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Load and Resistance Factor Design for Steel[a]

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Introduction

This paper will outline a research project that had as its aim the development of design criteria for steel building structures. These criteria were to be in a Load and Resistance Factor Design (LRFD) format rather than in the customary Allowable Stress form, and the load and resistance factors were to be determined through the use of first-order probabilistic principles.

The load and resistance factor design criterion is expressed by the following general formula:

$$\phi R_n \geq \sum_{k=1}^{j} \gamma_k Q_{km} \qquad (1)$$

The left side of the formula relates to the resistance (capacity) of the structure while the right side characterizes the loading acting on it.

The resistance side of the design criterion consists of the product ϕR_n, in which R_n = the "nominal resistance," and ϕ = the "resistance factor." The nominal resistance is the resistance computed according to a formula in a structural code and it is based on nominal material and cross-sectional properties. The resistance factor ϕ, which is always less than unity, together with R_n reflects the uncertainties associated with R. The factor ϕ is dimensionless and R_n is a generalized force: bending moment, axial force, or shear force associated with a limit state of strength or serviceability. Interaction equations, e.g., between axial force and bending, may also be used to define R_n for appropriate limit states.

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The loading side of the design criterion expressed by Eq. 1 is the sum of products γQ_m, in which Q_m = the "mean load effect," and γ = the corresponding "load factor." Here γ is dimensionless and Q_m is a generalized force (i.e., bending moment, axial force or shear force) computed for the mean loads for which the structure is to be designed. The γ-factors reflect potential overloads and the uncertainties inherent in the calculation of the load effects. The summation sign in Eq. 1 denotes the combination of load effects from different load sources. For example, if only dead load and live load effects are considered

$$\sum_{k=1}^{j} \gamma_k Q_{km} = \gamma_D Q_{Dm} + \gamma_L Q_{Lm} \quad \text{(2)}$$

in which Q_{Dm} and Q_{Lm} are the mean dead and live load effects, respectively; and γ_D and γ_L are the corresponding load factors.

In the Load and Resistance Factor Design, one expression of the type given in Eq. 1 is utilized for each set of load combinations that need to be considered. The nominal resistance always relates to a specific "limit state." Two classes of limit states are pertinent to structural design: the "maximum strength" (or "ultimate") limit state, and the "serviceability" limit state. Violation of a strength limit state implies "failure" in the sense that a clearly defined limit of structural usefulness has been exceeded, but this does not necessarily involve actual collapse. In the case of a structural system with "compact" beams this means that a plastic mechanism has formed. Other strength limit states for steel structures are: frame instability, lateral-torsional or local instability, incremental collapse, etc. Serviceability limit states include excessive deflection, excessive vibration, and premature yielding or slip.

The objective of this project was to develop the design criteria shown in Eq. 1 for different structural steel elements under a number of load combinations in a consistent way, taking into account the inherent uncertainties of the resistances and load effects. A "first-order" probabilistic design procedure (2,5,20,31) was used to determine the values of ϕ, R_n, γ, and Q_m. This is a simplified method that uses only two statistical parameters, i.e., mean values and coefficients of variation of the relevant variables and a relationship β between them, called the "safety index."

This paper will give a brief review of the salient features of, and the recommendations resulting from, a research project performed at Washington University during the period from 1969 to 1976. The major findings of the work are contained in eight detailed Washington University reports: one report (8) gives the general basis for the design criteria; five reports present the development of criteria for various structural elements, e.g., beams (10), plate-girders (11), connections (13), composite beams (14), and beam-colums (12); one report deals with wind and snow loads (9); and the final report contains the proposed LRFD design criteria together with a detailed commentary (15). The progress of the research was reported from time-to-time in oral presentations at conferences, workshops, and symposia, and a number of papers were published on one or another aspect of the work (16,17,28,29).

The research was sponsored by the American Iron and Steel Institute (AISI), and it was guided by an advisory task force consisting of researchers with backgrounds in probability, loads, steel structures, and of design engineers.

The project received additional input from various specialists (connections, plate-girders, composite beams, code logic, and decision theory) and an early version of the design criteria was tested in two large design offices. The result of the work, a set of design criteria for steel building structures fabricated from hot-rolled steel elements (15) is thus the product of many minds and it represents a creative interaction of diverse specialists. These criteria are at present (1978) under review by the Specification Advisory Committee of the American Institute of Steel Construction with the aim of adopting them as an alternate design procedure for the AISC Specification (33).

The work performed for steel building structures does not stand on its own, but it is interrelated with many similar efforts in other countries and for other materials (1,5,7,31). Probability based LRFD criteria have been adopted in Canada for hot-rolled and cold-formed steel structures (1,32,34), the basic guide-lines for European national codes have been formulated (7), and research on the development of similar procedures is underway for reinforced concrete and wood structures (29). Experience gained from one effort is transmitted to newer projects, and the concepts of the applications of probability, statistics, optimization, and decision theory have become increasingly more sophisticated (6,18,20, 21,22,25,30). Thus the field of design methodology research is very active and changes occur rapidly. However, in the development of criteria for use in specifications there must come a time when the basis for determination of the elements of the design criteria has to be set, regardless of later more superior formulations. Thus the criteria described in this paper are not now (1978) based on the most advanced methodology. No doubt, newer criteria will be more sophisticated. The point, however, is that the fundamental basis of all such presently contemplated design codes is all the same: first-order probability theory and calibration to existing codes.

Selection of Model

The probabilistic design format used to develop the LRFD criteria for steel structures is due to Cornell (5). This format was selected because of its simplicity and its ability to treat all uncertainties in the design problem in a consistent manner. The format is explained briefly in the following.

Structural safety is a function of the resistance, R, of the structure and of the load effect, Q, acting on it; R and Q are random variables. An example of the definition of safety is given in Fig. 1(a), where the frequency distribution of the random variable of $R - Q$, called the safety margin, is shown and survival is indicated by $R - Q > 0$. The probability of failure p_F of a structural element according to the representation of Fig. 1(a) is equal to

$$p_F = P[(R - Q) < 0] \quad \text{(3)}$$

An equivalent representation of structural safety is shown in Fig. 1(b) where the probability distribution of ln (R/Q) is given. In this case the probability of failure is

$$p_F = P\left[\ln\left(\frac{R}{Q}\right) < 0\right] \quad \text{(4)}$$

The format according to Fig. 1(b) was adopted for developing the LRFD criteria.

If the "standardized variate" U is introduced, in which

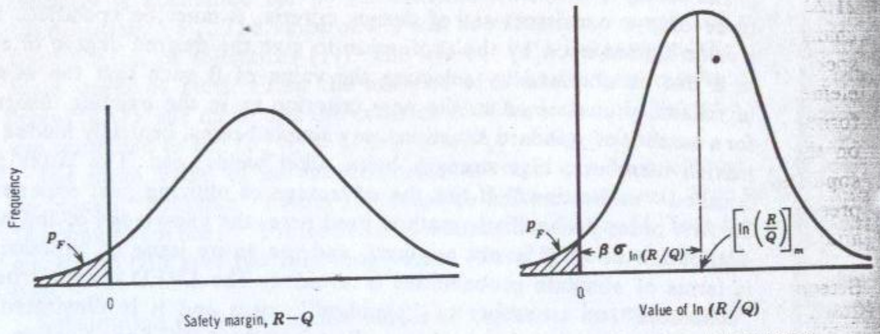
$$U = \frac{\ln\left(\dfrac{R}{Q}\right) - \left[\ln\left(\dfrac{R}{Q}\right)\right]_m}{\sigma_{\ln(R/Q)}} \quad \text{(5)}$$

in which $[\ln(R/Q)]_m$ and $\sigma_{\ln(R/Q)}$ are the mean and standard deviation of the natural logarithm of the ratio (R/Q), then from Eq. 5

$$p_F = P\left[\ln\left(\frac{R}{Q}\right) < 0\right]$$

$$= P\left\{U < -\frac{\left[\ln\left(\dfrac{R}{Q}\right)\right]_m}{\sigma_{\ln(R/Q)}}\right\} = F_U\left\{-\frac{\left[\ln\left(\dfrac{R}{Q}\right)\right]_m}{\sigma_{\ln(R/Q)}}\right\} \quad \text{(6a)}$$

Here F_U is the cumulative distribution function of the standardized variate



(a) PROBABILISTIC MODEL

(b) DEFINITION OF SAFETY INDEX

FIG. 1.—Definitions of Structural Safety

U. The quantity $[\ln(R/Q)]_m/\sigma_{\ln(R/Q)}$ defines the reliability of the element; thus it is called the "safety index" β. If the probability distribution of (R/Q) were known, β would directly indicate a value of the probability of failure. In practice, the probability distribution of R/Q is unknown and only the first two statistical moments of R and Q are estimated. In the first-order probabilistic design method used here, β is only a relative measure of reliability; a constant value of β effectively fixes the reliability as a constant for all similar structural elements.

The expression for the safety index β, from Eq. 6*a*, i.e.,

$$\beta = \frac{\left[\ln\left(\dfrac{R}{Q}\right)\right]_m}{\sigma_{\ln(R/Q)}} \quad \text{(6b)}$$

can be simplified by using first-order probability theory into:

$$\beta \approx \frac{\ln\left(\dfrac{R_m}{Q_m}\right)}{\sqrt{V_r^2 + V_Q^2}} \quad \text{(7)}$$

in which R_m and Q_m are the mean values of the resistance and the load effect, and V_R and V_Q are the corresponding coefficients of variation. Eq. 7 can be rearranged and expressed as a first-order probabilistic design criterion:

$$R_m \geq \theta\, Q_m \quad \text{(8)}$$

in which θ, the central safety factor, is given by

$$\theta = \exp\left(\beta\sqrt{V_R^2 + V_Q^2}\right) \quad \text{(9)}$$

It can be seen that the central safety factor combines the uncertainties of both the resistance and the load effects. It would be advantageous if the central safety factor θ could be split into factors so that resistance factors for different structural elements (e.g., beams, columns, and fasteners) could be determined independently of the loading uncertainties, once a value of the safety index is chosen, and that the load factors on different types of loads could be evaluated independently of each other and of the type of structural element. Using the linear approximation to the square root term proposed by Lind (20), the design criterion (Eq. 8) may be written as:

$$\exp(-\bar{\alpha}\beta V_R)\, R_m \geq \exp(\bar{\alpha}\beta V_Q)\, Q_m \quad \text{(10)}$$

Here $\bar{\alpha}$ is a numerical constant equal to 0.75. The right-hand side can be further separated to allow an independent treatment of the effects of the different types of loads, as shown subsequently. In what follows, an examination of the resistance and the load effect models used in LRFD is given.

Resistance

The randomness in the resistance R of a structural element arises because of the variabilities inherent in the mechanical properties of the materials, of variations in dimensions (tolerances), and because of the uncertainties in the theory underlying the design definition of member strength. The member resistance R often takes the following product form:

$$R = R_n\, M\, F\, P \quad \text{(11)}$$

in which R_n = the nominal code specified resistance, and R, M, F, and P are random variables. The dimensions of R_n are limit state moments, axial forces or shears and M, F, and P are thus nondimensional. The product form of Eq. 11 was chosen for illustration because many relationships in steel design are of this form. It will be assumed that the random variables M, F, and P are uncorrelated; the coefficient of variation of the resistance, V_R, is written approximately as:

$$V_R \approx \sqrt{V_M^2 + V_F^2 + V_P^2} \quad \text{(12)}$$

in which V_M, V_F, and V_P are the coefficients of variation of M, F, and P, respectively.

The random variable M represents the variation in material strength or stiffness. The statistical parameters, M and V_M, may be obtained by routine tests. The random variable F characterizes the uncertainties in "fabrication." The term "fabrication" includes the variations in geometrical properties introduced by rolling, fabrication tolerances, welding tolerances, initial distortions, erection variations, and the like. The variations are the differences between the ideal designed member and the member in the structure after erection.

The random variable P, called the "professional" factor, reflects the uncertainties of the assumptions used in determining the resistance from "design" models. These uncertainties could be the result of using approximations for theoretically exact formulas, or of the assumptions such as perfect elasticity, perfect plasticity, homogeneity, or "beam" theory instead of the "theory of elasticity." Enough data are not yet available to assess the parameters P_m and V_P in all situations but, where available, comparisons between "design" predictions and test results, or between "design" predictions and approximate or exact theoretical formulas could be used to estimate them. Future research is expected to provide the data to evaluate P_m and V_P in these cases where current information is sparse or absent.

Load Effects

In the following the load effects that refer to strength limit state are examined. It is assumed here that only dead and live load effects are present. Other load combinations are examined later.

The load effect Q for combined dead and live gravity loading will be assumed to have the form:

$$Q = E(c_D A\,D + c_L B\,L) \quad \ldots \quad (13)$$

The terms in Eq. 13 are defined as follows: D and L = random variables representing dead and live load intensities, respectively; c_D and c_L = deterministic influence coefficients that transform the load intensities to load effects (e.g., moment, shear, and axial force); A and B = random variables reflecting the uncertainties in the transformation of loads into load effects; and E = a random variable representing the uncertainties in structural analysis. The corresponding mean values are D_m, L_m, A_m, B_m, and E_m and the coefficients of variation are V_D, V_L, V_A, V_B, and V_E, respectively.

Dead load is assumed to be constant throughout the life of a structure, but it varies somewhat from structure to structure in a population of identical structures. Live load varies randomly from structure to structure in a class of identical structures, and it varies randomly with time in a particular structure. From the combination of dead load and live load under study, information on the maximum live load that occurs in the lifetime of a structure is required. The random variables D and L include the uncertainties in idealizing the loads, which vary randomly in space and time, by equivalent uniformly distributed or concentrated design loads. Common examples are: (1) Live loads are randomly distributed over a floor area but are idealized as uniformly distributed loads; and (2) localized loads are idealized as concentrated loads. The random variables A and B account for the uncertainties in the transformation of idealized design loads into load effects such as moments, shears, and axial forces. Their variation

characterizes the differences between the actual and computed internal forces in the structure.

The variable E includes the uncertainties in modelling a three-dimensional real structure of complex geometry and behavior into a set of members and connections of fixed geometry and stipulated behavior. It also accounts for the uncertainties induced by approximate or simplified structural analyses in lieu of complicated and refined theories (e.g., assumption of inflexion points, spring-mass systems in vibration analysis).

The mean and the coefficient of variation of Q are derived as:

$$Q_m = c_D A_m D_m + c_L B_m L_m \quad \ldots \quad (14)$$

and $$V_Q \simeq \sqrt{V_E^2 + \frac{c_D^2 A_m^2 D_m^2 (V_A^2 + V_D^2) + c_L^2 B_m^2 L_m^2 (V_B^2 + V_L^2)}{(c_D A_m D_m + c_L B_m L_m)^2}} \quad \ldots \quad (15)$$

The mean value of E is assumed to be unity in Eqs. 14 and 15.

Calibration

The safety index, β, is a relative measure of the structural safety. In order to develop a consistent set of design criteria, β must be specified. It can be a value agreed upon by the profession to give the desired degree of reliability, or it can be obtained by selecting the value of β such that the same degree of reliability is attained in the new criterion as in the existing design method for a number of standard situations, say simple beams, centrally loaded columns, tension members, high-strength bolts, fillet welds, etc. The latter procedure is called "calibration." It has the advantage of utilizing past experience; with the first order probabilistic method used here, the knowledge of the probability distribution of R/Q is not required, and the entire issue of expressing safety in terms of absolute probabilities is avoided. The LRFD design criterion was "calibrated" to a number of "standard" cases and it is illustrated here for simple beams designed according to Part 2 of the AISC Specification.

A simply supported beam under uniformly distributed dead and live loads, adequately braced and "compact," will require a plastic modulus Z given by

$$Z = \frac{1.7\,[c_D D_c + c_L L_c(1-\rho)]}{F_y} \quad \ldots \quad (16)$$

in which F_y = the specified minimum yield stress of the grade of steel used, c_D and c_L are influence coefficients equal to $sl^2/8$ (in which s = the beam spacing, and l = the span). The code specified dead load is D_c; L_c is the uniformly distributed code live load intensity specified in American National Standards Institute (ANSI) A58.1—1972 for the type of occupancy; and ρ is the ANSI (3) live load reduction given by:

$\rho = 0$ for $A_T \leq 150$ sq ft (14 m²);

$\rho = 0.0008\,A_T$ for $150 \leq A_T \leq 750$ sq ft (70 m²);

$$\rho = 0.60 \text{ for } A_T > 750 \text{ sq ft (70 m}^2\text{); or } \rho = 0.23\left(1 + \frac{D_c}{L_c}\right) \quad \ldots \quad (17)$$

whichever is smaller. The tributary area is A_T.

A value of β can be obtained by selecting a "standard" design situation and requiring that the LRFD criterion produce the same design (i.e., the same plastic section modulus) as the AISC specification. Instead of limiting the calibration to one preselected data point, a complete spectrum of design situations, characterized by different tributary areas and dead loads, is studied here in order to select a representative value of the safety index.

The safety index β is computed from Eq. 7 for the value of the plastic modulus Z, which is required by the AISC specification. The various components of Eq. 7 will now be determined.

The mean resistance R_m is equal to

$$R_m = Z_m (F_{sy})_m \left(\frac{\text{Test}}{\text{Prediction}}\right)_m \quad (18)$$

It will be assumed that the mean plastic modulus Z_m is equal to its nominal (handbook) value (i.e., $F_m = 1.0$ in Eq. 10) and furthermore, calibration is achieved by using Z from Eq. 16, the AISC value. The variability of the section modulus is accounted for by an estimate of the coefficient of variation of "fabrication," V_F. The value of V_F will be assumed to equal 0.05 representing good control of tolerances (19). The symbol $(F_y)_m$ represents the mean static yield stress of steel. From the analysis of available test results on the yield stress (8,17), the mean and the coefficient of variation of the static yield stress are estimated to be: $(F_{sy})_m = 1.05\, F_y$ and $V_{F_{sy}} = V_M = 0.10$.

The mean value of the test-to-prediction ratio can be estimated from tests on braced "compact" beams subjected to uniform moment. This term in Eq. 18 accounts for the P ("Professional") contribution to Eq. 11. Test data presented elsewhere (10) give the following values (Test/Prediction)$_m$ = 1.02 and V_P = 0.06.

Substitution of the estimated values of the mean quantities given previously into Eq. 18 and using Z from Eq. 16, results in

$$R_m = 1.82\,[c_D D_C + c_L L_c (1 - \rho)] \quad (19)$$

The coefficient of variation of the resistance, V_R is calculated to be 0.13 (Eq. 12).

In order to calculate the values of Q_m and V_Q, the components of Eqs. 14 and 15 were studied. The following values were chosen (8) to represent reasonable estimates on the basis of data or judgement: $E_m = 1.0$; $V_E = 0.05$; $A_m = 1.0$; $V_A = 0.04$; $B_m = 1.0$; $V_B = 0.20$ (24); $D_m = D_c$; and $V_D = 0.04$ (19,30).

The mean L_m and the standard deviation σ_L of the lifetime maximum live load for office occupancy (expressed in units of psf) are given by McGuire and Cornell (23) as:

$$L_m = 14.9 + \frac{763}{\sqrt{A_I}} \quad (20)$$

$$\sigma_L = \sqrt{11.3 + \frac{15{,}000}{A_I}} \quad (21)$$

in which A_I = the "influence area" equal to twice the tributary area A_T for floor beams. The value of $V_L = \sigma_L / L_m$ is calculated by substituting the preceding values in Eq. 15.

The determination of β is accomplished by knowing R_m, Q_m, V_R, and V_Q and using Eq. 7. The values of L_m and $V_L = \sigma_L/L_m$ are taken from Eqs. 20 and 21; L_c, the specified minimum uniformly distributed live load for office type occupancy is, according to ANSI A58.1—1972 (3), equal to 50 psf (2394 N/m^2). Thus the variables affecting β, according to the preceding estimated

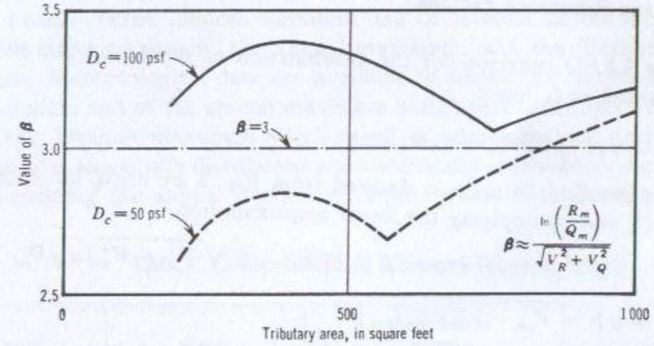


FIG. 2.—Variations of Safety Index for Simple Steel Beams Implied in Part 2, AISC Specification

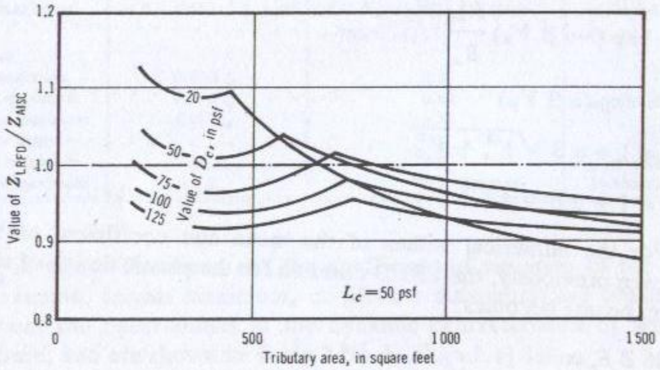


FIG. 3.—Comparison of AISC and LRFD Designs for Beams

set of data are D_c and A_T, the specified dead load intensity and the tributary area. The variation of β for different values of D_c and A_T is shown in Fig. 2. It was concluded from this study that β = 3.0 is a representative value for the safety index implied in the design of simply supported beams using Part 2 of the 1969 AISC Specification. Similar results were obtained from calibrating simple beams to Part 1 of the AISC Specification. Calibrations were also performed for centrally loaded columns and for high strength bolts and fillet welds. The results of these calibrations are presented elsewhere (8,13,16). The conclusion of the calibration exercise was that β = 3.0 and β = 4.5 was selected for members and for connectors, respectively, as the basis for developing load factors and resistance factors. The higher value of β for connectors reflects the fact that traditionally connections are designed stronger than the elements that are connected by them. The chosen β values represent, then, the link between the present code and the new code. Furthermore, a uniform value of β over the whole parameter domain will provide a more consistent reliability for the new criteria.

Load and Resistance Factors

The LRFD criterion for the combination of dead load and live load can be expressed as follows:

$$\phi R_n \geq \gamma_E(\gamma_D c_D D_m + \gamma_L c_L L_m) \quad \ldots \quad (22)$$

The preceding formula is derived from Eq. 8 by using the expressions for R and Q, and by applying the linear approximation:

$$\exp(-\alpha\beta V_R) R_m \geq \exp(\alpha\beta V_E)[(1 + \alpha\beta\sqrt{V_A^2 + V_D^2}) c_D D_m$$
$$+ (1 + \alpha\beta\sqrt{V_B^2 + V_L^2}) c_L L_m] \quad \ldots \quad (23)$$

in which $\alpha = 0.55$ gives a good approximation between Eqs. 23 and 8 (8) as determined by an error minimization process. Therefore, the resistance and load factors are given by:

$$\phi = \exp(-\alpha\beta V_R)\frac{R_m}{R_n} \quad \ldots \quad (24)$$

$$\gamma_E = \exp(\alpha\beta V_E) \quad \ldots \quad (25)$$

$$\gamma_D = 1 + \alpha\beta\sqrt{V_A^2 + V_D^2} \quad \ldots \quad (26)$$

$$\gamma_L = 1 + \alpha\beta\sqrt{V_B^2 + V_L^2} \quad \ldots \quad (27)$$

For the numerical values of the mean and coefficient of variation and β chosen previously, the LRFD criterion for the plastic design of simply supported steel beams becomes:

$$0.86\, Z F_y \geq 1.1\,[1.1\, c_D D_m + 1.4\, c_L L_m] \quad \ldots \quad (28)$$

in which $c_D = c_L = sl^2/8$; $D_m = d_c$; and L_m is given by Eq. 20. Eq. 28 is seen from Fig. 3 to calibrate reasonably well with the design criterion according to Part 2 of the AISC Specification. The ratios of the plastic moduli according to Eqs. 28 and 16 are plotted in Fig. 3 for various values of A_T and D_c. The LRFD format will generally require 0 to 10% smaller plastic moduli over most of the range of tributary areas and dead load intensities considered, except that larger beams are required for small dead loads and small tributary areas.

Resistance Factors for Different Structural Elements

The calculation of the resistance factor for a structural element is made in the following steps:

1. Selection of a formula or an algorithm for the nominal resistance of the element.
2. Calculation of the mean and the coefficient of variation of the resistance of the element using the relationship of step 1 and the available information on the mechanical properties of the material and test results on the element.
3. The resistance factor ϕ is calculated by

$$\phi = \exp(-\alpha\beta V_R)\frac{R_m}{R_n} \quad \ldots \quad (24)$$

4. In most cases the resistance of the structural element is expressed as a function of a characteristic variable, e.g., the slenderness parameter for columns. The resistance factor ϕ may also vary over the range of this variable.

A considerable part of the effort in the LRFD research project was devoted to the development of the ϕR_n term for the various structural elements that are included in a structural steel specification: tension members, compression members, beams and girders (including the limit states of plastic moment, lateral-torsional buckling, local buckling, shear, for rolled beams, plate-girders, hybrid beams and girders, composite beams), beam-columns, and various types of connections and connectors (8,10,11,12,13,14). Following are some representative samples of the results (15):

Tension Members (except eyebars).—Limit State: yielding

$$R_n = A_n F_y; \quad \phi = 0.88 \quad \ldots \quad (25)$$

Limit State: fracture

$$R_n = A_n F_u; \quad \phi = 0.74 \quad \ldots \quad (26)$$

in which A_n = the net area, and F_y and F_u = the specified mimimum yield stress and tensile stress, respectively.

Columns.—The following pertain to columns

$$R_n = A_g F_{cr} \quad \ldots \quad (27)$$

$$\phi = 0.86 \quad \text{for} \quad \lambda \leq 0.16$$
$$\phi = 0.90 - 0.25\lambda \quad \text{for} \quad 0.16 \leq \lambda \geq 1.0$$
$$\phi = 0.65 \quad \text{for} \quad \lambda \geq 1.0 \quad \ldots \quad (28)$$

in which A_g = the gross area and

$$F_{cr} = F_y(1 - 0.25\lambda^2) \quad \text{for} \quad \lambda \leq \sqrt{2}$$

$$F_{cr} = \frac{F_y}{\lambda^2} \quad \text{for} \quad \lambda \geq \sqrt{2} \quad \ldots \quad (29)$$

$$\lambda = \frac{KL}{r}\frac{1}{\pi}\sqrt{\frac{F_y}{E_s}} \quad \ldots \quad (30)$$

In Eq. 30, KL/r = the effective slenderness ratio, and E_s = the nodulus of elasticity.

The proposed LRFD criteria (15) contain all of the nominal resistance formulas and the corresponding ϕ-factors, arranged according to the logical scheme developed by Fenves and his associates (26) for efficient use in design specifications. Details of the derivations and the basic research data are given in the source documents, and these will be published in subsequent papers.

Load Combinations

The previous consideration was concerned with load effects due to live and dead load. The combination of other load effects can be considered by the same type of reasoning. For example, wind load effects could be included in the design criterion by adding its effect to the dead and live load effects, i.e.,

$$\phi R_n \geq \gamma_E(\gamma_D c_D D_m + \gamma_L c_L L_m + \gamma_W c_W W_m) \quad \text{. . . . . (31)}$$

in which γ_W = the load factor for wind and W_m = the mean wind load. The load factor γ_W can be shown to be (34)

$$\gamma_W = 1 + \alpha\beta\sqrt{V_C^2 + V_W^2} \quad \text{. . . . . (32)}$$

in which W = a random variable representing the wind load with all its environmental variations (W_m = the mean wind load intensity; and V_W = its coefficient of variation); and C = a factor reflecting the uncertainties in the process of translating the wind load intensity into load effects.

In studying the topic of load combinations, it is important to combine the different load effects properly to achieve a consistent level of reliability (8). Most load effects are random functions of time. When design is for resisting gravity loads, a possible load combination is the dead load (assumed to be constant in time) and the maximum live load in the lifetime of the structure. Load factors for this specific combination have been derived in the previous sections. Design against wind load combined with live load needs a different kind of thinking. It is unlikely that the lifetime maximum wind load will occur simultaneously with lifetime maximum live load. Furthermore, the time fluctuation of these two random functions exhibit different characteristics—rapid for wind and relatively slow for live loads. Assuming the two effects are uncorrelated, it is practical to add the lifetime maximum wind load effect to the sustained live load effect. Thus, the following are some of the important load combinations to be studied: (1) Dead load + lifetime maximum live load; (2) dead load + sustained live load + lifetime maximum wind load; (3) dead load + lifetime maximum live load + daily maximum wind load; (4) lifetime maximum wind load − dead load; and (5) dead load + lifetime maximum snow load.

For serviceability requirements such as drift and acceleration, the annual maximum wind load may have to be combined with the live load and dead load.

Different types of loads are included in the above list of load combinations. An examination of these loads follows.

Live Load.—Statistical information on live loads is obtained usually from live load surveys that give the live loads in the particular buildings surveyed at the time the surveys were made. From the load combination enumerated earlier, it is seen that the distribution of the lifetime maximum live load is also needed. Peir and Cornell (27) have modelled the live load as consisting of the superposition of two parts: The *sustained* live load, which remains on

the floor for a relatively long time until an occupancy change occurs, and the *transient* live load, which occurs infrequently but with a relatively high intensity and short duration. The sustained load includes furniture and normal working personnel. The transient or extraordinary live load may be caused by people crowding into a room. Peir has proposed models to derive the statistics of the lifetime maximum sustained load and of the extraordinary live load. Using the live load models pf Peir and the live load survey data of Mitchell and Woodgate (24), McGuire and Cornell (23) have derived the statistics of lifetime maximum live load, as shown in Table 1.

Wind Load.—Three random variables are of interest in the case of wind loads: The daily maximum, the annual maximum, and the lifetime maximum wind loads. Meteorological data are available to derive the distributions of the daily maximum and of the annual maximum wind speeds throughout the United States. The lifetime maximum wind speed is approximately derived as the maximum of n-identically distributed and statistically independent random variables representing the annual maximum values, where n is the lifetime of the

TABLE 1.—Statistics of Different Types of Loads

Load type (1)	Mean (2)	Coefficient of variation (3)	Comments (4)
Live load (office)			
Instantaneous	12 psf	$0.82[1 - 0.00113(A_T - 56)]$	$56 \leq A_T \leq 336$ ft
sustained		$0.56[1 - 0.0001865(A_T - 336)]$	$A_T > 336$ sq ft
Lifetime maximum	$14.9 + (763/\sqrt{A_I})$ psf	$\sqrt{11.3 + 15{,}000/A_I}/(14.9 + 763/\sqrt{A_I})$	$A_I = 2A_T$ for beam $A_I = 4A_T$ for column
Wind Load			
Daily maximum	$0.07X_{50}$	0.80	
Annual maximum	$0.49X_{50}$	0.60	X_{50} is the 50 yr wind load
Lifetime maximum	$1.17X_{50}$	0.27	
Roof snow load			
Annual maximum	X_m	0.8	X_m is calculated using
Lifetime maximum	$3.3X_m$	0.3 (typical)	Meteorological data

structure in years. The mean and the coefficient of variation of the wind load (daily maximum, annual maximum, or lifetime maximum) are obtained taking into account the uncertainties in the dynamic characteristics of wind and of the structure, and are shown in Table 2 (8).

Snow Load.—Meteorological data available in the form of annual maximum ground snow throughout the United States (35) are used to derive the statistics of the lifetime maximum ground snow load. The statistics of the roof snow load shown in Table 1 were calculated taking into consideration the variation between the roof and ground snow and other factors such as shape of the roof, wind direction, wind speeds, and temperature (9).

Load Factors.—The load factors on different loads in the load combinations enumerated previously are calculated using Eq. 1: the tentatively proposed LFRD criteria are (8,15):

Dead Load + Lifetime Maximum Live Load:

$$\phi R_n \geq 1.1\,[1.1\,c_D D_m + 1.4\,c_L L_m] \quad \text{. . . . . (33)}$$

Dead Load + Sustained Live + Lifetime Maximum Wind Load:

$$\phi R_n \geq 1.1\ [1.1\ c_D D_m + 2.0\ c_L L_{sm} + 1.6\ c_W W_m] \quad (34)$$

Lifetime Maximum Wind Load − Dead Load:

$$\phi R_n \geq 1.1\ [1.6\ c_W W_m - 0.9\ c_D D_m] \quad (35)$$

Dead Load + Sustained Live Load + Lifetime Maximum Snow Load:

$$\phi R_n \geq 1.1\ [1.1\ c_D D_m + 1.5\ c_S S_m] \quad (36)$$

Details of the derivations and data for these and other load factors are given in the source documents (9,10,11,12,13,14), and they are tabulated in the proposed LRFD criteria (15). Furthermore the Commentary contains suggestions for developing load factors for cases of loading that are unusual (15). The left sides of these formulas are calculated as explained in the preceding section.

Results

The central objective of this research was to develop design criteria that are consistent for different structural elements, and for different loads and load combinations, which are based on the information available on material strengths and loads, and which use probabilistic design concepts. In the following, some of the assumptions are examined and extensions and modifications are suggested.

Variation of Safety Index.—The LRFD formulation given here is preferable to those used in current design practice, because the uncertainties in design variables are included explicitly in the design criteria through the use of the coefficients of variation. Different designs will have approximately consistent reliabilities reflected in the choice of safety indices. In this project, the resistance and load factors were evaluated for a particular set of coefficients of variation and safety indices judged to represent current design practice. A major advantage of the LRFD formulation presented here is that the specification writer is able to select the values of the coefficients of variation and of the safety indices to reflect the particular conditions and location under his jurisdiction. Depending on the uncertainties in the mechanical properties of the materials available, or the quality of workmanship, etc. he may select the means and coefficients of variation other than those used in this project.

The specification writer can also influence the design criteria by changing the value of β. In this project, it was demonstrated that $\beta = 3.0$ represented approximately the reliability of presently designed simple beams and axially loaded columns. Future considerations may lead to a change, say $\beta > 3.0$, if it turns out that $\beta = 3.0$ gives a smaller reliability than what the profession considers desirable. For the present however, it is recommended that $\beta = 3.0$ be retained for the routine design situations.

The value of the safety index may be varied to account for the importance of the structure. There are some structures in which failure of one or a few critical elements may result in the total loss of the structure ("weakest link" type structures) in contrast to ductile or continuous structures ("parallel" type structures). Optimal levels of reliability for different types of structures could be obtained from an expected total cost optimization process (22). It could

be decided, as was done in the proposed LRFD criteria, that $\beta = 3.0$ in ordinary buildings, $\beta = 4.5$ for very important buildings, and $\beta = 2.5$ for temporary structures.

In this study, the LRFD criteria were obtained through calibration where only the reliabilities of cross sections were considered. It is possible to incorporate the statistical correlation between cross sections and among members and failure modes by suitably varying the value of the safety index β. The proposed LRFD formulation is versatile enough to incorporate these future developments in probabilistic design.

Modern Probabilistic Design Formats.—Probabilistic structural design is, at present, a very dynamic field of research. Recently new design formats that are theoretically exact and that could be simplified into LRFD format have been formulated (6,18). In the present LRFD project, the statistics of the resistances of different structural elements, of loads, and combination have been derived. These may be used as input to any one of the aforementioned design formats to come up with LRFD criteria as demonstrated by Cecen (4).

Internal Consistency in Design Criteria.—The design criteria for different structural elements are developed in such a manner as to avoid discontinuities in design rules for any element and to have continuity between different structural members based on their structural behavior. For example, at the extreme limits of the range of beam-columns, the design criteria should match with those for beams or for columns. A similar situation exists in regard to beams, plate girders, and hybrid girders.

Serviceability Limit States.—LRFD criteria for serviceability limit states such as excessive deflection, drift, or vibration can be also developed using the procedure outlined in this paper (8). Such serviceability criteria were developed for yield under service loads, slip of high-strength bolted joints, floor deflection and story drift, and the methods of handling these problems have been incorporated in the proposed LRFD criteria (15).

Conclusions

Load and Resistance Factor Design criteria for steel buildings are described in this paper. They are based on a first order probabilistic approach and on calibration to existing structures. The design criterion is

$$\phi R_n \geq \gamma_E(\gamma_D c_D D_m + \gamma_L c_L L_m + \gamma_W c_W W_m + \ldots) \quad (37)$$

in which ϕ = the resistance factor; γ_E, γ_D, γ_L, and γ_W are the factors on structural analysis, dead load, live load, and wind load, respectively; R_n = the nominal resistance; and D_m, L_m, and W_m are the mean dead, live, and wind load effects. The terms in Eq. 37 for various structural elements and for different load combinations have been evaluated in this project.

It was demonstrated in this project that practical but consistent rules can be developed in a first-order probabilistic sense; sufficient data are generally available. However, where data are inadequate, professional judgement has to be used but this judgmental process can proceed within a consistent and rational framework. It should be realized that since the work on this research is still in progress the load and resistance factors presented here may not represent final approved values.

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APPENDIX I.—REFERENCES

1. Allen, D. E., "Limit States Design—A Probabilistic Study," *Canadian Journal of Civil Engineering*, Vol. 2, No. 1, Mar., 1975.
2. Ang, A. H.-S., Cornell, C. A., "Reliability Bases of Structural Safety and Design", *Journal of the Structural Division*, ASCE, Vol. 100, No. ST9, Proc. Paper 10777, Sept., 1974, pp. 1755–1769.
3. "Building Code Requirements for Minimum Design Loads in Buildings," American National Standards Institute, ANSI A58.1-1972.
4. Cecen, H., "Reliability of Multi-Bay, Multi-Story Steel Frames," thesis presented to Washington University, at St. Louis, Mo., in 1974, in partial fulfillment of the requirements for the degree of Master of Science.
5. Cornell, C. A., "A Probability-based Structural Code," *Journal*, American Concrete Institute, Vol. 66, No. 12, Dec., 1969.
6. Ditlevsen, O., "Structural Reliability Analysis and the Invariance Problem," *Technical Report No. 22*, Solid Mechanics Division, University of Waterloo, Ontario, Canada, 1973.
7. "First Order Reliability Concepts for Design Codes," Committé Europeén du Beton, Bulletin d'Information No. 112, Munich, West Germany, July, 1976.
8. Galambos, T. V., and Ravindra, M. K., "Tentative Load and Resistance Factor Design Criteria for Steel Buildings," *Research Report No. 18*, Civil Engineering Department, Washington University, St. Louis, Mo., Sept., 1973.
9. Galambos, T. V., and Ravindra, M. K., "Load Factors for Wind and Snow Loads for Use in Load and Resistance Factor Design Criteria," *Research Report No. 34*, Civil Engineering Department, Washington University, St. Louis, Mo., Jan., 1976.
10. Galambos, T. V., and Ravindra, M. K., "Load and Resistance Factor Design Criteria for Steel Beams," *Research Report No. 27*, Civil Engineering Department, Washington University, St. Louis, Mo., Feb., 1976.
11. Galambos, T. V., and Ravindra, M. K., "Tentative Load and Resistance Factor Design Criteria for Steel Plate Girders," *Research Report No. 29*, Civil Engineering Department, Washington University, St. Louis, Mo., Feb., 1976.
12. Galambos, T. V., and Ravindra, M. K., "Tentative Load and Resistance Factor Design Criteria for Steel Beam-Columns," *Research Report No. 32*, Civil Engineering Department, Washington University, St. Louis, Mo., Feb., 1976.
13. Galambos, T. V., and Ravindra, M. K., "Tentative Load and Resistance Factor Design Criteria for Connections," *Research Report No. 33*, Civil Engineering Department, Washington University, St. Louis, Mo., Apr., 1976.
14. Galambos, T. V., and Ravindra, M. K., "Load and Resistance Factor Design Criteria for Composite Beams," *Research Report No. 44*, Civil Engineering Department, Washington University, St. Louis, Mo., Apr., 1976.
15. Galambos, T. V., and Ravindra, M. K., "Proposed Criteria for Load and Resistance Factor Design of Steel Building Structures," *Research Report No. 45*, Civil Engineering Department, Washington University, St. Louis, Mo., May, 1976.

16. Galambos, T. V., and Ravindra, M. K., "The Basis for Load and Resistance Factor Design Criteria for Steel Building Structures," *Canadian Journal of Civil Engineering*, Vol. 4, No. 2, 1977.
17. Galambos, T. V., and Ravindra, M. K., "Properties of Steel for Use in LRFD," *Journal of the Structural Division*, ASCE, Vol. 104, No. ST9, Proc. Paper 14009, Sept., 1978, pp. 1459–1468.
18. Hasofer, A. M., and Lind, N. C., "Exact and Invariant Second-Moment Code Format," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 100, No. EM1, Proc. Paper 10376, Feb., 1974, pp. 111–121.
19. Johnson, A. I., "Strength, Safety and Economical Dimensions of Structures," Royal Institute of Technology, Institute of Byggnadsstatik, Meddelanden, No. 12, Stockholm, 1953.
20. Lind, N. C., "Consistent Partial Safety Factors," *Journal of the Structural Division*, ASCE, Vol. 97, No. ST6, Proc. Paper 8166, June, 1971, pp. 1651–1669.
21. Lind, N. C., "Formulations of Probabilistic Design," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 103, No. EM2, Proc. Paper 12861, Apr., 1977, pp. 273–284.
22. Mau, S., and Sexsmith, R. G., "Optimum Design of Structures with a Minimum Cost Criterion," *Proceedings*, Third Canadian Congress of Applied Mechanics, Calgary, Canada, May, 1971.
23. McGuire, R. K. and Cornell, C. A., "Live Load Effects in Office Buildings," *Journal of the Structural Division*, ASCE, Vol. 100, No. ST7, Proc. Paper 10660, July, 1974, pp. 1351–1366.
24. Mitchell, G. R., and Woodgate, R. W., "A Survey of Floor Loadings in Office Buildings," *Report No. 25*, Construction Industry Research and Association, London, England, Aug., 1970.
25. Moses, F., "Reliability of Structural Systems," *Journal of the Structural Division*, ASCE, Vol. 100, No. ST9, Proc. Paper 10780, Sept., 1974, pp. 1813–1820.
26. Nyman, D. J., Mozer, J. D., and Fenves, S. J., "Decision Table Formulation of the Load and Resistance Factor Design Criteria," Department of Civil Engineering, Carnegie-Mellon University, Pittsburgh, Pa., Apr., 1977.
27. Peir, J-C. and Cornell, C. A., "Spatial and Temporal Variability of Live Loads," *Journal of the Structural Division*, ASCE, Vol. 99, No. ST5, Proc. Paper 9747, May, 1973, pp. 903–922.
28. Ravindra, M. K., and Galambos, T. V., "Load Factor Design of Steel Beams," presented at the November 2–3, 1974, ASCE Specialty Conference on Safety and Reliability of Metal Structures, held at Pittsburgh, Pa.
29. Ravindra, M. K., Lind, N. C., Siu, W. W. C., "Illustrations of Reliability-Based Design," *Journal of the Structural Division*, ASCE, Vol. 100, No. ST9, Proc. Paper 10779, Sept., 1974, pp. 1789–1811.
30. Rosenblueth, E., "Applications of Decision Theory," *Proceedings*, VII International Conference on Soil Mechanics and Foundation Engineering, Mexico, D.F., 1969.
31. Rosenblueth, E., and Esteva, L., "Reliability Basis for Some Mexican Codes," *Probabilistic Design of Reinforced Concrete Buildings*, Publication SP-31, American Concrete Institute, Detroit, Mich., 1972, pp. 1–42.
32. Schuster, R. M., "Cold Formed Steel Design Manual," University of Waterloo Press, Waterloo, Canada, 1975.
33. "Specification for the Design Fabrication and Erection of Structural Steel for Buildings," American Institute of Steel Construction, New York, N.Y., 1977.
34. "Steel Structures for Buildings—Limit States Design," CSA Standard S16.1-1974.
35. Thom, H. C. S., "Distribution of Maximum Annual Water Equivalent of Snow on the Ground," *Monthly Weather Review*, Vol. 94, No. 4, Apr., 1966.