Tribology

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Module No. # 06
Lecture No. # 40
Hydrostatic Bearings

Welcome to 40th lecture of video course on Tribology, topic of the present lecture is Hydrostatic Bearings. In my previous lecture, we studied porous bearing, bearing which has porous in a range of 16 to 36 percent. Porous are required to store the lubricant and whenever there is a need, the lubricant will be released and pumped back also. So, releasing and storing back the information is very important aspect of the porous bearings.

So, in first few slides we will again revise about the porous bearing, then we will try to connect porous bearing with hydrostatic bearings, there is a lot of similarity between these two bearings; subsequently, we will be doing some sort of a mathematical calculation to estimate load carrying capacity of hydrostatic bearing, one typical example will be covered in the present lecture.

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Porous Metal Bearings

- Small electric motors, household appliances, automotive accessories, etc.

Materials	Composition, %w	Pressure Limit, MPa		V, m/sec	PV MN/(m.sec)
		Static	Dynamic		
Bronze	Cu 90, Sn 10	59	28	6.1	1.8
Iron copper	F 90, Cu 10	140	28	1.1	1.4
Iron copper carbon	Fe 96, Cu 3, C 0.7	340	56	0.2	2.6
Bronze iron	F60 Cu 36, Sn 4	72	17	4.1	1.2
Aluminum		28	14	6.1	1.8

$\mu \sim 0.05$ to 0.15

So, let us start with the porous bearings, these are very popular in number of application, which have a relatively low load and relatively low speed. Most applications are like a small electric motors, naturally power is not very high to apply load, will be lesser. Similarly, we have a number of applications related to household appliances. Some automatic accessories,

which do not need much load or do not impose very high load. In those situations porous bearings can be utilized.

Typical porous bearing, which has been utilized or bronze related material, iron copper is mostly metal based porous bearing, iron copper carbon, bronze iron and aluminium, interesting thing is that, wherever there is a copper your thermal conductivity is higher and the speed of operation is relatively higher side you can see bronze and aluminium, they have a relative speed a 6.1 meter per second, but if I compare with percentage of iron, so even in this case, iron and copper is here, but copper has been reduced to 10 percent, in bronze its percentage is 90 percent, while an iron is a 90 percent, iron copper iron is 90 percent, so this is lower 1.1, but what is the advantage we are getting high M P, a high pressure limit for iron copper that is why 140 megapascal increasing iron percent with addition of carbon this maximum limit is increased from 140 to 340, so that is the advantage.

However, permissible speed of operation is reduced, that is the comparison, when we require very high speed operation not very high speed, is a relatively high speed operation, then we can choose a bronze, we can choose aluminium. When we required high load applications, we mentioned the porous bearings should not be used for heavy load, but the range in upper bond of that we can utilize iron based bearings with iron copper, iron copper carbon, it should have maximum limit 340 and 140.

Similarly, in this case the PV limit is been decided based on production of pressure and velocity. Now, question comes there are two columns of maximum pressure or pressure limit, one says, it is static other says, it is a dynamic, whenever there is a dynamic load, permissible limit is reduced; due to dynamism, all will not give sufficient time to leak out from the pores and go back in the pores, so that is why we require lesser Pressure compared to the static cases, this is the permissible limit for dynamic cases, if lesser may be say almost the 50 percent compared to static cases, these are the common materials, which we can utilize for porous bearings.

Now, let us see about the we say that, get formation about the coefficient of friction. Compared to the dry bearing, coefficient of friction will be lesser, because there is a supply of lubricant even the intermediary supply of lubricant, but is still the lubricant is there and what we are doing, we are trying to calculate average coefficient of friction and that gives a range of the 0.05 to 0.15, naturally this is preferable compared to the dry bearing, treatment design analysis is more or less same, a porous bearing as well as dry bearing.

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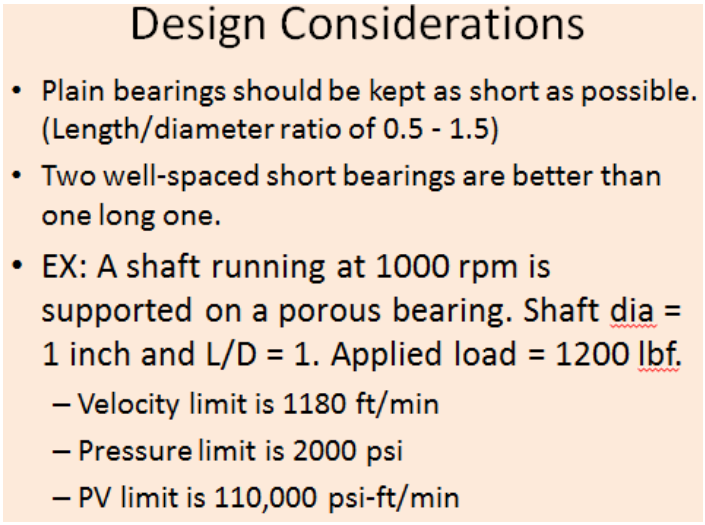

Let us take some example, for the porous bearing before considering that example, we are just giving some guidelines. So, that this bearings, porous bearings, whenever we select we prefer short bearing, short in the sense the length, diameter, ratio need to be lead to least small, we can keep in a range of 0.5 to 1.5 otherwise, many bearings are been used with many large L by D ratio, particularly for porous bearing, particularly for dry bearing we do not keep very high L by D ratio.

And if it is required, we cannot change the dimension, we cannot change the diameter, board diameter and we need to support relatively large load, then instead of one large bearing of long bearing we will prefer two short bearings that will give better solution that will be having some sort of adoptability to the misalignment.

Now, it is a point comes how to design a porous bearing? We can consider one example, a shaft running at the 1000 rpm, 1000 rotation per minute is supported on a porous bearing, shaft dia meter is given as a 1 inch; that means, 25.4 mm and L by D is equal to 1, whatever the diameter, it is a length is equal to same.

Now, applied load is been given in a bounds and that is a 1200 lbf, for this kind of bearing whichever the bearing we have been mentioning we are not mentioned the material, but relevant data have been given over here. So, velocity limit maximum velocity is 1180 feet per minute, pressure limit given is psi and that is equal to 2000 psi. We can convert and make a Pascal we can convert this in the atmospheric pressure, but for time being all the data have been given same unit; we will go ahead with this unit and the PV limit is been given as 110 k

psi feet per minute we can utilize the formula and see the result, whether the result is satisfactory or not.

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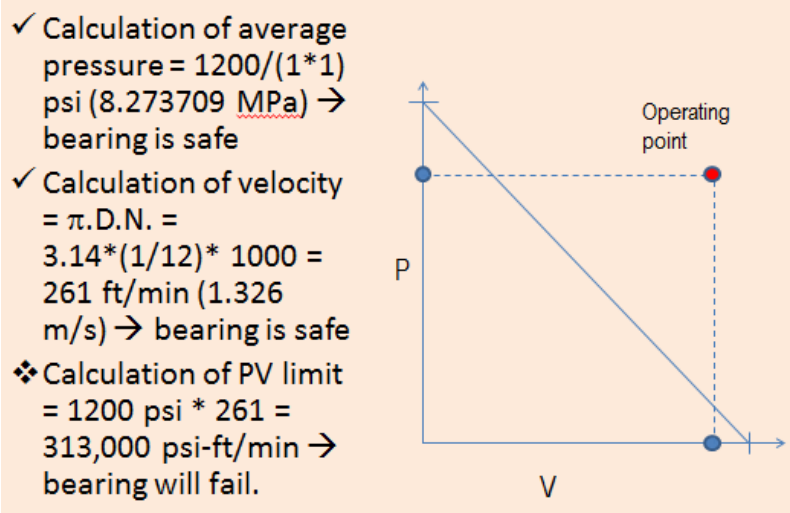

We will do first calculation, the calculation of average pressure load divided by a and that is turning out to be 8.27 M Pa and that is lesser than what has been recommended or given to us, second comes the velocity; velocity is also turning out to be lesser than what is been mentioned maximum limit 261 feet per minute is been calculated and which is lesser than permissible.

However PV limit, which is coming as a product is very high, it is a 313 kilo psi feet per minute and what is the permissible? Permissible is a 110 it is almost one third is permissible and this value is very high, when pressure was a 2000 psi and we are getting lesser than that and velocity limit was 1180 again what we are getting that is lesser, so that is what we are saying may be better, if bearing is same from pressure point of view, from velocity point of view and is same from PV limit point of view.

So, how to demonstrate that? So, lets see this is a pressure axis, this is a velocity axis and these are the terminal velocities, maximum allowable velocity and is a maximum pressure allowable pressure, if we connect this that is generally giving us a PV like PV approach.

Now, if this is maximum pressure which we are getting, this is a maximum velocity which we are getting. So, from obsolete point of view, from pressure point of view, the bearing is same, because generated pressure is a lesser than maximum allowable pressure; one velocity is also

lesser than permissible velocity, but we try to project this, this point operating point comes somewhere here which if far away, from permissible PV limit.

So, this bearing is going to fail, because of the heating, because of the bearing rate we will not be able to estimate, we will not be able to find the life of this bearing using PV limit or PV approach reason being, this bearing undergoing excess of wear is crossed PV limit and after that mild wear equations cannot be used.

So, this is a brief about the how to utilize a PV approach for the porous bearing and find out, whether the bearing is going to survive or not, if bearing is able to survive, then we can recommend we can find out what will be the life using the wear equation, but if it already crossing the PV limit; we will not be able to find, because bearing is going to fail or wear rate is much faster and ordinary orchid equation cannot be used for this purpose.

Now, this is about the operating point, I mentioned. We try to connect this porous bearing with hydrostatic bearing. It is very good idea to think porous bearing and hydrostatic bearing and co relation between that.

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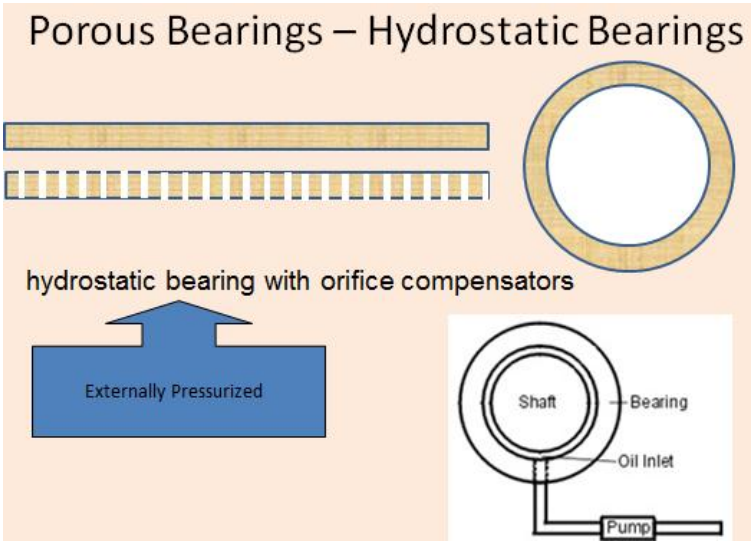

Well let me take some example, we say that I am trying to relate porous bearing with hydrostatic bearing and this is a porous bearing, this is a board of porous bearing in the outer diameter of porous bearing what we can do? We can develop this surface I can cut from 1 plane and stretch it in one plane. Now, this can be done like that. So, this slab is more like a development of this bearing or this cylinder, this hallow cylinder; this is development of this hallow cylinder.

Now, if I do a cross section if I cut any cutting plain from this slab what we will be able to see, there will be some pores, may be equally spaced or for the time being I am assuming this spacing may be not very uniform, but when we pass a cutting plain we are going to see some pores in that, this is a cross section, this is cross section that the cutting plain is touching this surface, but there is a hole over here and these holes are required to transfer oil from this void to the surface that is what, they are voids may be not connected completely, but they are connected one way or another way and the pressure get released and they release liquid and they are able to pass a liquid from one corner of the bearing to other corner of bearing from one surface of bearing to other surface of bearing.

And this is exactly structure of hydrostatic bearing, if I use a palm somewhere here try to connect all this pipes or we say I am assuming this are the pipes or the capillaries and feed or pump liquid lubricant or maybe say even we can compress or we can pass compressed gasses from these pores those will reach to other side. And that is what we are trying to convey that, this kind of a structure can be related to hydrostatic bearing or we can say, this kind of a arrangement is a hydrostatic bearing with orifice compensator, there are number of orifice in this and there is a compensator what is the meaning of compensator?

See, when we require we have only one palm and we want to connect all the holes with that palm these orifice will work as a compensator otherwise, we will be requiring individual pump for individual hole or individual orifice, we will be requiring too many pumps in those situations that is why we go ahead with this kind of orifice compensator to that there is some pressure loss, but there is a stability of operation I can say, whenever the heard of hydrostatic comes, there is a need to give external pressure or in other words, we can say externalize pressure, externally pressurized bearing or hydrostatic bearing.

They are synonymous and for externally pressurized we require some kind of arrangement like this, say there is a piping, there is a pump, liquid is getting sucked in a pump and getting forced in a bearing and there is a inlet, oil inlet is a pumping the liquid, when there is liquid is getting pumped naturally shaft will be moved will be displaced from the point of contact, because the liquid is getting pumped in that. I am sure the pumped liquid will be having slightly larger value compared to minimum value, which is required to lift the shaft.

And that is what, all about the hydrostatic bearing, now question comes, do we require only one hole or we require too many holes? In reality, we require too many holes to make it stable to provide a stable operation.

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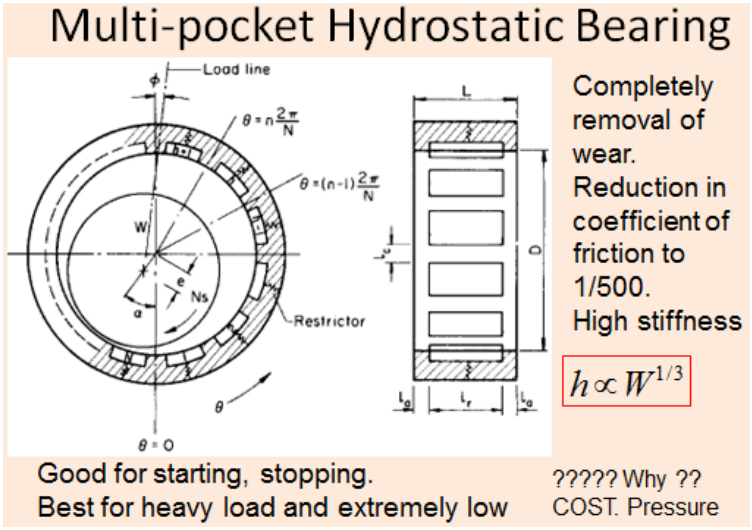

And this is what we use a word multi-pocket hydrostatic bearing; you can see there are pockets, there are restrictors mean like what we are shown in previous slide with a porous bearing, those are restrictors orifice and can may be capillary action also. And we are using this pockets; these pockets act as a reservoir and avoid any pulsation in liquid, if we do not use this, there will be lot of pulsation, lot of instability, lot of turbulence to avoid that, we are using this kind of pockets it is more like a bath tub for the bearing, the localized domain.

And this is the one side view, this is the other side view we are able to show over here, pockets are not from one end to other end we provide we will be doing mistake reason being, because of this pocket, liquid will be simply moved out, it will not be acting as a reservoir, it will be simpler by an basic canals sort of thing from one side liquid is coming and other side is going out without any support to the shaft.

So, that is why we require a slot, there is no through and through slot, when the liquid comes it should remain in that slot, it should not be simply skipped from surface. So, that is a major purpose of this, now it can be, we use a word multi-pocket, it can be 6 pocket or 8 pockets or 12 pockets, generally we use even number of pockets.

So, that there is basic stability, what is the meaning of basic stability? So, whatever the force comes from this direction, there should be something opposite to that; that means, we can regulate motion of the shaft, whatever the force we are providing from one side, there should be opposite force and overall, control algorithm which will be required to position the shaft will be simpler, if we use odd numbers, then algorithm may be slightly more difficult, but for an even number algorithm is very simpler.

So, what we say this kind of hydrostatic bearings remove completely wear, no wear chance, completely shaft is levitated, no metal to metal contact. If there is no metal to metal contact, naturally there will not be any wear and generally permissible lowest speed relatively speed in this kind of bearing is very small.

In addition, coefficient of friction is almost a 1 by 500 times of dry friction it is significantly low and that is a major advantage. In addition, they show high stiffness, because this kind of the bearings additionally required positioning the shaft and wherever we required positioning the shaft, even there is a change in the load applied on the shaft or supporting surfaces, they should not be much variation in the position of the shaft that is why we require very high stiffness or in other words, I can represent this film thickness at the any point, film thickness at any point need to be related with a W naturally, if the W is increasing, thickness is going down is going to decrease and that is having a 1 by 3 or say, it is a cubic root of this, it should be minus 1 by 3.

So, it is a cubic root of W, sensitivity is much smaller it is not a very high or when the sensitivity towards the deflection is very small, naturally stiffness is very high and we are saying that, we are talking about the high stiffness that is a major advantage of hydrostatic bearing.

In addition, there are some other advantages we say this kind of bearings is good for starting and stopping. This has been utilized, even when we consider the hydro dynamic bearing may be next lecture, then we will be able to find there are always problems with hydro dynamic bearing during starting and stopping and here we are using the word over here this hydrostatic bearing is good for starting and stopping, naturally we can hybridize hydrostatic and hydro dynamic bearing to make the overall the best bearing, a combination gives the best results.

We require good characteristics or the initial point we require good characteristics under running conditions; one bearing provides a good characteristics other they start and stop other bearing gives a very good characteristics during the running time and we want both, we want win-win situation naturally we need to go with hybridization of this bearing. That is what the emphasis has been given, this bearings are good for starting and stopping and this is a major point for the hydrostatic bearing or we say this bearings are the best, these bearings are the best for heavy load or we say we can use the word extremely heavy load, load which cannot be supported by any other source.

And requirement, if the requirement is also that extremely low friction, take an example of big size telescopes; rotation may be required a few degree, not a single rolling element bearing not a hydro dynamic bearing or any other bearing can be utilized, rotation is very small and almost negligible coefficient of friction is a demand if we require accuracy of even the 0.1 degree in those situation we do not have any other alternate other than hydrostatic bearings, you can think about the electromagnetic bearings and all, but compared to that these bearings are better option.

Now, the interesting question comes. So, many advantages have hydrostatic bearings, why every bearing is not hydrostatic bearing, why do I use a rolling element bearing, why do I think about the hydro dynamic bearing, why I do think about the dry bearing, everywhere there should be hydrostatic bearing, because it gives a low friction, it gives negligible coefficient of friction, it gives a very high stiffness, it is very good for starting and stopping.

Whatever situation you give we are going to get best results, but the cost; the cost of this bearings are very high, actual control which is required for this kind of bearing is also important to maintain high stiffness, always we need to believe on a capillary or compensators and also orifice compensators or we may be requiring otherwise, the constant volume pump.

Another thing, pressure fill, which is required to feed the, to lift the shaft is generally very high, it can be in a M Pa, it can be 150 bar, 200 bars. So, if I have to supply that high pressure, naturally I will be requiring a big pump, very high pressure lines and we know very well high pressure lines reliability is a major issue, if it burst, if there is any crack whole unit will collapse. So, reliability will be a major issue and if you want very reliable operation we need to go add with additional units. So, cost will be very high compared to our ordinary bearings. So, this is not a first option and this should be always a last option, because we are always cost conscious.

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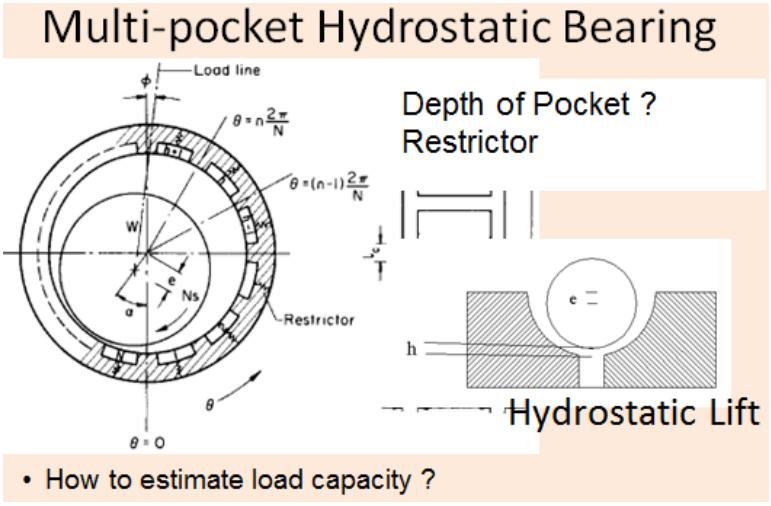

Now, this says a multi-pocket and question comes, what should be the depth of pocket and what is restricted? We say that, depth of pocket can be even 50 to 100 times of film thickness, so it acts as a reservoir, it acts as a stabilizing agent, there should not be any turbulence and there is a slight variation in that, because of the abundant liquid in this pocket, there will not be much variation.

So, depth of pocket should be around 50 to 100 times of the film thickness, which is the minimum film thickness requirement. What is the 1 micron? And depth of pocket should be 100 microns, what is 2 micron? It should be the 200 microns or more than that, but minimum value of I am talking about the 50 to 150 to 100 times of the film thickness.

Then, comes what is the restrictor? We are able to see, these are the restrictors, this can be the orifice or it can be capillary it can be capillary, there is a possibility of chocking, because of the some third particle and all and then, whole operation will stop. So, either way we require a very good filter with a capillary or otherwise, we can use orifice, which are not very sensitive towards the environment in that.

So, depth of pocket need to be 50 to 100 times minimum value, restricted can be capillary action or can be orifice, it can be utilized in those things, the question comes, how to start calculation, how to think about the load carrying capacity of this bearings, what will be the relation between load and film thickness, what will be relation between the power supply or may be even oil supply or supply pressure and film thickness, how to design this bearings, how to think about the designing this bearings.

I will take very simple example, see assuming this big shaft is supported only in one pocket for the time being, once we know the analysis for one pocket we can go ahead with a vector algebra for the multi pockets, relations are not going to change, only the supply condition are going to change or different orientation the porous will we can resolve it, we can find out overall stability analysis based on that, but first point is start with a one pocket and this is been shown over here, again this is the one pocket over here or we say that is the hydrostatic lift we are pumping the liquid from here because of that pressurized liquid.

This shaft is getting up or we say that, it is getting rest just for the convenience I showed so big value in this is the 90 degree extent, if I assume there is a line of excess symmetry over here, axis over here, vertical and from 0 to 90 degree, this side is 0 to 90 degrees this side in actual case, may be much lesser than that and we need to account actual angle.

For time being, we are using the 90 degree and to estimate the load carrying capacity we need to go back to our Reynolds equation, which we have done or which we have derived in earlier relation or at least we go ahead with initial equation, what we say the flow equation we want to know, how much lubricant supply will be required to levitate the shaft, we are going to pump the liquid from the pump.

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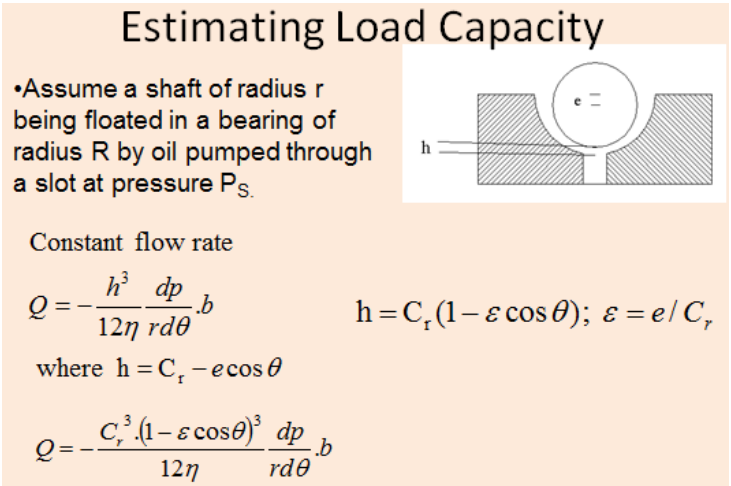

So, for that purpose we are estimating we are assuming a shaft of course, the dimension is much smaller, but it can be much larger than this, the shaft has a radius of r being floated is levitated in a bearing which has a radius capital R, what will be the clearance, clearance will be the capital R minus small r, that will be the clearance between bearing and the shaft is been floated by oil, which is been pumped through a slot at the pressure P_s.

We say the supply pressure is P_s assuming some supply pressure maximum value is a P_s and geometry is been defined, radius of the shaft is been given, radius of the bearing is been given and we can find out the clearance based on this. In addition to that, we have shown somewhere here this symbol says e; that means, there is bearing center and the shaft center. So, the shaft center may be lower than this, because there is slight deviation and bearing center is above. So, there is eccentricity in some value naturally it will be lesser than clearance value.

Now, we will start with the flow equation, because we require how much flow is, how much liquid need to be pumped, this relation can be given in terms of film thickness, in terms of viscosity, in terms of pressure gradient, because there will be some sort of pressure loss, when it goes through the this kind of pockets. And it is been depending on a, it is depending also on a length of the bearing. We are assuming the even the pocket will not be through and through from one slot or one side to other side, for time being we are assuming, for calculation purpose we are assuming the b is that otherwise, we need to calculate actual depth of pocket.

So, if it is the 80 percent of overall length of the bearing we should account 0.8 b otherwise, if it is 60 percent we should account 60 percent depends, if we do the optimization on final results will come, what kind of the length will be more suitable for our operation. So, Q can be given in terms of a film thickness that is the h cube divided by twelve η dp by rd θ, this is a r is a rd θ is giving a one axis may be the dx line and into the length, here naturally about a variable it is a h over here and h is given by the C_r, which is the difference between capital R minus small r that is a C_r minus e cos θ, because we know film thickness is going to vary with θ and I am assuming this line is a mid point when the θ is equal to 0.

So, θ may vary in anti-clock wise and will vary in clock wise, when it varies in anti-clock wise, it can be treated as a negative direction and in clock wise it can be treated as positive direction, but when we talk about the cosθ, negative θ, positive θ does not make any sense and has the same value it is not going to change.

So, what is been given here the C_r minus e cos θ and we saw that, this point film thickness will be minimum, other than this film thickness will be larger and that is been reflected over here, cos θ is a 0 that is a C_r minus e will be the minimum value and we know as a θ increases from 0 to 90 degree, the value of the cos θ is going to decrease and value of the cos θ is the lesser than 1, when it is been multiplied naturally the film thickness is going to increase or in

other words, minimum film thickness is going to be at the point, where the theta is 0 and that is a been represented by this equation.

Now, we can go ahead one more step we say that, we are not interested in a absolute value of e, we are more interested in epsilon; an epsilon is a ratio of eccentricity to radial clearance. We know very well, the maximum value of epsilon will be 1 and minimum value epsilon will be 0; however, in the present situation there is a possibility the shaft center goes beyond the bearing center, goes more above the bearing center.

So, there is a possibility that e turns out to be negative. So, we are saying the e will be positive, when the shaft center is below the bearing center, e will be negative, when the shaft center is above the bearing center, there is a possibility we pump too much liquid, supply too much pressure naturally this shaft will go up higher than bearing center or we may say that, load carrying capacity is more than the load applied on that, naturally it will go beyond the bearing center.

And this is a common situation, when we are using or we are talking about the multi-pocket bearings. So, we need to account negative epsilon or we say negative value of eccentricity also. So, with this kind of thing we can take a C_r as a common, in bracket what will be 1 minus epsilon cos θ again, when the cos θ is 0 naturally this will be maximum value 1 minus epsilon or when cos θ is a 90 degree, naturally this will turn out to be maximum value, overall this will be 0 and this will be 1.

And in this way, we can find we can estimate film thickness, we can find out the Q, if we know what is a relation of p with the kind of the θ, what is a relation within p and θ or if we know the θ, we can find out what will be the relation between p and h, where film thickness is going to continuously vary over here. So, for that purpose what we can start, we can say that this is already θ over here, we can substitute value of a h in terms of theta, C_r is a constant for one situation for one equilibrium epsilon is also will remain constant, it is not going to change; for static case for one particular case, epsilon will remain constant.

So, we substitute C_r, this epsilon is here and there is a cos θ and we have differential term theta. So, we can rearrange this equation we can integrate pressure over θ to find out pressure distribution with a θ.

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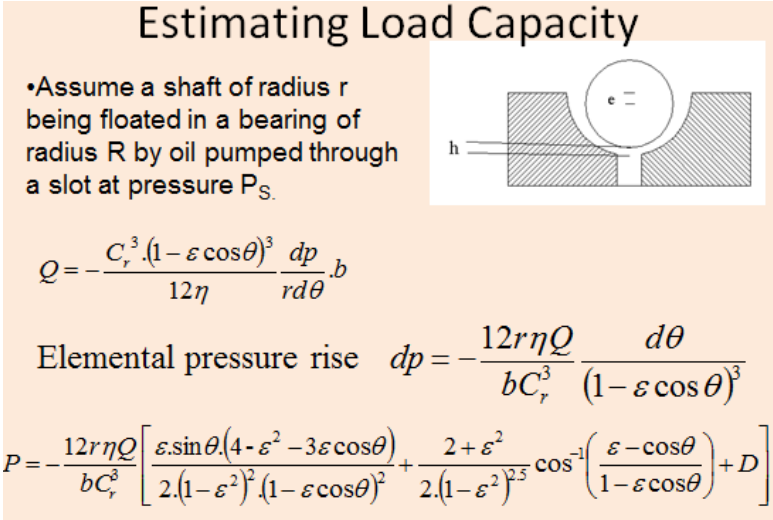

Now, we can do that rearranging it something like this we say elemental pressure rises at any point can be given as a d p in terms of this constant that is the 12 into radius into viscosity into flow rate, here we are assuming we are going to supply at the constant volume. This is a mm cube or meter cube per unit second, this volume rate is a constant.

However, we can design similar bearing, making pressure as a constant at the supply point or at the restriction point. We have our choice for the better one is, constant volume that is why we are assuming here, the volume remains constant I know this much volume is there and then, we need to get the results, length of the bearing is constant, C_r cube is constant. So, this is a constant factor.

Now, what we need to do integration, d theta divided by 1 minus epsilon cos θ, there is a theta term here and we need to do integration and naturally we need to follow some sort of a table or we can go ahead with a replacement in substitution to get the results what would we have done in fluid lubrication part of this course.

We go ahead with that, slightly longer derivation comes I am not deriving complete equations, because we have done already two times in this kind of thing and know I am just giving as it results. Say, the pressure P is given this constant will remain constant and this term 1 minus epsilon cosθ cube, there will be given in this way, there is a constant of integration that is a D over here.

And this equation is been represented in terms of epsilon and sin theta, if I know that epsilon, I can really find out what will be the P at the various levels. Now, there are number of terms

and we can solve it, we can say the D can be obtained, D can be obtained by using the input condition.

We know at a θ is equal to 0, P is equal to Ps, P is equal to supply pressure at the slot or P is equal to the pressure at the slot entrance that can be utilized to find out, what is the D over here that is the one thing or we can use other boundary condition and we can say that, the theta is equal to 90 degree, when theta is a 90 degree, pressure will be 0 that is another boundary condition we can utilize.

If we know the supply pressure, I can find out from that or we I do not know the supply pressure I can say, we know very well the 90 degree or the exit when the pocket is coming out naturally pressure will not be there, pressure will be 0, both the condition can be realized to get the results, whichever I think easier one is a 0 condition directly substitute 0. And so theta is this 90 degree we can get the results or theta is a theta 1, where the pressure is maximum pressure is coming to the 0. We can use any condition we get the results.

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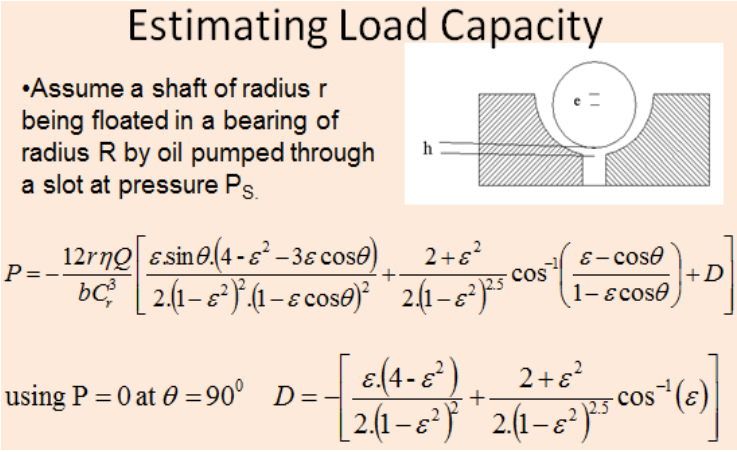

And when, as I mentioned the simpler one is a pressure it is 0 as θ is equal to 90 degree as per our assumption. Now, the using this P is equal to 0 and theta is equal to 90 degree, what we get the D in terms of completely epsilon, now we can substitute this value over here and if we were not knowing initially what is the P_s, we can find out directly P_s by substituting theta is equal to 0.

Once I know this D, substitute in this relation and if I know the epsilon and I know the parameter, I can find out what will be the pressure or we say or we decided how much liquid should be pumped per unit second, then we should be able to find out, what will be the Ps,

what will be the supply pressure, which is been pumped from the pump and may be it is coming at the exit of the slot.

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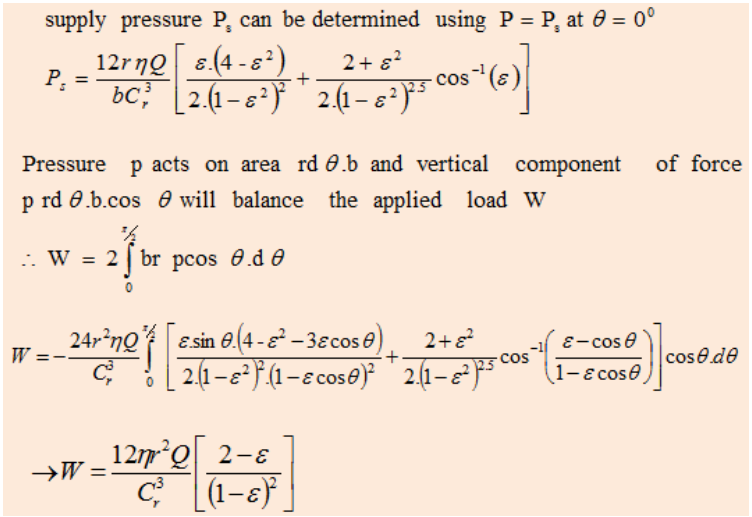

Using this conditions we can find that P_s, this is completely based on epsilon or in other words, change in epsilon is going to change the pressure supply that is why we say that, we are supplying with the constant volume and this P_s is going to change with the changing in epsilon and will decrease or will increase depending on the value of epsilon.

The question comes, we started this for finding the load carrying capacity of the bearing, but we are not yet reached to the bearing load carrying capacity, what we are discussing only about the pressure; pressure relation naturally to find out the load carrying capacity we need to integrate over the area and that is done like this, we say pressure P acts on area r d theta, if I am trying to integrate assuming the uniform variation; that means into length of the bearing as I mentioned not necessary, we need to account overall length in this case and we can think pocket length also.

So, pressure P acts on the area this one and a vertical component naturally when we are talking about bearing, there will be always two components, we are talking about the polar coordinate; there will be one term component containing cos θ terms or that component will contain a sin theta terms.

And generally, when we are talking about the middle of the symmetry what will happen? 2 sin θ terms will cancel with each other or in other words, what I am trying to convey, if I draw line over here sin theta term will be this side, sin theta terms will be this side. So, half of

this portion and half of this portion, this side portion those will be cancelled and what about the cosθ terms along this will be added of and that is been shown over here cos θ terms have been added to whatever the pressure relation have been added, while sin θ terms are going to cancel each other. So, that is why we do not require calculation of sin θ terms, we are only consternated on cos θ term.

And consider see the boundary condition, it is a 0 to π by 2, because you are taking half one we assume the symmetry about the sides and theta is 0 at a center line or the middle point. So, when we do or we substitute pressure relation which has been shown in previous slides we will be able to find out what is W.

If we know that W, which is been applied we will be able to find out pressure relation directly and that comes somewhere here, we substitute whatever we have calculated in previous slide, substitute here and integrate it. When we do integration, what we are going to get something like this, that is a huge term is going to get converted to in very small term. Of course, here 24 and 12 naturally one half of that, this is only the 2 minus epsilon divided by 1 minus epsilon square, very short term, easy to understand, easy to remember and easy to plot also. The huge term, which is required for the derivation can be simplified in this relation for constant volume.

So, this W is been represented in terms of viscosity or we say that, when we pump liquid with a high viscosity, load carrying capacity is going to be higher side, we are going for larger radius, load capacity is going to be pumped too much liquid, load carrying capacity is going to be higher. If we reduce the clearance, load carrying capacity will be higher and of course, remaining case is epsilon.

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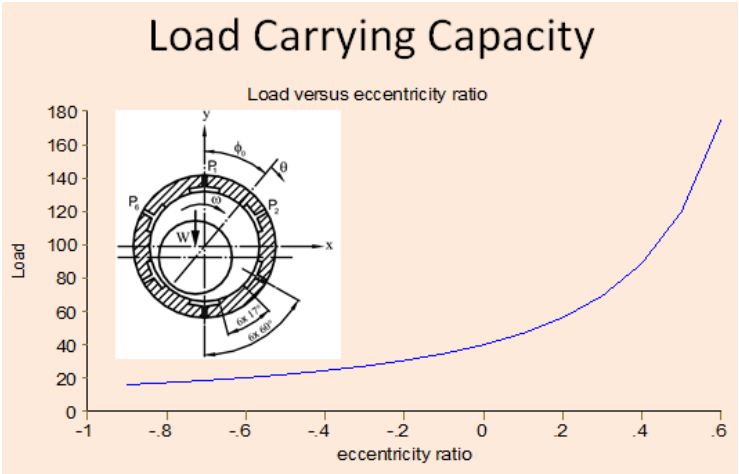

Now, let me plot, see I am talking about the negative eccentricity ratio also, I am talking about the positive eccentricity ratio also, you can see the load, the sensitivity when we are talking the positive sensitivity or positive eccentricity is increasing drastically that is a nice thing as it goes closer and closer load carrying capacity is going to be increased phenomenally.

And generally we say that, this kind of a initial approximation will not be able to provide, when we talk about eccentricity more than 0.9 or even the 0.8 also that is why it has been cut to the 0.6, when you go for 0.8 l we will be getting very high extent high value of load carrying capacity and beyond that, reliability of this relation will decrease, no linearity is not been accounted properly we require regression analysis for that purpose that is why we say that, we can if we try to find out using this relation high eccentricity, what will be getting is unnecessary very high load carrying capacity.

I remember one example we say that, when eccentricity was a 0.93 and we estimated the error, which was obtained when the load was more than 2000 percent, it is a phenomenally high error that is why we do not use a very simple expressions for high eccentricity value, because non level is going to increase very high and this bearing of this expression, your assumption was that bearings are not touching each other, there is a low inter locking will never give eccentricity 0.999 something load carrying capacity will be always on higher side. So, it will not reach to that high level of eccentricity.

Now, this is a given result you say that, negative side is also mentioned over here that is a minus 0.2 point, minus 0.4, minus 0.6 you can see that, shaft is levitated beyond its bearing center and this is important, when we are talking about the multi resist groove. Now, what we say that, we can start with one groove we have, if we know about value of eccentricity or we know the position we can find out, what will be the eccentricity for this slot, what will be eccentricity for this slot, what will be eccentricity for this slot.

And we are talking always about the even number of slots. So, when whatever we are pumping liquid from here similarly we are pumping from that side we can find out the force balance or whatever eccentricity, this side or that side that is going to give force component in this direction, force component in that direction overall this shaft going to be located somewhere under static condition and that is a beauty of hydrostatic bearing, location point is always giving the good results very good results to control the position of shaft, hydrostatic bearings have given the best results.

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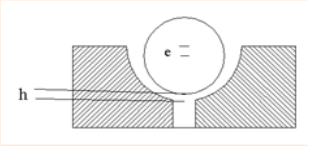
Ex: A 101.6 mm diameter journal rests in a bearing of diameter 101.9 mm. Lubricant of 30 mPa.s is supplied under pressure through a groove at the lowest point of the bearing. Length of bearing is 152.4 mm and applied load is 16 kN. What inlet pressure and flow rate are needed to raise the journal 0.0508 mm.

$$\rightarrow W = \frac{12\eta r^2 Q}{C_r^3}\left[\frac{2-\varepsilon}{(1-\varepsilon)^2}\right]$$



$W = 65000N$

$\eta = 30\times10^{-3}\ Pa.s$

$r = 0.5\times101.6mm$

$C_r = 0.5(101.9-101.6)$

$\varepsilon = 1-\frac{0.0508}{C_r}$

So, we can say to understand this one simple example, say there is a bearing having a diameter of 101.6 mm diameter, we say journal is a the load bearing is a journal diameter is a 101.6 mm is a resting in a bearing of diameter 101.9 mm.

So, this difference will be the diametral difference, diametral clearance will be given by this; if you want the radial clearance naturally it will be 50 percent of this. And it is been lubricated, viscosity of the viscous liquid, which has a viscosity as a 30 millipascal second or 30 into 10 is to minus 3 Pascal second this bearing is subjected to some groove pressure or the supply pressure, when supply pressure is been given or we say liquid is been supplied with some pressure P s at the lowest point.

So, whatever the example we consider, the bearing example is similar kind of thing; length of bearing is defined as a 152.4 mm and applied load on the bearing or we say the applied load on the shaft, which need to beard by the bearing is 16000 or 16000 Newton 16 kilo Newton.

So, what is the question say, what inlet pressure is, what will be supply pressure and what is the flow rate, what inlet pressure and flow rate are needed to raise this journal, this journal by 0.0508 mm. So, indirectly this is going to give as eccentricity ratio, bearing dimension have been defined, viscosity is been defined, bearing length has completely defined and the applied load is been given to us, you say this is a example related to this kind of configuration in this point, ratio will be supplied this shaft is going to get levitated, the levitation is given as 0.0508 mm.

And we can use the load relation, because the load is been given to us, it is a 16000 Newton load is been given, viscosity is defined, radius of the shaft is given, C_r is given, eccentricity is

been defined. So, what we can get from this equation? This Q, we can determine what will be the value of the Q for this 16000 Newton load, you can do that.

So, W is a 16000, viscosity is a 30 into 10 is to minus 3 Pascal second, radius is a 15 percent of this that is the 0.5 into 101.6 mm, C_r is a 50 percent of this difference. So, 50.5 into 101.9 minus 101.6 mm and then, the eccentricity will be 1 minus because we tell that it is touching here, eccentricity will be equal to 1. So, this ratio will be 1 minus whatever the position divided by C_r and that is a going to give the eccentricity ratio from here. And naturally if it is a above this or we say this value is more than the radial clearance, then we need to sum up, eccentricity will be more than 1 or will be otherwise a negative side we can do or we can directly calculate from this also, if it is more than that then value will turn out to be negative that is fine for us.

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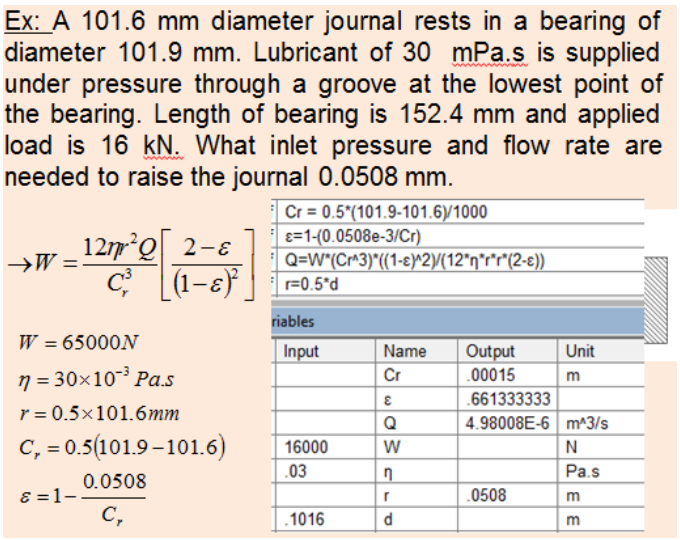

Now, we can use the simple algorithm, in this case it is been defined the way it is shown here 0.5 into 101.9 minus 101.6 and it is been converted to the meters, this unit is nano and we need to convert in meters, because we are dealing with the Pascal and Pascal it is Newton per meter square. So, that is been given over here, then we have epsilon that is also been defined over here that is here we say that, this value C r already been converted to the meters we need to convert this value in meters also.

So, that is a 0.0508 into 10 is to minus 3 because of the meters, C_r is already in meters. So, can calculate the results and this Q is given by rearranging the equation, Q can be given as a W into C_r cube into 1 minus epsilon square divided by 12η r square into bracket 2 minus epsilon and that is been given here, W C_r cube 1 minus epsilon square divided by 12ηr square

and 2 minus epsilon is been given over here and we say that, radius is a 50 percent of diameter.

And this is a parameter, input parameter we know the 16000 has been defined, viscosity is given as a 30 millipascal second or 0.03 and diameter of a shaft is been given that is a 0.1016 meter, when we solve it, what we are going to get, the radial clearance is roughly 0.15 mm that is a 0.00015 meter and epsilon when we calculate using this relation is turning out to be 0.66 and we can say that, this is still in a easy range we can calculate using our analytical expressions. Of course, if it is this value is turning out to be more than 0.9, naturally we will not be able to utilize this relation, because whatever we get it will not be reliable.

And based on that, we can find out, what will be value of Q in meter cube per unit second, this much flow rate has been supplied or will be given to sustain this load with this eccentricity. So, that is what is been asked, what will be the flow rate has been calculated however, this pressure has not been calculated using this relation. So, we need a second relation for pressure supply or supply pressure.

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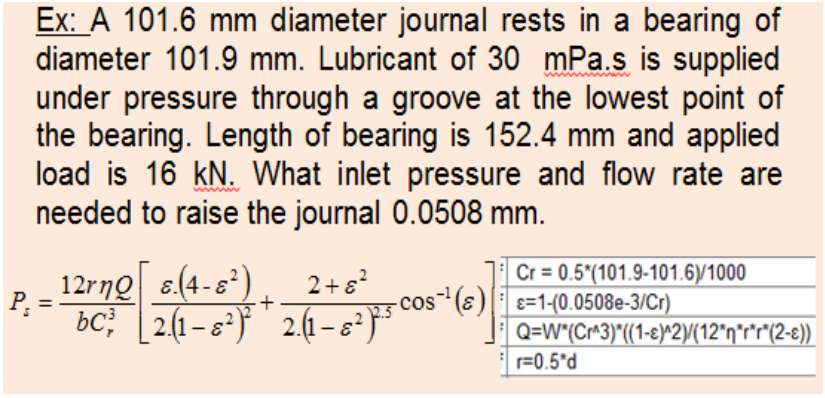

And that is given over here something like a supply pressure, P_s can be given in terms of 12r η Q, Q is known and b here in a previous relation we are not using b, but now we are using b here, the length of the bearing or the length of the pocket is been utilized completely here and that this relation is in terms of epsilon, we know epsilon we can directly use resolution and get the overall results.

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Solution

Cr = 0.5*(101.9-101.6)/1000
ε=1-(0.0508e-3/Cr)
Q=W*(Cr^3)*((1-ε)^2)/(12*η*r*r*(2-ε))
r=0.5*d
Ps = 12*r*η*Q/(b*(Cr^3))*(0.5*ε*(4-sqε)/((1-sqε)^2)+0.5*(2+sqε)/((1-sqε)^2.5)*acos(ε))
sqε=ε*ε

riables

Input	Name	Output	Unit	Comment
	Cr	.00015	m	
	ε	.661333333		
	Q	4.98008E-6	m^3/s	
16000	W		N	
.03	η		Pa.s	
	r	.0508	m	
.1016	d		m	
	Ps	1429774.25	Pa	
.1524	b		m	
	sqε	.437361778		

Now, this is a overall solution this is a C_r, epsilon, Q, r, P s has been defined, P_s has been given over here and just for simplifying this expression instead of writing epsilon into epsilon what we have done is sqe or sq epsilon is a epsilon into epsilon it has been utilized in number of times, so 1sqe, 1sq epsilon sq epsilon, sq epsilon here, sq epsilon here, sq epsilon here. So, instead of doing a four times calculation for we are agreeing an algorithm, we will do only one time calculation, remaining time we are substituting this results.

So, this will be giving the slightly faster results we know this overall solution comes much lesser than 1 second, so one this system is not going to simplify much, may be 1 micro second lesser, second than that or may be some change, but it will be always a good practice to obtain this instead of doing the four time calculation, do one time calculation and substitute for the remaining time.

Now, when we see the inputs; this load input, viscosity input, diameter of shaft is an input and length of the bearing as input, what are the output in clearance which we find using this relation; first relation that is in meters or 0.00015 meter, epsilon, Q which we find in meter cube per seconds, then radius and then, there is a P_s, this is a critical parameter it is in a Pascal say, they are the 6 is a almost a 1.4 mega Pascal or we need to supply pressure more than 14 bars that is a huge pressure 14 bar to levitated shaft.

And what is been supported only 16 kilo Newton load, not very large load. So, in that we again we are assuming here 90 degree expand of the cavity, it will not be 90 degree. We will continue with journal bearing operation and maybe we will try to relate this hydrostatic bearing with a hydro dynamic bearing, because hydrostatic bearing may not be very useful in the absolute sense in the major areas and n number of applications they are very specify

applications for the hydrostatic bearing, for the hydro dynamic bearing it is very journal application used almost in every machineries, that can be coupled with hydrostatic bearing.

And we will continue with hydrostatic bearing in our next lecture, we will continue with hydro dynamic bearing after that, thank you, thanks for your attention.