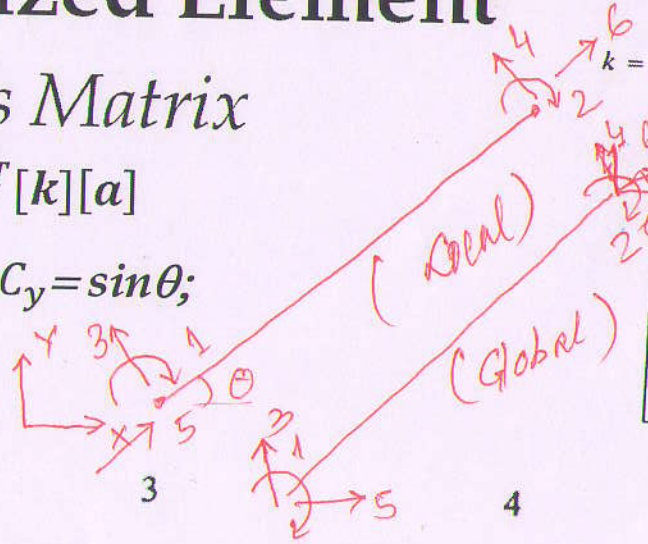
Frame Member

- **Generalized Element**
  - *Stiffness Matrix*

$$[\bar{k}] = [a]^T[k][a]$$

$$C_x = \cos\theta\,;\; C_y = \sin\theta;$$



$$k = \begin{array}{c} \\ \frac{4EI}{L} \\ \frac{2EI}{L} \\ \frac{6EI}{L^2} \\ \frac{6EI}{L^2} \\ 0 \\ 0 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{6EI}{L^2} & 0 & 0 \\ \frac{2EI}{L} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{6EI}{L^2} & 0 & 0 \\ \frac{6EI}{L^2} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{12EI}{L^3} & 0 & 0 \\ \frac{6EI}{L^2} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{12EI}{L^3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{AE}{L} & -\frac{AE}{L} \\ 0 & 0 & 0 & 0 & -\frac{AE}{L} & \frac{AE}{L} \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

$$\bar{k} = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \frac{4EI}{L} & & & & & \\ \frac{2EI}{L} & \frac{4EI}{L} & & & & \\ -\frac{6EI}{L^2}C_x & -\frac{6EI}{L^2}C_x & \frac{12EI}{L^3}C_x^2 + \frac{AE}{L}C_y^2 & & \text{sym.} & \\ \frac{6EI}{L^2}C_x & \frac{6EI}{L^2}C_x & -\frac{12EI}{L^3}C_x^2 - \frac{AE}{L}C_y^2 & \frac{12EI}{L^3}C_x^2 + \frac{AE}{L}C_y^2 & & \\ \frac{6EI}{L^2}C_y & \frac{6EI}{L^2}C_y & \left(-\frac{12EI}{L^3} + \frac{AE}{L}\right)C_xC_y & \left(\frac{12EI}{L^3} - \frac{AE}{L}\right)C_xC_y & \frac{12EI}{L^3}C_y^2 + \frac{AE}{L}C_x^2 & \\ -\frac{6EI}{L^2}C_y & -\frac{6EI}{L^2}C_y & \left(\frac{12EI}{L^3} - \frac{AE}{L}\right)C_xC_y & \left(-\frac{12EI}{L^3} + \frac{AE}{L}\right)C_xC_y & -\frac{12EI}{L^3}C_y^2 - \frac{AE}{L}C_x^2 & \frac{12EI}{L^3}C_y^2 + \frac{AE}{L}C_x^2 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$