{T_k^{(2)} < t} Z_k - \mathbb{E}^G[Z] N_t \right) + \frac{\lambda_0 - a}{(\delta - \mathbb{E}^G[Z])} \left(\frac{1 - e^{-\delta t}}{t} \right) - \frac{\delta}{(\delta - \mathbb{E}^G[Z])} \left(\frac{\epsilon_t}{t} \right) \\ &= I_1(t) + I_2(t) + I_3(t) + I_4(t) + I_5(t) + I_6(t) - I_7(t). \end{aligned} \quad (10)$$

Now, the required convergence in probability is achieved by showing that first or second moment of $I_i(t)$ converge to 0 as $t \rightarrow \infty$, for each $i = 1, 2, 3, 4, 5, 6, 7$.

For the first term, i.e., $I_1(t)$, note that $N_t - \int_0^t \lambda_s ds$ is a martingale. By applying Jensen's inequality $\left(N_t - \int_0^t \lambda_s ds\right)^2$ becomes a sub-martingale. Furthermore, using Doob's decomposition, we obtain

$$\mathbb{E} \left(N_t - \int_0^t \lambda_s ds \right)^2 = \mathbb{E} \left(\int_0^t \lambda_s ds \right) \quad (11)$$

and by using Lemma 3.2, we get

$$\mathbb{E} \left[\left(\frac{N_t - \int_0^t \lambda_s ds}{t} \right)^2 \right] = \frac{1}{t^2} \mathbb{E} \left(\int_0^t \lambda_s ds \right) = \frac{1}{t^2} \int_0^t \mathbb{E}[\lambda_s] ds \leq \frac{1}{t} \sup_{t \geq 0} \mathbb{E}[\lambda_t] \rightarrow 0 \quad (12)$$

as $t \rightarrow \infty$, which implies $\mathbb{E}[I_1^2(t)] \rightarrow 0$ as $t \rightarrow \infty$.

Recalling that $\{M_t, t \geq 0\}$ is a standard Poisson process, which implies $\frac{M_t}{t} \rightarrow \rho$ a.s. as $t \rightarrow \infty$, consequently, $I_2(t) \rightarrow 0$ in probability as $t \rightarrow \infty$. Now in order to use the zero mean property of Itô integral, i.e.,

$$\mathbb{E} \left(\frac{\sigma}{(\delta - \mathbb{E}^G[Z])} \int_0^t (1 - e^{-\delta(t-\tau)}) \sqrt{\lambda_\tau} dW_\tau \right) = 0, \quad (13)$$

firstly, we need to show that

$$\mathbb{E} \left(\frac{\sigma^2}{(\delta - \mathbb{E}^G[Z])^2} \int_0^t (1 - e^{-\delta(t-\tau)})^2 \lambda_\tau d\tau \right) < \infty. \quad (14)$$

Since, using [Lemma 3.2](#)

$$\begin{aligned} \mathbb{E} \left(\frac{\sigma^2}{(\delta - \mathbb{E}^G[Z])^2} \int_0^t (1 - e^{-\delta(t-\tau)})^2 \lambda_\tau d\tau \right) &= \frac{\sigma^2}{(\delta - \mathbb{E}^G[Z])^2} \int_0^t (1 - e^{-\delta(t-\tau)})^2 \mathbb{E}[\lambda_\tau] d\tau \\ &\leq \frac{\sigma^2}{(\delta - \mathbb{E}^G[Z])^2} \sup_{t \geq 0} \mathbb{E}[\lambda_t] \int_0^t (1 - e^{-\delta(t-\tau)})^2 d\tau < \infty \end{aligned}$$

which implies that the integral in Eq. (14) is finite and Eq. (13) holds, which further conclude that

$$\mathbb{E}[I_3(t)] = \frac{1}{t} \mathbb{E} \left(\frac{\sigma}{(\delta - \mathbb{E}^G[Z])} \int_0^t (1 - e^{-\delta(t-\tau)}) \sqrt{\lambda_\tau} dW_\tau \right) = 0$$

as $t \rightarrow \infty$. For $I_4(t)$, note that

$$\begin{aligned} \mathbb{E}[I_4^2(t)] &= \frac{1}{t^2} \frac{1}{(\delta - \mathbb{E}^G[Z])^2} \mathbb{E} \left(\sum_{T_i^{(1)} < t} Y_i - \mathbb{E}^H[Y] M_t \right)^2 \\ &= \frac{1}{t^2} \frac{1}{(\delta - \mathbb{E}^G[Z])^2} \mathbb{E} \left(\sum_{T_i^{(1)} < t} (Y_i - \mathbb{E}^H[Y]) \right)^2 \\ &= \frac{1}{t^2} \frac{1}{(\delta - \mathbb{E}^G[Z])^2} (\mathbb{E}[M_t] \text{Var}^H[Y]) \\ &= \frac{1}{t} \frac{\rho}{(\delta - \mathbb{E}^G[Z])^2} \text{Var}^H[Y] \end{aligned}$$

using [Assumption 3.1](#), i.e., $\text{Var}^H[Y] < \infty$, we obtain $\lim_{t \rightarrow \infty} \mathbb{E}[I_4^2(t)] = 0$. For $I_5(t)$, using [Lemma 3.2](#), we have

$$\begin{aligned} \mathbb{E}[I_5^2(t)] &= \frac{1}{t^2} \frac{1}{(\delta - \mathbb{E}^G[Z])^2} \mathbb{E} \left(\sum_{T_k^{(2)} < t} Z_k - \mathbb{E}^G[Z] N_t \right)^2 \\ &= \frac{1}{t^2} \frac{1}{(\delta - \mathbb{E}^G[Z])^2} (\mathbb{E}[N_t] \text{Var}^G[Z]) \\ &= \frac{1}{t^2} \frac{1}{(\delta - \mathbb{E}^G[Z])^2} \left(\int_0^t \mathbb{E}[\lambda_\tau] d\tau \right) \text{Var}^G[Z] \\ &\leq \frac{1}{t} \frac{\text{Var}^G[Z]}{(\delta - \mathbb{E}^G[Z])^2} \sup_{t \geq 0} \mathbb{E}[\lambda_t] \end{aligned}$$

and by using [Assumption 3.1](#), i.e., $\text{Var}^G[Z] < \infty$, it implies that $\lim_{t \rightarrow \infty} \mathbb{E}[I_5^2(t)] = 0$. For $I_6(t)$, we have

$$I_6(t) = \frac{\lambda_0 - a}{(\delta - \mathbb{E}^G[Z])} \left(\frac{1 - e^{-\delta t}}{t} \right) \rightarrow 0 \text{ as } t \rightarrow \infty.$$

For I_7 , simplifying and taking into account Eq. (7), we obtain

$$\begin{aligned} \mathbb{E}[I_7(t)] &= \frac{1}{t} \frac{1}{(\delta - \mathbb{E}^G[Z])} \mathbb{E} \left(\sum_{T_i^{(1)} < t} Y_i e^{-\delta(t-T_i^{(1)})} + \frac{1}{\delta} \sum_{T_k^{(2)} < t} Z_k e^{-\delta(t-T_k^{(2)})} \right) \\ &= \frac{1}{t} \frac{1}{(\delta - \mathbb{E}^G[Z])} \left(\rho \mathbb{E}^H[Y] \int_0^t e^{-\delta(t-\tau)} d\tau + \mathbb{E}^G[Z] \int_0^t e^{-\delta(t-\tau)} \mathbb{E}[\lambda_\tau] d\tau \right) \\ &\leq \frac{1}{(\delta - \mathbb{E}^G[Z])} \left(\rho \mathbb{E}^H[Y] + \mathbb{E}^G[Z] \sup_{t \geq 0} \mathbb{E}[\lambda_t] \right) \left(\frac{1 - e^{-\delta t}}{\delta t} \right) \rightarrow 0 \end{aligned}$$

as $t \rightarrow \infty$. Summing up, we have shown that, for $i = 1, 2, 3, 4, 5, 6, 7$, either first or second moment of $I_i(t)$ converges to 0 as $t \rightarrow \infty$, which implies that $I_i(t) \rightarrow 0$ in probability as $t \rightarrow \infty$ by using Markov's inequality. Consequently, substituting all the results obtained so far in Eq. (10), we conclude that

$$\frac{N_t}{t} \rightarrow \frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \text{ in probability as } t \rightarrow \infty. \quad \square$$

Theorem 4.2 (Central Limit Theorem). Under the [Assumption 3.1](#), we have

$$\frac{N_t - \left(\frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \right) t}{\sqrt{t}} \rightarrow N(0, \sigma_1^2) \text{ in distribution as } t \rightarrow \infty,$$

where

$$\sigma_1^2 = \frac{(\delta a + \rho \mathbb{E}^H[Y]) (\delta^2 + \text{Var}^G[Z] + \sigma^2)}{(\delta - \mathbb{E}^G[Z])^3} + \frac{\rho ((\mathbb{E}^H[Y])^2 + \text{Var}^H[Y])}{(\delta - \mathbb{E}^G[Z])^2}. \quad (15)$$

Proof. Using the Eq. (10), defined in Proof of [Theorem 4.1](#), we can deduce that

$$\begin{aligned} & \frac{N_t - \left(\frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \right) t}{\sqrt{t}} \\ &= \frac{\delta}{(\delta - \mathbb{E}^G[Z])} \left(\frac{N_t - \int_0^t \lambda_s ds}{\sqrt{t}} \right) + \frac{1}{\sqrt{t}} \frac{1}{(\delta - \mathbb{E}^G[Z])} \left(\sum_{T_k^{(2)} < t} Z_k - \mathbb{E}^G[Z] N_t \right) + \frac{\mathbb{E}^H[Y]}{(\delta - \mathbb{E}^G[Z])} \left(\frac{M_t - \rho t}{\sqrt{t}} \right) \\ &+ \frac{1}{\sqrt{t}} \frac{1}{(\delta - \mathbb{E}^G[Z])} \left(\sum_{T_i^{(1)} < t} Y_i - \mathbb{E}^H[Y] M_t \right) + \frac{1}{\sqrt{t}} \left(\frac{\sigma}{(\delta - \mathbb{E}^G[Z])} \int_0^t (1 - e^{-\delta(t-\tau)}) \sqrt{\lambda_\tau} dW_\tau \right) \\ &+ \frac{\lambda_0 - a}{(\delta - \mathbb{E}^G[Z])} \left(\frac{1 - e^{-\delta t}}{\sqrt{t}} \right) - \frac{\delta}{(\delta - \mathbb{E}^G[Z])} \left(\frac{\epsilon_t}{\sqrt{t}} \right) \\ &= \frac{M_1(t)}{\sqrt{t}} + \frac{M_2(t)}{\sqrt{t}} + \frac{M_3(t)}{\sqrt{t}} + \frac{M_4(t)}{\sqrt{t}} + \frac{M_5(t)}{\sqrt{t}} + \frac{\lambda_0 - a}{(\delta - \mathbb{E}^G[Z])} \left(\frac{1 - e^{-\delta t}}{\sqrt{t}} \right) - \frac{\delta}{(\delta - \mathbb{E}^G[Z])} \left(\frac{\epsilon_t}{\sqrt{t}} \right). \end{aligned} \quad (16)$$

By following the same approach as demonstrated in [Theorem 4.1](#), we can derive the result that

$$\frac{1}{\sqrt{t}} \left(\frac{\lambda_0 - a}{(\delta - \mathbb{E}^G[Z])} (1 - e^{-\delta t}) - \frac{\delta}{(\delta - \mathbb{E}^G[Z])} \epsilon_t \right) \rightarrow 0 \text{ in probability as } t \rightarrow \infty. \quad (17)$$

Furthermore, consider $M_1(t), M_2(t), M_3(t), M_4(t), M_5(t)$ as defined in Eq. (16) are martingales and thus, their sum. Additionally, the quadratic variation of $M_1(t) + M_2(t) + M_3(t) + M_4(t) + M_5(t)$ is same as the quadratic variation of

$$\frac{\delta}{(\delta - \mathbb{E}^G[Z])} N_t + M_2(t) + \frac{\mathbb{E}^H[Y]}{(\delta - \mathbb{E}^G[Z])} M_t + M_4(t) + M_5(t). \quad (18)$$

Note that,

$$\begin{aligned} & \frac{\delta}{(\delta - \mathbb{E}^G[Z])} N_t + M_2(t) + \frac{\mathbb{E}^H[Y]}{(\delta - \mathbb{E}^G[Z])} M_t + M_4(t) + M_5(t) \\ &= \frac{1}{(\delta - \mathbb{E}^G[Z])} \left(\sum_{T_k^{(2)} < t} (\delta + Z_k - \mathbb{E}^G[Z]) \right) + \frac{1}{(\delta - \mathbb{E}^G[Z])} \left(\sum_{T_i^{(1)} < t} (\mathbb{E}^H[Y] + Y_i - \mathbb{E}^H[Y]) \right) + M_5(t), \end{aligned}$$

which has quadratic variation

$$\begin{aligned} & \frac{1}{(\delta - \mathbb{E}^G[Z])^2} \sum_{T_k^{(2)} < t} (\delta + Z_k - \mathbb{E}^G[Z])^2 + \frac{1}{(\delta - \mathbb{E}^G[Z])^2} \sum_{T_i^{(1)} < t} (\mathbb{E}^H[Y] + Y_i - \mathbb{E}^H[Y])^2 \\ &+ \frac{\sigma^2}{(\delta - \mathbb{E}^G[Z])^2} \int_0^t (1 - e^{-\delta(t-\tau)})^2 \lambda_\tau d\tau. \end{aligned}$$

It can be observe that

$$\begin{aligned} & \frac{1}{t} \mathbb{E} \left[\int_0^t \lambda_\tau \left| e^{-2\delta(t-\tau)} - 2e^{-\delta(t-\tau)} \right| d\tau \right] = \frac{1}{t} \mathbb{E} \left[\int_0^t (2e^{-\delta(t-\tau)} - e^{-2\delta(t-\tau)}) \lambda_\tau d\tau \right] \\ & \leq \frac{1}{t} \int_0^t (2e^{-\delta(t-\tau)} - e^{-2\delta(t-\tau)}) d\tau \cdot \sup_{t \geq 0} \mathbb{E}[\lambda_t] \rightarrow 0 \end{aligned} \quad (19)$$

as $t \rightarrow \infty$. From Eq. (12), it has been proved that $\frac{N_t - \int_0^t \lambda_s ds}{t} \rightarrow 0$ in probability as $t \rightarrow \infty$, which implies

$$\frac{1}{t} \int_0^t \lambda_s ds \rightarrow \frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \text{ in probability as } t \rightarrow \infty. \quad (20)$$

Consequently, using the LLN, we obtain

$$\begin{aligned} & \frac{1}{(\delta - \mathbb{E}^G[Z])^2} \left[\frac{1}{t} \sum_{T_k^{(2)} < t} (\delta + Z_k - \mathbb{E}^G[Z])^2 + \frac{1}{t} \sum_{T_i^{(1)} < t} (\mathbb{E}^H[Y] + Y_i - \mathbb{E}^H[Y])^2 + \sigma^2 \cdot \frac{1}{t} \int_0^t \lambda_\tau d\tau \right] \\ &= \frac{1}{(\delta - \mathbb{E}^G[Z])^2} \left[\frac{N_t}{t} \cdot \frac{1}{N_t} \sum_{T_k^{(2)} < t} (\delta + Z_k - \mathbb{E}^G[Z])^2 + \frac{M_t}{t} \cdot \frac{1}{M_t} \sum_{T_i^{(1)} < t} (\mathbb{E}^H[Y] + Y_i - \mathbb{E}^H[Y])^2 + \sigma^2 \cdot \frac{1}{t} \int_0^t \lambda_\tau d\tau \right] \\ &\rightarrow \frac{1}{(\delta^2 - \mathbb{E}^G[Z])^2} \left[\left(\frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \right) \mathbb{E}^G[(\delta + Z_k - \mathbb{E}^G[Z])^2] + \rho \mathbb{E}^H[(\mathbb{E}^H[Y] + Y_i - \mathbb{E}^H[Y])^2] + \sigma^2 \left(\frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \right) \right] \\ &= \frac{1}{(\delta - \mathbb{E}^G[Z])^2} \left[\left(\frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \right) (\delta^2 + \text{Var}^G[Z]) + \rho ((\mathbb{E}^H[Y])^2 + \text{Var}^H[Y]) + \sigma^2 \left(\frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \right) \right] \end{aligned} \quad (21)$$

in probability as $t \rightarrow \infty$. Thus, by using the Martingale Central Limit Theorem, we obtain

$$\frac{N_t - \left(\frac{\delta a + \rho \mathbb{E}^H[Y]}{\delta - \mathbb{E}^G[Z]} \right) t}{\sqrt{t}} \rightarrow N(0, \sigma_1^2) \quad (22)$$

in distribution as $t \rightarrow \infty$ where σ_1^2 as defined in Eq. (15). $\square$

Before moving on to the results related to the LDP for N_t , let us consider a sequence of probability measures $(P_n)_{n \in \mathbb{N}}$ defined on a topological space $\mathbf{X}$. The sequence satisfies the LDP with rate function $I^* : \mathbf{X} \rightarrow [0, \infty]$, if I^* is lower semi-continuous and for any measurable set B in $\mathbf{X}$, the following inequalities hold

$$-\inf_{x \in B^0} I^*(x) \leq \liminf_{n \rightarrow \infty} \frac{1}{n} \log P_n(B) \leq \limsup_{n \rightarrow \infty} \frac{1}{n} \log P_n(B) \leq -\inf_{x \in \bar{B}} I^*(x),$$

where B^0 represents the interior of B (also denoted as $\text{int}(B)$) and $\bar{B}$ represents its closure.

Theorem 4.3 (Large Deviation Principle). Under the Assumption 3.1, $P\left(\frac{N_t}{t} \in \cdot\right)$ satisfies the LDP with rate function $I^*(x)$ given by

$$I^*(x) = \sup_{\theta \leq \theta_m} \left\{ \theta x - \delta a g(\theta) - \rho \left(\mathbb{E}^H[e^{g(\theta)Y}] - 1 \right) \right\} \quad (23)$$

where for $\theta \leq \theta_m$, $g(\theta)$ is the smaller solution of

$$-\delta g(\theta) + \frac{1}{2} \sigma^2 g(\theta)^2 + (\mathbb{E}^G[e^{g(\theta)Z}] - 1) + \theta \delta = 0,$$

and $\theta_m = \frac{1}{\delta} \left(\delta g_m - \frac{1}{2} \sigma^2 g_m^2 - (\mathbb{E}^G[e^{g_m Z}] - 1) \right)$, where g_m is the unique positive solution of $\delta = \sigma^2 g_m + \mathbb{E}^G[Z e^{g_m Z}]$.

Now, we examine the LDP of $P\left(\frac{N_t}{t} \in \cdot\right)$ using the Gärtner–Ellis Theorem (see Definition 2.3.5 and Theorem 2.3.6 in Dembo and Zeitouni (1998)).

Proof. In order to obtain the LDP, let us first find the moment generating function of N_t . From Eq. (1), we have that

$$d\lambda_t = \delta(a - \lambda_t)dt + \sigma \sqrt{\lambda_t} dW_t + Y_t dM_t + Z_t dN_t$$

which can be rewrite as

$$N_t = \lambda_t - \lambda_0 - \delta \int_0^t (a - \lambda_s) ds - \sigma \int_0^t \sqrt{\lambda_s} dW_s - \int_0^t Y_s dM_s - \int_0^t (Z_s - 1) dN_s. \quad (24)$$

Now by Eq. (24), note that $\mathbb{E}[e^{\theta N_t} | \lambda_0 = \lambda] = e^{-\theta \lambda} u(\theta, t, \lambda)$ where

$$u(\theta, t, \lambda) = \mathbb{E} \left[e^{\theta \lambda_t - \theta \delta \int_0^t (a - \lambda_s) ds - \sigma \theta \int_0^t \sqrt{\lambda_s} dW_s - \theta \int_0^t Y_s dM_s - \theta \int_0^t (Z_s - 1) dN_s} | \lambda_0 = \lambda \right].$$

Using the Feynman–Kac formula, u satisfies the following partial differential equation

$$\frac{\partial u}{\partial t} = \delta a \frac{\partial u}{\partial \lambda} - \delta \lambda \frac{\partial u}{\partial \lambda} + \frac{1}{2} \sigma^2 \lambda \frac{\partial^2 u}{\partial \lambda^2} + \rho \left(\int_0^\infty u(\theta, t, \lambda + y) dH(y) - u(\theta, t, \lambda) \right) + \lambda \left(\int_0^\infty u(\theta, t, \lambda + z) dG(z) - u(\theta, t, \lambda) \right) + u \delta \theta \lambda,$$

with initial condition given as $u(\theta, 0, \lambda) = e^{\theta \lambda}$. Taking into account the affine structure (see for instance Errais et al. (2010)), we have $u(\theta, t, \lambda) = e^{A(t; \theta) \lambda + B(t; \theta)}$, where $A(t; \theta)$ and $B(t; \theta)$ satisfy the following differential equations

$$A'(t; \theta) = -\delta A(t; \theta) + \frac{1}{2} \sigma^2 A^2(t; \theta) + \left(\int_0^\infty e^{A(t; \theta) z} dG(z) - 1 \right) + \theta \delta,$$

$$B'(t; \theta) = \delta a A(t; \theta) + \rho \left(\int_0^\infty e^{A(t; \theta)y} dH(y) - 1 \right),$$

$$A(0; \theta) = \theta, \quad \text{and} \quad B(0; \theta) = 0.$$

It can be observe that $\lim_{t \rightarrow \infty} A(t; \theta) = g(\theta)$, where $g = g(\theta)$ satisfies the equation given by

$$-\delta g(\theta) + \frac{1}{2} \sigma^2 (g(\theta))^2 + (\mathbb{E}^G [e^{g(\theta)Z}] - 1) + \theta \delta = 0. \quad (25)$$

If Eq. (25) admits a solution then the limit exist, otherwise, $\lim_{t \rightarrow \infty} A(t; \theta) = \infty$. Now we claim that $g(\theta)$ is the smaller solution of the Eq. (25) for $\theta \leq \theta_m$, where

$$\begin{aligned} \theta_m &= \frac{1}{\delta} \left(\sup_{g \in \mathbb{R}^+} \left\{ \delta g - \frac{1}{2} \sigma^2 g^2 - (\mathbb{E}^G [e^{gZ}] - 1) \right\} \right) \\ &= \frac{1}{\delta} \left(\delta g_m - \frac{1}{2} \sigma^2 g_m^2 - (\mathbb{E}^G [e^{g_m Z}] - 1) \right) \end{aligned} \quad (26)$$

and g_m is a unique positive solution of $\delta = \sigma^2 g_m + \mathbb{E}^G [Z e^{g_m Z}]$.

Define the function $\theta = F(g) = \frac{1}{\delta} \left(\delta g - \frac{1}{2} \sigma^2 g^2 - (\mathbb{E}^G [e^{gZ}] - 1) \right)$. Now solving Eq. (25) is equivalent to solving $\theta = F(g)$. Since the function $F(g)$ is strictly concave, it has at most one critical point. The critical point is given by

$$F'(g) = 0 \iff \delta = \sigma^2 g + \mathbb{E}^G [Z e^{gZ}].$$

Since, $\delta > \mathbb{E}^G [Z]$ and the right-hand side is strictly increasing in g , this equation admits a unique positive solution, denoted by g_m . Therefore, g_m is the unique global maximizer of $F(g)$, and hence F attains its global maximum at $g = g_m$.

Consequently, it follows that for $\theta < \theta_m$, the equation $\theta = F(g)$ has two distinct solutions, and hence, Eq. (25) admits two distinct solutions. For $\theta = \theta_m$, Eq. (25) has a unique solution, corresponding to the maximum point g_m .

Furthermore, by defining

$$\Gamma(g) = -\delta g + \frac{1}{2} \sigma^2 (g)^2 + (\mathbb{E}^G [e^{gZ}] - 1) + \theta \delta,$$

it follows that, when $\theta < 0$, we have $\Gamma(0) = \delta \theta < 0$. Since, $\Gamma(g) \rightarrow +\infty$ as $g \rightarrow \pm\infty$ and Γ is strictly convex, then the equation $\Gamma(g) = 0$ has exactly two solutions where one is negative and one is positive. Therefore, $g(\theta)$ is the unique negative solution of $\Gamma(g) = 0$ or of Eq. (25). When $0 \leq \theta \leq \theta_m$, both solutions of Eq. (25), lie on the non-negative side. In this case, $g(\theta)$ is defined as the smaller non-negative solution of Eq. (25).

Now, we claim that $\theta_m > 0$. Since, for sufficiently small $g > 0$ (as defined in Eq. (26)) and using Assumption 3.1 (iii), we get

$$\delta g - \frac{1}{2} \sigma^2 g^2 - (\mathbb{E}^G [e^{gZ}] - 1) = (\delta - \mathbb{E}^G [Z]) g + O(g^2) > 0.$$

Consequently, we deduce that $\theta_m > 0$ for sufficiently small $g > 0$.

Therefore, we get that for $\theta \leq \theta_m$, $\lim_{t \rightarrow \infty} A(t; \theta) = g(\theta)$ which further implies that for $\theta \leq \theta_m$

$$\lim_{t \rightarrow \infty} B'(t; \theta) = \delta a g(\theta) + \rho (\mathbb{E}^H [e^{g(\theta)Y}] - 1).$$

Thus, for $\theta \leq \theta_m$ using L'Hôpital's rule, we obtain

$$I(\theta) = \lim_{t \rightarrow \infty} \frac{1}{t} \log \mathbb{E} [e^{\theta N_t} | \lambda_0 = \lambda] = \delta a g(\theta) + \rho (\mathbb{E}^H [e^{g(\theta)Y}] - 1). \quad (27)$$

Now to obtain the LDP, we will use Gärtner–Ellis Theorem and to this end, we have to check the, so-called, essential smoothness condition. Define, $D_I = \{\theta : I(\theta) < \infty\}$ and recall that for $\theta \leq \theta_m$, $I(\theta) < \infty$ which implies that the interior of D_I contains a nonempty neighborhood of the origin, i.e., $\text{int}(D_I) \neq \emptyset$. Also, from Eq. (25) we have that $g(\cdot)$ is differentiable at θ for $\theta < \theta_m$, which implies that $I(\theta)$ is differentiable for $\theta < \theta_m$. Now differentiate Eq. (25) with respect to θ , we obtain

$$\frac{\partial g(\theta)}{\partial \theta} = \frac{\delta}{\delta - \sigma^2 g(\theta) - \mathbb{E}^G [Z e^{g(\theta)Z}]}.$$

Since, $g \uparrow g_m$ as $\theta \uparrow \theta_m$, where g_m is a unique positive solution of $\delta = \sigma^2 g_m + \mathbb{E}^G [Z e^{g_m Z}]$, therefore, as $\theta \uparrow \theta_m$, we get $\frac{\partial g(\theta)}{\partial \theta} \rightarrow \infty$. Now differentiate both the sides of the Eq. (27) w.r.t. θ , we obtain

$$\frac{\partial I(\theta)}{\partial \theta} = (\delta a + \rho \mathbb{E}^H [Y e^{g(\theta)Y}]) \frac{\partial g(\theta)}{\partial \theta}.$$

Now considering $\frac{\partial g(\theta)}{\partial \theta} \rightarrow \infty$, we deduce that $\frac{\partial I(\theta)}{\partial \theta} \rightarrow \infty$ as $\theta \uparrow \theta_m$. Since, we have proved that I satisfies the essential smoothness condition, thus by Gärtner–Ellis Theorem, $P\left(\frac{N_t}{t} \in \cdot\right)$ satisfies the LDP with rate function $I^*(x)$ as defined in Eq. (23). $\square$

Remark 4.4. Note that, if in Eq. (1), $Y_i = \alpha$ a.s. and $Z_k = \beta$ a.s. where $\alpha, \beta > 0$, then the results obtained in Theorems 4.1 and 4.2 are consistent with the limit theorems for dynamic contagion process with CIR intensity without random marks as done in Pandey and Selvamuthu (2025).

Remark 4.5. Note that, if in Eq. (1), $Y_i = 0$ a.s. and $Z_k = \alpha$ a.s. where $\alpha > 0$, then Theorems 4.1, 4.2 and 4.3, correspond to the asymptotic results of the CIR process with Hawkes jumps as presented in Zhu (2014a).

Remark 4.6. Note that, if in Eq. (1), $\sigma = 0$, $Y_i = 0$ a.s. and $Z_k = \alpha$ a.s. where $\alpha > 0$, then Theorems 4.1, 4.2 and 4.3, correspond to the asymptotic results of the Hawkes process with exponential exciting functions as discussed in Daley and Vere-Jones (2003), Bacry et al. (2013), Bordenave and Torrisi (2007), respectively.

5. Conclusions and future work

This paper discussed the dynamic contagion process where the intensity is modeled through a CIR process with an i.i.d. sequence of random marks. The CIR process is widely used to exhibit key features such as mean reversion and volatility proportional to the square root of the intensity. As a result, the proposed model's ability to capture mean-reverting behavior, volatility clustering, and inter-connectedness makes it a useful quantitative tool for modeling contagious arrivals of events and improving risk management practices. Future research in this area may focus on extending the underlying model by incorporating general exciting functions. In such settings, the Markov property would no longer hold, which requires the development of new proof techniques to address the more complex dependence structure.

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Data availability

No data was used for the research described in the article.

References

- Akhtari, B., Li, H., 2024. The Cox-Ingersoll-Ross process under volatility uncertainty. *J. Math. Anal. Appl.* 531, 127867.
- Bacry, E., Delattre, S., Hoffmann, M., Muzy, J.F., 2013. Some limit theorems for Hawkes processes and application to financial statistics. *Stochastic Process. Appl.* 123 (7), 2475–2499.
- Bordenave, C., Torrisi, G.L., 2007. Large deviations of Poisson cluster processes. *Stoch. Models* 23 (4), 593–625.
- Brémaud, P., Massoulié, L., 1996. Stability of nonlinear Hawkes processes. *Ann. Prob.* 24 (3), 1563–1588.
- Cox, J.C., Ingersoll, Jr., J.E., Ross, S.A., 1985. A theory of the term structure of interest rates. *Econometrica* 53 (2), 385–407.
- Daley, D.J., Vere-Jones, D., 2003. *An Introduction To the Theory of Point Processes*, second ed. *Elementary Theory and Methods*, vol. I and II, Springer, New York.
- Dassios, A., Jang, J., Zhao, H., 2019. A generalised CIR process with externally-exciting and self-exciting jumps and its applications in insurance and finance. *Risks* 7 (4), 103.
- Dassios, A., Zhao, H., 2011. A dynamic contagion process. *Adv. Appl. Prob.* 43 (3), 814–846.
- Dassios, A., Zhao, H., 2017. A generalised contagion process with an application to credit risk. *Int. J. Theor. Appl. Financ.* 20 (1), 1–33.
- Dembo, A., Zeitouni, O., 1998. *Large Deviations Techniques and Applications*, second ed. Springer, New York.
- Duffie, D., Gârleanu, N., 2001. Risk and valuation of collateralized debt obligations. *Financ. Anal. J.* 57 (1), 41–59.
- Errais, E., Giesecke, K., Goldberg, L.R., 2010. Affine point processes and portfolio credit risk. *SIAM J. Financ. Math.* 1, 642–665.
- Feller, W., 1951. Two singular diffusion problems. *Ann. Maths.* 54 (1), 173–182.
- Hawkes, A.G., 1971. Spectra of some self-exciting and mutually exciting point processes. *Biometrika* 58 (1), 83–90.
- Hawkes, A.G., Oakes, D., 1974. A cluster process representation of a self-exciting process. *J. Appl. Prob.* 11 (3), 493–503.
- Jang, J., 2007. Jump diffusion processes and their applications in insurance and finance. *Ins. : Maths. Eco.* 41 (1), 62–70.
- Jarrow, R.A., Turnbull, S.M., 1995. Pricing derivatives on financial securities subject to credit risk. *J. Financ.* 50 (1), 53–85.
- Longstaff, F.A., Rajan, A., 2008. An empirical analysis of the pricing of collateralized debt obligations. *J. Financ.* 63 (2), 529–563.
- Pandey, S., Selvamuthu, D., 2025. Limit theorems for Cox-Ingersoll-Ross process with externally and self-exciting jumps and application to finance. *Stoch. Dyn.* 25 (3 & 4), 2550017.
- Pandey, S., Selvamuthu, D., Tardelli, P., 2025. Asymptotic results for dynamic contagion processes with different exciting functions and application to risk models. *J. Math. Anal. Appl.* 547, 129392.
- Pasricha, P., Selvamuthu, D., 2019. A Markov modulated dynamic contagion process with application to credit risk. *J. Stat. Phys.* 175, 495–511.
- Selvamuthu, D., Pandey, S., Tardelli, P., 2023. Limit theorems for an extended inverse Hawkes process with general exciting functions. *Stat. Prob. Lett.* 197, 109817.
- Yao, N., Xiao, M., 2018. Limit theorems for non-Markovian marked dynamic contagion processes. *J. Math. Anal. Appl.* 464 (1), 693–706.
- Zhu, L., 2013. Central limit theorem for nonlinear Hawkes processes. *J. Appl. Prob.* 50 (3), 760–771.
- Zhu, L., 2014a. Limit theorems for a Cox-Ingersoll-Ross process with Hawkes jumps. *J. Appl. Prob.* 51 (3), 699–712.
- Zhu, L., 2014b. Process-level large deviations for nonlinear Hawkes point processes. *Ann. Inst. H. Poincaré Prob. Stat.* 50 (3), 845–871.
- Zhu, L., 2015. Large deviations for Markovian nonlinear Hawkes processes. *Ann. Appl. Prob.* 25 (2), 548–581.