A New Approach for the Design of Fast Output Sampling Feedback Controller

S. Janardhanan*
Email : janas@ee.iitb.ac.in

B. Bandyopadhyay*
Email : bijnan@ee.iitb.ac.in

Abstract— **This paper presents a new approach for the calculation of the fast output sampling based feedback controller for a MIMO linear time invariant system. Further, the method avoids the constraints imposed on the state feedback gain in the original algorithm.**

I. INTRODUCTION

The problem of pole placement for a linear time-invariant system is a well studied one. Many methods are available for the design of a state feedback gain F that when applied to the system (Φ_τ, Γ_τ) would make the closed loop system $(\Phi_\tau + \Gamma_\tau F)$ stable. However, the system states are not always available for measurement. Hence, the system output has to be used to design a suitable controller. Static output feedback has the problem that it cannot guarantee a stable closed loop system[1]. Dynamic compensators may increase the complexity of the system.

With the *Fast output sampling* (FOS) approach proposed by Werner and Furuta in [2], it is generically possible to realize a given state feedback gain. For an FOS gain L to realize the effect of state feedback gain F, find L such that $(\Phi_\tau + \Gamma_\tau L\mathbf{C})$ is stable. The FOS controller obtained by the above method requires only constant gains and hence is easier to implement online. However, this method too imposes constraints such as the state feedback F should be such that $(\Phi_\tau + \Gamma_\tau F)$ is non-singular. The proposed algorithm eliminates this constraint and also initial state estimation error.

The paper is organised in the following manner. Section II deals with a brief introduction of fast output sampling followed by the presentation of the proposed algorithm in section III. Section IV gives a numerical example to check the proposed algorithm followed by the conclusions.

*The authors are affiliated to the Interdisciplinary Programme in Systems and Control Engineering, Indian Institute of Technology Bombay, Powai, Mumbai - 400076

II. FAST OUTPUT SAMPLING

Let $(\Phi_\tau, \Gamma_\tau, C)$ be the discrete-time representation of an nth order continuous time MIMO $p \times m$ system sampled at an interval τ, and (Φ, Γ, C) the same system sampled at $\Delta = \tau/N$, $N \geq \nu$, the observability index. Let the former representation be controllable and the latter be observable. Then the system would be

$$\begin{aligned} x(k+1) &= \Phi_\tau x(k) + \Gamma_\tau u(k) \\ y_{k+1} &= C_0 x(k) + D_0 u(k) \end{aligned} \tag{1}$$

where the matrices y_k, C_0 and D_0 are as defined in [2]. Assume that a state feedback gain has been designed such that $(\Phi_\tau + \Gamma_\tau F)$ has no eigenvalues at the origin. For this state feedback one can define the fictitious measurement matrix

$$\mathbf{C} = (C_0 + D_0 F)(\Phi_\tau + \Gamma_\tau F)^{-1} \tag{2}$$

which satisfies the equation $y_k = \mathbf{C}x_k$. Now consider the control input structure

$$u(t) = \begin{bmatrix} L_0 & L_1 & \cdots & L_{N-1} \end{bmatrix} \begin{bmatrix} y(k\tau - \tau) \\ y(k\tau - \tau + \Delta) \\ \vdots \\ y(k\tau - \Delta) \end{bmatrix} = Ly_k \tag{3}$$

If L satisfies the equation

$$L\mathbf{C} = F \tag{4}$$

it would realize the effect of F in the system described by Eqn. (1).

A. CLOSED LOOP STABILITY

For $0 < t \leq \tau$ the input $u(t)$ cannot be computed from Eqn. (3) as output measurements are not available for $t < 0$. If initial state x_0 is known one can take $u_0 = Fx_0$. However, the error in this estimate (Δx_0) would cause the value of u_0 will differ by $\Delta u_0 = F\Delta x_0$ from the truly required control signal $Fx(0)$. If $\Delta x_0 \neq 0$, the assumption $u_k = Fx_k$ and therefore $y_k = \mathbf{C}x_k$ do not hold, and the effect of initial error Δu_0 will propagate through the closed loop response of the system. One can verify that this closed loop dynamics are governed by

$$\begin{aligned} \begin{bmatrix} x_{k+1} \\ \Delta u_{k+1} \end{bmatrix} &= \begin{bmatrix} \Phi_\tau + \Gamma_\tau F & \Gamma_\tau \\ 0 & \Psi \end{bmatrix} \begin{bmatrix} x_k \\ \Delta u_k \end{bmatrix} \\ \Delta u(k) &= u_k - Fx_k, \Psi = LD_0 - F\Gamma_\tau \end{aligned} \quad (5)$$

Thus we have the eigenvalues of the closed loop system under a fast output sampling control law in Eqn. (3) as those of $\Phi_\tau + \Gamma_\tau F$ together with those of $LD_0 - F\Gamma_\tau$. Both of these subsystems need to be stable for the FOS feedback to function successfully.

III. PROPOSED ALGORITHM

The proposed algorithm incorporates a slight modification of the above strategy. Consider Eqn. (1). Here, the matrix C_0 is of dimension $pN \times n$, and since the system is observable, it is of rank n. Therefore, $C_0^T C_0$ would be an invertible $n \times n$ square matrix. Thus, from Eqn. (1),

$$C_0^T y_{k+1} = C_0^T \left(C_0 x(k) + D_0 u(k) \right)$$

wherefrom the value of $x(k)$ and $x(k+1)$ may be calculated as

$$\begin{aligned} x(k) &= \left(C_0^T C_0 \right)^{-1} C_0^T \left(C_0^T y_{k+1} - D_0 u(k) \right) \\ x(k+1) &= \Phi_\tau \left(C_0^T C_0 \right)^{-1} C_0^T y_{k+1} \\ &\quad + \left(\Gamma_\tau - \Phi_\tau \left(C_0^T C_0 \right)^{-1} C_0^T D_0 \right) u(k) \end{aligned}$$

or

$$\begin{aligned} x(k) &= \Phi_\tau \left(C_0^T C_0 \right)^{-1} C_0^T y_k \\ &\quad + \left(\Gamma_\tau - \Phi_\tau \left(C_0^T C_0 \right)^{-1} C_0^T D_0 \right) u(k-1) \end{aligned} \quad (6)$$

Thus, the state feedback control $u(k) = Fx(k)$ can be realized by substituting for $x(k)$ from Eqn. (6) as

$$\begin{aligned} u(k) &= F\Phi_\tau \left(C_0^T C_0 \right)^{-1} C_0^T y_k \\ &\quad + F \left(\Gamma_\tau - \Phi_\tau \left(C_0^T C_0 \right)^{-1} C_0^T D_0 \right) u(k-1) \end{aligned}$$

By not assuming a control structure a priori, the technique eliminates the non-singularity constraint of $(\Phi_\tau + \Gamma_\tau F)$ as imposed in [2] as it does not need to invert the matrix. Due to the same reason, the error in the assumed initial state also does not propagate in the proposed control algorithm.

IV. NUMERICAL EXAMPLE

Consider the discrete time system

$$\begin{aligned} X(k+1) &= \begin{bmatrix} 1.011 & 0.117 \\ 0.234 & 1.362 \end{bmatrix} X(k) + \begin{bmatrix} 0.006 \\ 0.117 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 1 & 1 \end{bmatrix} X(k) \end{aligned}$$

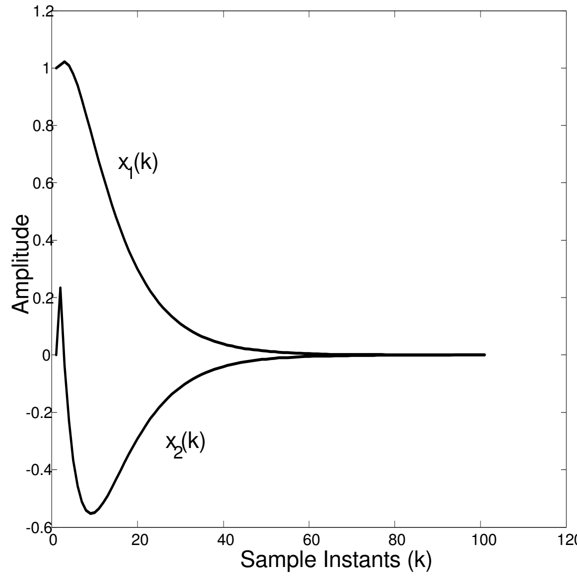

Figure 1 – Closed loop state response using the proposed control law.

A stabilizing state feedback gain $F = \begin{bmatrix} -3.712 & -5.578 \end{bmatrix}$ is designed. Using the proposed algorithm, for a value of $N = 3$, the control input is found to be

$$u(k) = \begin{bmatrix} 18.137 & -0.027 & 20.1 \end{bmatrix} y_k + 0.1447 u(k-1)$$

For an initial condition of $X(0) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$, and an initial estimate $X_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, the controller is able to recover and stabilize the system in spite of the large estimation error. The simulation results are presented in Fig. (1)

V. CONCLUSION

A new method for the computation of fast output sampling control has been presented in this paper. The method uses previous control input $u(k-1)$ to compute the present control input $u(k)$. In this process, it avoids the inversion of the closed loop matrix $(\Phi_\tau + \Gamma_\tau F)$ and hence eliminates the constraint on F for the former's invertibility. Moreover, the procedure also has an inherent property of non-propagation of initial state estimation error. This property has been illustrated through the simulation result presented.

References

[1] V. L. Syrmos, C. T. Abdallah, P. Dorato and K. Grigoriadis, " Static Output Feedback–A Survey", *Automatica,* Vol. 33, Issue 2, pp. 125-137, Feb. 1997

[2] H. Werner, K. Furuta, "Simultaneous stabilization based on output measurement", *Kybernetika,* Issue 31, pp. 395-411, 1995.