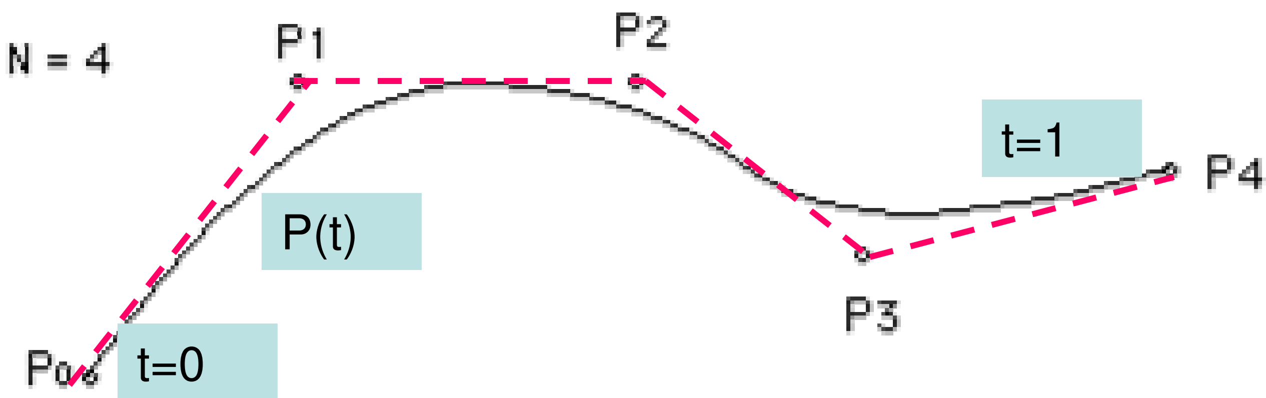
Bezier Surfaces

- Recall how we constructed a Bezier Curve

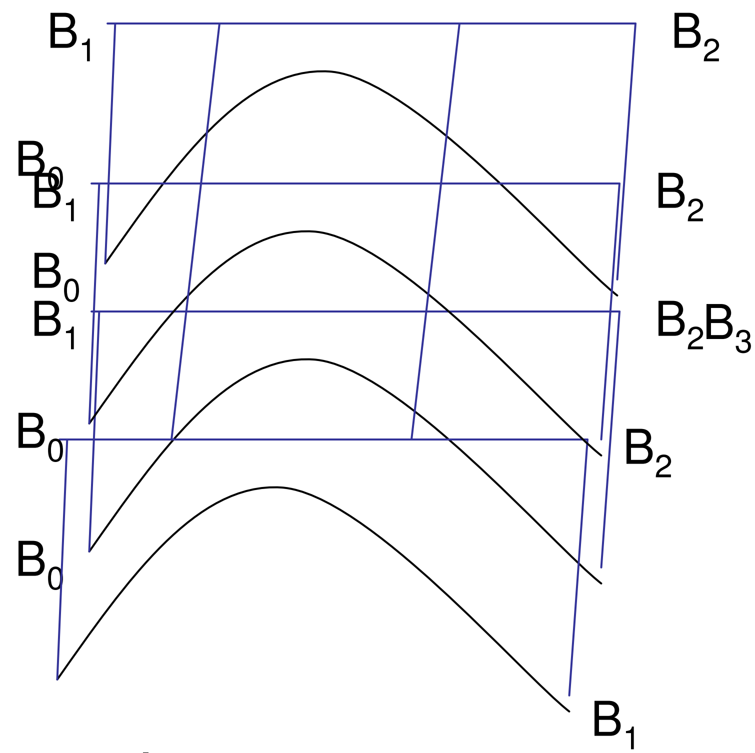
P(t) is a continuous function in 3 space defining the curve with N discrete control points B_i. t=0 at the first control point (i=0) and t=1 at the last control point (i=N).



Bezier Surface

As in the case of a Bezier curve, it is required to relax strict requirements of cubic spline for a flexible design. A Bezier surface provides this flexibility

The Cartesian or tensor product Bezier surface is represented as:



$$Q(u,w) = \sum_{i=0}^{n} \sum_{j=0}^{m} B_{i,j} J_{n,i}(u) K_{m,j}(w)$$

$$J_{n,i}(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i}$$

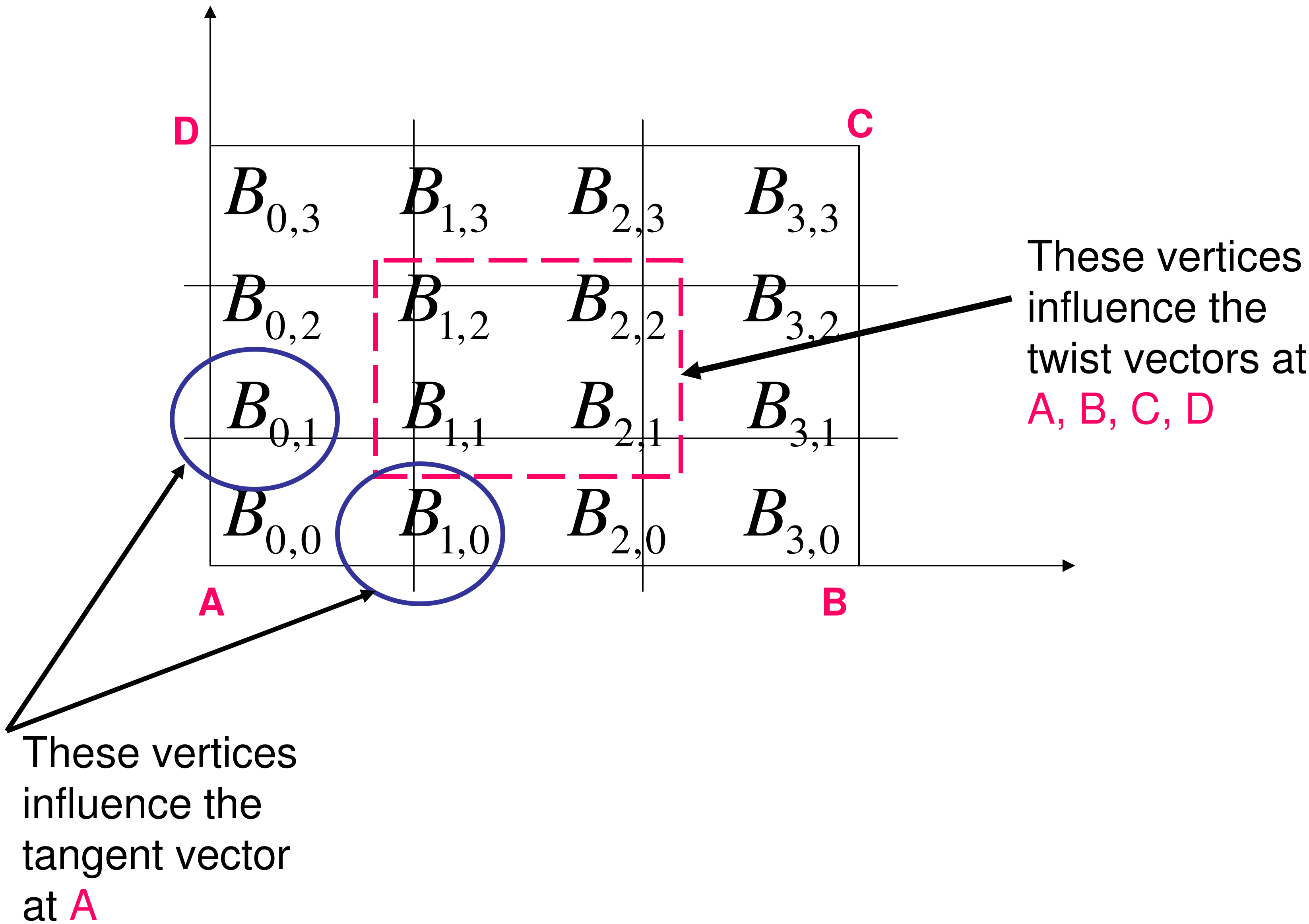
$$K_{m,j}(w) = \frac{m!}{j!(m-j)!} w^j (1-w)^{m-j}$$

B_{ij} are the vertices of the defining polygon net

Properties of Bezier Surfaces

- The degree of the surface in each polynomial direction is one less than the number of defining polygon vertices in that direction
- The continuity of the surface in each parametric direction is two less than the number of defining polygon vertices
- The surface generally follows the shape of the defining polygon net.
- The surface is contained in the convex hull of the polygon net.
- The surface is invariant under an affine transform
- Each of the boundary curve in this case is a Bezier curve

Shape Control



Matrix formulation of Bezier Surface

We can think of a matrix formulation now for Bezier surface. Consider a similar formulation as used in Bezier curve

$$Q(u,w) = [U]N[B][M][W]$$

Where

$$[U] = [U^n \quad U^{n-1} \quad \ldots \quad 1]$$

$$[W] = [W^n \quad W^{n-1} \quad \ldots \quad 1]^T$$

$$B = \begin{Bmatrix} B_{0,0} & \cdots & \cdots & B_{0,m} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ B_{n,0} & \cdots & \cdots & B_{n,m} \end{Bmatrix}$$

Matrix formulation of Bicubic Bezier Surface

We have 4X4 polygon net for a bicubic Bezier surface

$$Q(u,w) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} B_{0,0} & B_{0,1} & B_{0,2} & B_{0,3} \\ B_{1,0} & B_{1,1} & B_{1,2} & B_{1,3} \\ B_{2,0} & B_{2,1} & B_{2,2} & B_{2,3} \\ B_{3,0} & B_{3,1} & B_{3,2} & B_{3,3} \end{bmatrix}$$

$$\times \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w^3 \\ w^2 \\ w \\ 1 \end{bmatrix}$$

$$Q(u,w) = \sum_{i=0}^{n} \sum_{j=0}^{m} B_{i,j} J_{n,i}(u) K_{m,j}(w)$$

➢ Derivatives of a Bezier Surface

We find the derivatives w.r.t u,w and the cross derivative

$$Q(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}B_{i,j}J_{n,i}(u)K_{m,j}(w)$$

$$Q_u(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}B_{i,j}J'_{n,i}(u)K_{m,j}(w)$$

$$Q_w(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}B_{i,j}J_{n,i}(u)K'_{m,j}(w)$$

$$Q_{uw}(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}B_{i,j}J'_{n,i}(u)K'_{m,j}(w)$$

$$Q_{uu}(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}B_{i,j}J''_{n,i}(u)K_{m,j}(w)$$

$$Q_{ww}(u,w)=\sum_{i=0}^{n}\sum_{j=0}^{m}B_{i,j}J_{n,i}(u)K''_{m,j}(w)$$