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Effect of boundary absorption on dispersion in Casson fluid flow in an annulus: application to catheterized artery

Received: 23 April 2007 / Revised: 7 February 2008 / Published online: 4 June 2008
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Abstract The combined effect of annular gap, yield stress and irreversible boundary reaction on the dispersion process in a Casson fluid flow is studied using generalized dispersion model. The study describes the development of dispersive transport following the injection of a tracer in terms of the three effective transport coefficients viz. absorption, convection and dispersion coefficients. The combined effect of annular gap, yield stress and wall absorption parameter on the above three effective transport coefficients is discussed. It is observed that the absorption coefficient is independent of the yield stress of the fluid and depends on the annular gap and wall absorption parameter. It is also observed that the asymptotic convection, dispersion coefficients are dependent on the yield stress of the fluid, annular gap and wall absorption parameter. The effect of the flow parameters on the mean concentration is studied. Application of this model for understanding the dispersion of solute in blood in a catheterized artery is discussed.

1 Introduction

The study of dispersion of a tracer in an annulus when it is either irreversibly absorbed or undergoes an exchange process at the boundary has many applications in the fields of chemical engineering, environmental dynamics, biomedical engineering and physiological fluid dynamics. The studies on natural convection mass/heat transfer in an annulus bounded by two horizontal cylinders have applications in nuclear reactors, recovery of geothermal energy and exothermal chemical reaction [1, 2]. An annular chromatographic method is used for separation of metals, sugars and proteins, etc. [3–5]. An example from clinical medicine is the dye or indicator dilution technique, which is a common practice among physiologists. In these techniques, catheters are used to inject the dye and to withdraw blood samples for the purpose of measurements. Also, the study of blood-tissue exchange in the physiological system is done by multiple indicator techniques. The dispersion of mass or heat in blood vessels is influenced by the conductive blood vessel walls as well as the inserted catheter. Modelling of this situation is usually done by considering the flow as the annular flow between the inserted catheter and blood vessel wall [6–9].

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The dispersion of a bolus of a passive tracer in a straight circular tube was first studied by Taylor [10]. Later Gill and Sankarasubramanian [11] studied the unsteady convective diffusion problem for laminar flow in a circular tube by means of a series solution. This model is widely referred to as the generalized dispersion model. This analysis was also extended to incorporate the interphase transport effects by Sankarasubramanian and Gill [12]. They concluded that the three effective transport coefficients, namely the exchange, the convection and the dispersion coefficients were all influenced by the interphase transport. Rao and Deshikachar [13] extended the exact analysis of Gill and Sankarasubramanian [11] to study the unsteady convective diffusion of a solute in a fully developed flow in an annulus. Davidson and Schroter [14] studied the pattern of dispersion and uptake of an inhaled slug of tissue soluble gas within a bronchial wall as an assembly of straight rigid tubes with absorbing wall of finite thickness. Phillips et al. [15] studied the transport of a tracer substance through a wall layer consisting of a tube containing flowing fluid surrounded by a wall layer in which the tracer was soluble. Jayaraman et al. [16] extended the theoretical model of Davidson and Schroter [14] for the dispersion of solute in a fluid flowing through a curved tube with absorbing walls by using a mathematical model of an infinitely long conduit defined by two concentric curved circular pipes. Balasubramanian et al. [17, 18] studied the retention effects upon dispersion in secondary flows. Sarkar and Jayaraman [19] modelled the effect of irreversible boundary reaction on the dispersion of a tracer in steady annular flow, with possible application to the catheterized artery. In a subsequent study, Sarkar and Jayaraman [20] analysed the effect of permeable wall characteristics in the study of the dispersion in an oscillatory flow in an annulus.

The study of dispersion through non-Newtonian fluids has abundant applications in polymer and biochemical processing and in cardiovascular system. A brief review on dispersion in power law fluids was given by Agarwal and Jayaraman [21]. Sharp [22] studied the dispersion in non-Newtonian fluids (Casson, Bingham plastic and power law fluids) through conduits. Dash et al. [23] analysed the shear augmented unsteady dispersion of a solute in a Casson fluid flowing in a conduit using the generalized dispersion model. Nagarani et al. [24] extended this study by incorporating the effect of wall absorption and observed that the rate of dispersion is significantly influenced by the wall absorption parameter and yield stress of the fluid. In a recent paper, Nagarani et al. [25] studied the dispersion process in Casson fluid flow in an annulus and discussed the combined effect of non-Newtonian rheology of the fluid and annular gap on the dispersion process.

The present paper deals with the study of dispersion of solute in a Casson fluid flowing in an annulus with interphase mass transfer at the outer boundary. The generalized dispersion model of Sankarasubramanian and Gill [12] is used to study the dispersion process. The effect of wall absorption parameter, yield stress and annular gap on the three dispersion coefficients, namely absorption, convection and dispersion coefficients and mean concentration is investigated. The application of the model to the dispersion in a catheterized artery is discussed.

2 Mathematical formulation

Figure 1 shows the schematic diagram of the annular geometry. The radius of the outer tube is a and that of the inner tube is ka with $k < 1$. For a fully developed laminar flow of a Casson fluid in an annulus, the convective diffusion equation, which describes the local concentration C of a solute as a function of axial distance z , radial distance r and time t in non-dimensional form is given by

$$\frac{\partial C}{\partial t} + w \frac{\partial C}{\partial z} = \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{Pe^2} \frac{\partial^2}{\partial z^2} \right) C \quad (1)$$

with the non-dimensional variables

$$C = \frac{\bar{C}}{C_0}, \quad w = \frac{\bar{w}}{w_0}, \quad r = \frac{\bar{r}}{a}, \quad z = \frac{D_m \bar{z}}{a^2 w_0}, \quad t = \frac{D_m \bar{t}}{a^2}, \quad Pe = \frac{a w_0}{D_m} \quad (2)$$

where w is the non-dimensional axial velocity of the fluid, D_m is the coefficient of molecular diffusion (molecular diffusivity), which is assumed to be constant, C_0 is the reference concentration, Pe is the Peclet number and w_0 is the characteristic velocity. The variables with bars represent the corresponding dimensional quantities.

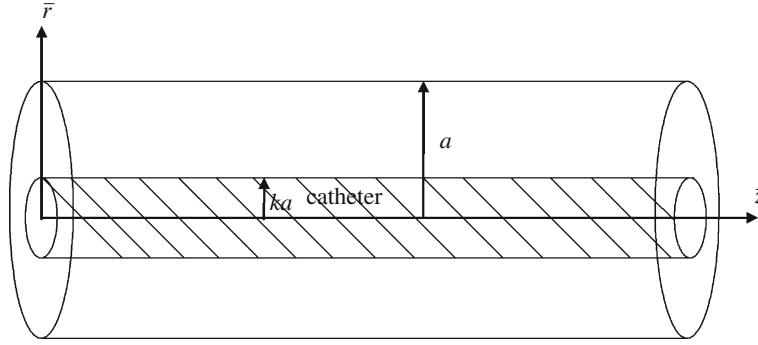


Fig. 1 Schematic diagram of a catheterized artery

2.1 Initial and boundary conditions

2.1.1 Initial condition

At any instant of time, the amount of tracer left in the system, its convective velocity and the extent of shear distribution depend upon the initial discharge. The initial distribution is taken at $t = 0$ as the case when a solute of mass m is introduced instantaneously at the plane $z = 0$, uniformly over the cross-section of a circle of radius “ da ” (where $k < d \leq 1$), concentric, in the annular region, with the tube. Thus, the initial distribution of the solute assumed in a variable separable form in terms of the non-dimensional quantities is given by

$$C(0, z, r) = \psi(z)B(r) \quad (3)$$

with

$$\psi(z) = \delta(z)/((d^2 - k^2)Pe) \quad (4.1)$$

and

$$B(r) = \begin{cases} 1 & k < r \leq d \\ 0 & d < r \leq 1 \end{cases} \quad (4.2)$$

where $\delta(z)$ denotes the Dirac delta function.

2.1.2 Boundary conditions

In the current model, we consider the heterogeneous reaction mechanism occurring at the outer wall ($r = 1$) of the tube so that

$$\frac{\partial C}{\partial r}(t, z, 1) = -\beta C(t, z, 1), \quad (5)$$

where β is the non-dimensional wall reaction parameter.

In addition, we have the boundary condition at $r = k$

$$\frac{\partial C}{\partial r}(t, z, k) = 0 \quad (6)$$

describing the impermeable wall of the inner tube and

$$C(t, \infty, r) = \frac{\partial C}{\partial z}(t, \infty, r) = 0. \quad (7)$$

2.2 Constitutive equation and velocity distribution for a Casson fluid model

The constitutive equation for a Casson fluid relating stress (τ) and strain rate ($\frac{\partial w}{\partial r}$) in non-dimensional form is given by

$$\tau^{1/2} = \tau_y^{1/2} + \left(-\frac{\partial w}{\partial r}\right)^{1/2} \quad \text{if } \tau \geq \tau_y \quad (8.1)$$

$$\frac{\partial w}{\partial r} = 0 \quad \text{if } \tau \leq \tau_y \quad (8.2)$$

where

$$\tau = \frac{\bar{\tau}}{\mu(w_0/a)} \quad (8.3)$$

and

$$\tau_y = \frac{\bar{\tau}_y}{\mu(w_0/a)} \quad (8.4)$$

where τ_y is the yield stress of the fluid, $\bar{\tau}$ and $\bar{\tau}_y$ are the dimensional shear stress and yield stress, respectively, and μ the coefficient of viscosity of the fluid. The above relations between the shear stress (τ) and shear rate ($\frac{\partial w}{\partial r}$) are appropriate for positive values of τ and negative values of $\frac{\partial w}{\partial r}$. The equivalent form of these relations for a more general situation, where the shear stress and shear rate can change sign (as in the case of annular flows [26–28]) may be written as

$$\frac{\partial w}{\partial r} = - \left[1 + \frac{\tau_y}{|\tau|} - 2 \frac{\tau_y^{1/2}}{|\tau|} \right] \tau \quad \text{if } |\tau| \geq \tau_y \quad (9.1)$$

and

$$\frac{\partial w}{\partial r} = 0 \quad \text{if } |\tau| \leq \tau_y. \quad (9.2)$$

From Eq. (9) it is seen that the flow of a Casson fluid in an annular region has three phases such that in the central core region the velocity profile is flat and hence forms the plug flow region. In this plug flow region, the shear stress does not exceed the yield stress τ_y , and the fluid does not flow by itself, but is merely carried along by the fluid in the two adjacent shear flow regions as a solid body with a constant velocity, the plug flow velocity. If the plug flow region is represented by $\lambda_1 \leq r \leq \lambda_2$ where $k \leq \lambda_1$, $\lambda_2 \leq 1$, and the two shear flow regions by $k \leq r \leq \lambda_1$ and $\lambda_2 \leq r \leq 1$, then the Casson's constitutive Eq. (9) in these regions can be written as

$$\frac{\partial w}{\partial r} = -\tau + \tau_y - 2\tau_y^{1/2} |\tau|^{1/2} \quad \text{if } k \leq r \leq \lambda_1 \quad (10.1)$$

$$\frac{\partial w}{\partial r} = 0 \quad \text{if } \lambda_1 \leq r \leq \lambda_2 \quad (10.2)$$

$$\frac{\partial w}{\partial r} = -(\tau + \tau_y - 2\tau_y^{1/2} \tau^{1/2}) \quad \text{if } \lambda_2 \leq r \leq 1. \quad (10.3)$$

where λ_1 and λ_2 are the yield plane locations. The velocity distribution in these three regions is given by Dash et al. [29]

$$w(r) = w^+(r) = \left[2\lambda^2 \log\left(\frac{r}{k}\right) - (r^2 - k^2) + 2\Lambda(r - k) - 4\Lambda^{\frac{1}{2}} \int_k^r \left(\frac{\lambda^2 - r^2}{r}\right)^{\frac{1}{2}} dr \right] \quad \text{if } k \leq r \leq \lambda_1 \quad (11.1)$$

$$w = w^-(r) = w_p = w^+(\lambda_1) = w^{++}(\lambda_2) = \text{constant} \quad \text{if } \lambda_1 \leq r \leq \lambda_2 \quad (11.2)$$

$$w(r) = w^{++}(r) = \begin{cases} (1 - r^2) + 2\lambda^2 \log r + 2\Lambda(1 - r) - 4\Lambda^{\frac{1}{2}} \int_r^1 \left(\frac{r^2 - \lambda^2}{r} \right)^{\frac{1}{2}} dr \\ \text{if } \lambda_2 \leq r \leq 1 \end{cases} \quad (11.3)$$

where

$$\Lambda = \lambda_2 - \lambda_1 = \frac{\tau_y}{2} \quad (12.1)$$

is the width of the plug flow region and

$$\lambda^2 = \lambda_1 \lambda_2. \quad (12.2)$$

The superscripts “+” and “+ +” represent the shear flow regions $k \leq r \leq \lambda_1$ and $\lambda_2 \leq r \leq 1$, respectively, and the superscript “-” represents the plug flow region $\lambda_1 \leq r \leq \lambda_2$. From Eq. (12.1) and from Eq. (11.2) and using $\lambda^2 = \lambda_1 \lambda_2$ from Eq. (12.2), we obtain

$$\begin{aligned} & 3\Lambda^2 - (1 + k)(1 - k + 2\Lambda) + 6\Lambda\lambda_1 - 2\lambda_1(\lambda_1 + \beta) \ln \left\{ \frac{k(\lambda_1 + \Lambda)}{\lambda_1} \right\} \\ & - 4\Lambda^{\frac{1}{2}} \left[\int_k^{\lambda_1} \left\{ \frac{\lambda_1(\lambda_1 + \Lambda) - r^2}{r} \right\}^{\frac{1}{2}} dr - \int_{\lambda_1 + \Lambda}^1 \left\{ \frac{r^2 - \lambda_1(\lambda_1 + \Lambda)}{r} \right\}^{\frac{1}{2}} dr \right] = 0 \end{aligned} \quad (13)$$

where $\Lambda = \frac{\tau_y}{2}$ which is an integral equation to be solved for λ_1 numerically by using Regula-Falsi method, and λ_2 can be obtained from Eq. (12.1), once λ_1 is known.

3 Method of solution

In order to solve the convective-diffusion equation (1) along with the associated sets of initial and boundary conditions (3)–(7), we express the concentration following Sankarasubramanian and Gill [12] as

$$C(t, r, z) = \sum_{n=0}^{\infty} f_n(t, r) \frac{\partial^n C_m}{\partial z^n} \quad (14)$$

where the average concentration C_m is expressed as

$$C_m = \frac{\int_0^{2\pi} \int_k^1 C r dr d\theta}{\int_0^{2\pi} \int_k^1 r dr d\theta} = \frac{2}{(1 - k^2)} \int_k^1 C r dr. \quad (15)$$

We assume that the distribution of mean concentration C_m can be described by the generalized dispersion model truncated after three terms as

$$\frac{\partial C_m}{\partial t} = K_0(t)C_m + K_1(t) \frac{\partial C_m}{\partial z} + K_2(t) \frac{\partial^2 C_m}{\partial z^2} \quad (16)$$

where $K_0(t)$, the absorption coefficient, accounts for the non-zero solute flux at the tube wall and will be negative in this problem indicating the depletion of the solute in the system with time caused by the irreversible reaction occurring at the tube wall, $K_1(t)$, the convection coefficient, accounts for the velocity of the reactive tracer, and $K_2(t)$, the dispersion coefficient, provides the modification in the convective dispersion due to absorption.

Multiplying Eq. (1) by $\frac{2r}{1 - k^2}$ and integrating with respect to r from k to 1 yield

$$\frac{\partial C_m}{\partial t} = \frac{1}{Pe^2} \frac{\partial^2 C_m}{\partial z^2} + \frac{2}{1 - k^2} \frac{\partial C}{\partial r}(t, z, 1) - \frac{2}{1 - k^2} \frac{\partial}{\partial z} \int_0^1 w(t, r) C(t, z, r) r dr. \quad (17)$$

Introducing Eqs. (15) and (16) into Eq. (17) and equating the coefficients of $\frac{\partial^n C_m}{\partial z^n}$, $n = 0, 1, 2, \dots$ we obtain

$$K_n(t) = \frac{\delta_{n2}}{Pe^2} + \frac{2}{1-k^2} \frac{\partial f_n}{\partial r}(t, 1) - \frac{2}{1-k^2} \int_k^1 f_{n-1}(t, r) w(t, r) r dr, \quad n = 0, 1, 2, \dots \quad (18)$$

where δ denotes the Kronecker delta.

Using Eqs. (15), (16) in Eq. (1) and equating the coefficients of $\frac{\partial^n C_m}{\partial z^n}$, $n = 0, 1, 2, \dots$, the following set of differential equations for f_n is obtained as

$$\frac{\partial f_n}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f_n}{\partial r} \right) - w(t, r) f_{n-1} + \frac{1}{Pe^2} f_{n-2} - \sum_{i=0}^n K_i f_{n-i}, \quad n = 0, 1, 2, \dots \quad (19)$$

where $f_{-1} = 0 = f_{-2}$.

The initial and boundary conditions on C_m and f_n are obtained from Eqs. (3)–(7) as below.

From Eqs. (3), (4) and (15), we get

$$C_m(0, z) = \frac{2}{1-k^2} \int_k^1 r C(0, r, z) dr = \frac{2\psi(z)}{1-k^2} \int_k^1 r B(r) dr \quad (20.1)$$

which gives

$$f_0(0, r) = \frac{1-k^2}{2} \frac{B(r)}{\int_k^1 r B(r) dr} \quad (20.2)$$

and

$$f_n(0, r) = 0 \quad n = 1, 2, \dots \quad (20.3)$$

$$\frac{\partial f_n}{\partial r}(t, 1) = -\beta f_n(t, 1) \quad n = 0, 1, 2, \dots \quad (20.4)$$

$$\frac{\partial f_n}{\partial r}(t, k) = 0 \quad \text{for } n = 0, 1, 2, \dots \quad (20.5)$$

$$\int_k^1 f_n(t, r) r dr = \frac{1-k^2}{2} \delta_{n0}, \quad \text{for } n = 0, 1, 2, \dots \quad (20.6)$$

The coupled Eqs. (18) and (19) are solved one after the other to obtain the solution of (f_n, K_n) for $n = 0, 1, 2$.

3.1 Evaluation of $f_0(t, r)$ and $K_0(t)$

The functions f_0 and absorption coefficient K_0 are independent of the velocity field and hence can be solved directly.

From Eq. (19) for $n = 0$, we obtain

$$\frac{\partial f_0}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f_0}{\partial r} \right) - f_0 K_0. \quad (21)$$

Introducing the transformation

$$f_0(t, r) = e^{-\int_0^t K_0(\eta) d\eta} g_0(t, r) \quad (22)$$

Eq. (21) takes the form

$$\frac{\partial g_0}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial g_0}{\partial r} \right). \quad (23)$$

The initial and boundary conditions on g_0 are

$$g_0(0, r) = \frac{1 - k^2}{2} \frac{B(r)}{\int_k^1 r B(r) dr}, \quad (24)$$

$$\frac{\partial g_0}{\partial r}(t, 1) = -\beta g_0(t, 1), \quad (25)$$

$$\frac{\partial g_0}{\partial r}(t, k) = 0. \quad (26)$$

The solution of g_0 satisfying the initial and boundary conditions (24)–(26) is given by

$$g_0(t, r) = \sum_{n=0}^{\infty} \frac{A_n}{J_1(\mu_n k)} E_0(\mu_n r) e^{-\mu_n^2 t} \quad (27)$$

where

$$E_0(\mu_n r) = [J_1(\mu_n k) Y_0(\mu_n r) - Y_1(\mu_n k) J_0(\mu_n r)] \quad (28)$$

and μ_n are the roots of the transcendental equation

$$\mu_n [Y_1(\mu_n k) J_1(\mu_n) - Y_1(\mu_n) J_1(\mu_n k)] + \beta [Y_0(\mu_n) J_1(\mu_n k) - Y_1(\mu_n k) J_0(\mu_n)] = 0, \quad (29)$$

and

$$A_n = \frac{\mu_n^2 (1 - k^2) J_1(\mu_n k)}{(\mu_n^2 + \beta^2) E_0^2(\mu_n) - k^2 \mu_n^2 E_0^2(\mu_n k)} \frac{\int_k^1 r B_1(r) E_0(\mu_n r) dr}{\int_k^1 r B_1(r) dr} \quad (30)$$

and J_0 , J_1 and Y_0 , Y_1 are the Bessel functions of first kind and second kind of order zero and one, respectively.

From Eq. (20.6) we have

$$\int_k^1 r f_0(t, r) dr = \frac{1 - k^2}{2}. \quad (31)$$

Using the transformation Eq. (22) and Eq. (31) we obtain

$$e^{-\int_0^t K_0(\eta) d\eta} = \frac{1 - k^2}{2 \int_k^1 g_0(t, r) r dr}. \quad (32)$$

From Eqs. (32) and (22) the expression $f_0(t, r)$ can be obtained as

$$f_0(t, r) = \frac{1 - k^2}{2} \frac{g_0(t, r)}{\int_k^1 g_0(t, r) r dr}, \quad (33)$$

which can be written as

$$f_0(t, r) = \frac{1 - k^2}{2} \frac{\sum_0^{\infty} \frac{A_n}{J_1(\mu_n k)} E_0(\mu_n r) e^{-\mu_n^2 t}}{\sum_0^{\infty} \frac{A_n}{\mu_n J_1(\mu_n k)} e^{-\mu_n^2 t} [J_1(\mu_n k) Y_1(\mu_n) - Y_1(\mu_n k) J_1(\mu_n)]}. \quad (34)$$

Using Eqs. (17) and (34) the absorption coefficient $K_0(t)$ is obtained as

$$K_0(t) = - \frac{\sum_0^\infty \frac{A_n \mu_n}{J_1(\mu_n k)} [J_1(\mu_n k) Y_1(\mu_n) - Y_1(\mu_n k) J_1(\mu_n)] e^{-\mu_n^2 t}}{\sum_0^\infty \frac{A_n}{\mu_n J_1(\mu_n k)} [J_1(\mu_n k) Y_1(\mu_n) - Y_1(\mu_n k) J_1(\mu_n)] e^{-\mu_n^2 t}}. \quad (35)$$

The expressions for $f_0(t, r)$ and $K_0(t)$ reduce to those derived by Sankarasubramanian and Gill [12] in the limiting case as $k \rightarrow 0$.

3.2 Asymptotic expansions for f_n 's and K_n 's for $n = 0, 1, 2, \dots$ for steady flow

Due to the coupling between $f_n(t, r)$ and $K_n(t)$ the solution of the transformed problem for, say $f_1(t, r)$ would result in a complicated integral equation in $K_1(t)$. The higher order functions are even more difficult to solve. Hence, we restrict ourselves to the asymptotic steady state representations of $f_n(t, r)$ and $K_n(t)$ for the case of steady flow and we will obtain solutions (f_n, K_n) , $n = 0, 1, 2$ for large times, so that the dispersion model defined in Eq. (16) is a representation of the asymptotic results under steady state conditions.

As $t \rightarrow \infty$, Eqs. (34) and (35) give the following asymptotic representation for f_0 and K_0 .

$$f_0(\infty, r) = \frac{1 - k^2}{2} \frac{\mu_0 E_0(\mu_0 r)}{[J_1(\mu_0 k) Y_1(\mu_0) - Y_1(\mu_0 k) J_1(\mu_0)]} \quad (36)$$

$$K_0(\infty) = -\mu_0^2 \quad (37)$$

where μ_0 is the first root (lowest in magnitude) of Eq. (29).

For large times and using Eq. (37), the Eq. (19) reduces to

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f_n}{\partial r} \right) + \mu_0^2 f_n = w(t, r) f_{n-1} - \frac{1}{P e^2} f_{n-2} + \sum_{i=1}^n K_i f_{n-i}. \quad (38)$$

The corresponding boundary conditions on $f_n(r)$ are

$$\frac{\partial f_n}{\partial r}(r = 1) = -\beta f_n(1) \quad n = 1, 2, \dots \quad (39.1)$$

$$\frac{\partial f_n}{\partial r}(r = k) = 0 \quad n = 1, 2, \dots \quad (39.2)$$

and

$$\int_k^1 f_n(r) r dr = 0 \quad n = 1, 2, \dots \quad (39.3)$$

For large times, the Eq. (18) for the K_n 's reduces to

$$K_n = \frac{\delta_{n2}}{P e^2} + \frac{2}{1 - k^2} \frac{\partial f_n}{\partial r}(1) - \frac{2}{1 - k^2} \int_k^1 f_{n-1}(r) w(r) r dr \quad n = 1, 2, \dots \quad (40)$$

The use of the solvability condition in Eq. (38) gives the expression for K_n as

$$K_n = \frac{\int_k^1 \left[\frac{1}{P e^2} f_{n-2} - w(t, r) f_{n-1} - \sum_{i=1}^{n-1} K_i f_{n-i} \right] E_0(\mu_0 r) r dr}{\int_k^1 f_0 E_0(\mu_0 r) r dr}. \quad (41)$$

For $n = 1$, the expression for the asymptotic convection coefficient K_1 can be obtained as

$$K_1 = \frac{-2\mu_0^2}{(\mu_0^2 + \beta^2) E_0^2(\mu_0) - k^2 \mu_0^2 E_0^2(\mu_0 k)} \int_k^1 w(r) r E_0^2(\mu_0 r) dr. \quad (42)$$

For large times the differential equation for f_1 from Eq. (38) can be written as

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f_1}{\partial r} \right) + \mu_0^2 f_1 = [w(r) + K_1] f_0. \quad (43)$$

Using Eqs. (42) and (43) the solution for f_1 satisfying the corresponding boundary conditions (39) with $n = 1$ is given by

$$f_1(r) = \sum_{n=0}^{\infty} B_n \frac{E_0(\mu_n r)}{J_1(\mu_n k)}. \quad (44)$$

The coefficient B_0 is obtained in terms of B_j ($j = 1, 2, \dots$).

With the help of the condition (39.3) with $n = 1$ and from Eq. (44) we get

$$B_0 = \frac{-J_1(\mu_0 k)}{\int_k^1 r E_0(\mu_0 r) dr} \sum_{n=1}^{\infty} B_n \frac{\int_k^1 r E_0(\mu_n r) dr}{J_1(\mu_n k)}. \quad (45)$$

Using Eqs. (44) and (45) the expression for f_1 can be written as

$$f_1 = \sum_{n=1}^{\infty} \frac{B_n}{J_1(\mu_n k)} \left[E_0(\mu_n r) - \frac{\int_k^1 r E_0(\mu_n r) dr}{\int_k^1 r E_0(\mu_0 r) dr} E_0(\mu_0 r) \right] \quad (46)$$

where B_n 's are given by

$$B_n = \frac{\int_k^1 [w(r) + K_1] f_0(r) r E_0(\mu_n r) dr}{(\mu_0^2 - \mu_n^2) J_1(\mu_n k)}. \quad (47)$$

The asymptotic dispersion coefficient $K_2(t)$ can be evaluated from Eq. (41) as

$$K_2 = \frac{1}{Pe^2} - \frac{\int_k^1 [w(r) + K_1] f_1 E_0(\mu_0 r) r dr}{\int_k^1 f_0 E_0(\mu_0 r) r dr}. \quad (48)$$

The expression for the mean concentration C_m is obtained by solving Eq. (16) with the help of boundary conditions (3), (7) and using Eqs. (4.1) as

$$C_m(t, z) = \frac{1}{2Pe\sqrt{\pi\xi}} \exp \left[\zeta - \frac{z_1^2}{4\xi} \right] \quad (49)$$

where

$$\zeta(t) = \int_0^t K_0(\eta) d\eta, \quad (50.1)$$

$$z_1(z, t) = z + \int_0^t K_1(\eta) d\eta, \quad (50.2)$$

$$\xi(t) = \int_0^t K_2(\eta) d\eta. \quad (50.3)$$

4 Results and discussion

The objective of the present study is to investigate the dispersion process following the injection of a chemically active tracer in a solvent flowing through an annulus with reactive outer boundary. The fluid flowing through the annulus is modelled as a Casson fluid. The derivative expansion method of Sankarasubramanian and Gill [12] is applied to analyse the dispersion process. The effect of the rheological parameter (yield stress) τ_y , the wall absorption parameter β and the annular gap on the three effective transport coefficients viz. absorption, convection and dispersion coefficients is studied.

In the present study, the ratio of the radius of the inner cylinder to the radius of the outer cylinder k is varied from 0.1 to 0.4. The value of the wall absorption parameter β is taken in the range of 0.01–100. The yield stress τ_y is taken in a range of 0.05–0.2. For each value of k and β the eigenvalues μ_n for $n = 0, 1, 2, \dots, 100$ are computed from Eq. (29) by using Newton–Raphson method. The integrals involved in Eqs. (42), (47) and (48) are evaluated numerically using Simpson's rule. The summations in Eqs. (34), (35) and (46) are calculated to six decimal places, hence the number of eigenvalues used in the evaluation of the transport coefficient varies from 15 to 30 for the variation of k and β . The analysis shows that the absorption coefficient $K_0(t)$ is independent of the rheological parameter τ_y of the fluid and depends on the annular gap and wall absorption parameter β . The asymptotic convection coefficient K_1 and asymptotic dispersion coefficient K_2 depend on all the three parameters τ_y, k, β . In the absence of yield stress, i.e. when the fluid is Newtonian and in the limiting case as $k \rightarrow 0$, the results are found to match with those of [12]. The results reduce to those of [13] when $\beta = 0$ and $\tau_y = 0$, and to those of [24] in the limiting case as $k \rightarrow 0$.

4.1 Absorption coefficient $K_0(t)$

Due to the irreversible reaction occurring at the outer boundary, the entire amount of the solvent does not remain constant. The coefficient $K_0(t)$, accounting for the non-zero solute at the outer wall, is negative to account for the depletion of solute in the system.

It is seen from Fig. 2a–c that when the radius d of the injected solute is 1 the absorption coefficient $-K_0(t)$ reaches its steady value very fast. For smaller values of the radius of the initial solute, $-K_0(t)$ takes more time to reach its asymptotic value.

When $d \neq 1$, $-K_0(t)$ is zero initially and becomes prominent when the solute gets diffused towards the wall. This can also be seen from the equation

$$\begin{aligned} K_0(0) &= -\frac{2}{1-k^2}\beta \quad (d=1) \\ &= 0 \quad (k \leq d < 1). \end{aligned}$$

For larger values of the absorption parameter ($\beta = 100$), the steady state is reached at earlier instances of time ($t = 0.2$) and more time ($t \approx 1$) is required to reach the steady state for small values of the absorption parameter ($\beta = 0.01$ and 1).

4.2 Asymptotic absorption coefficient K_0

The variation of the asymptotic absorption coefficient $-K_0$ with β for different values of k is shown in Fig. 3. It is observed that the insertion of a catheter (of size $k = 0.4$) enhances the absorption coefficient to 9.37 from a value 5.67 in the absence of a catheter ($k = 0$), when the wall absorption parameter is 100. Hence, it can be interpreted that the absorption of solutes at the wall is favoured in annular flows. This might be due to the decrease in the annular gap, i.e., when the size of the inner cylinder increases, the particles are pushed towards the wall, thus the transverse gradients in the system with wall reaction tend to increase transverse diffusion. $-K_0$ increases from 0.0202 to 5.8705 as the wall absorption parameter varies from 0.01 to 100 for k as small as 0.1, as was noticed by Sankarasubramanian and Gill [12] in a tubular flow.

4.3 Asymptotic convection coefficient K_1

A plot of the asymptotic convection coefficient $-K_1$ against the wall absorption parameter β is given in Fig. 4a for different values of k . In the case of a Newtonian fluid and when no solute flux is at the wall i.e.,

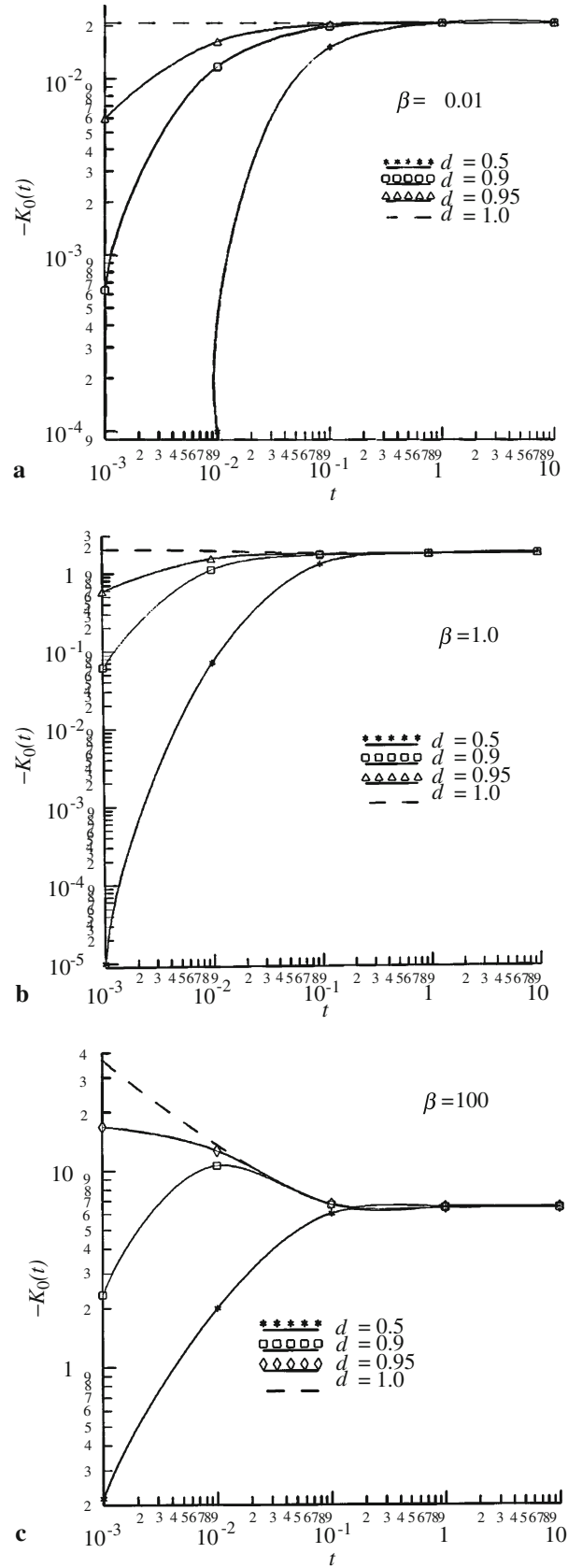


Fig. 2 Variation of the negative absorption coefficient $-K_0(t)$ with time t when $k = 0.2$

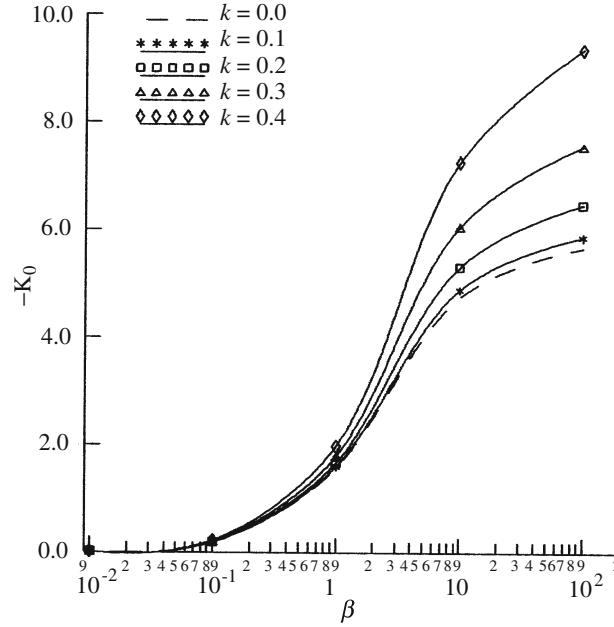


Fig. 3 Variation of the negative asymptotic absorption coefficient $-K_0$ with wall absorption parameter β

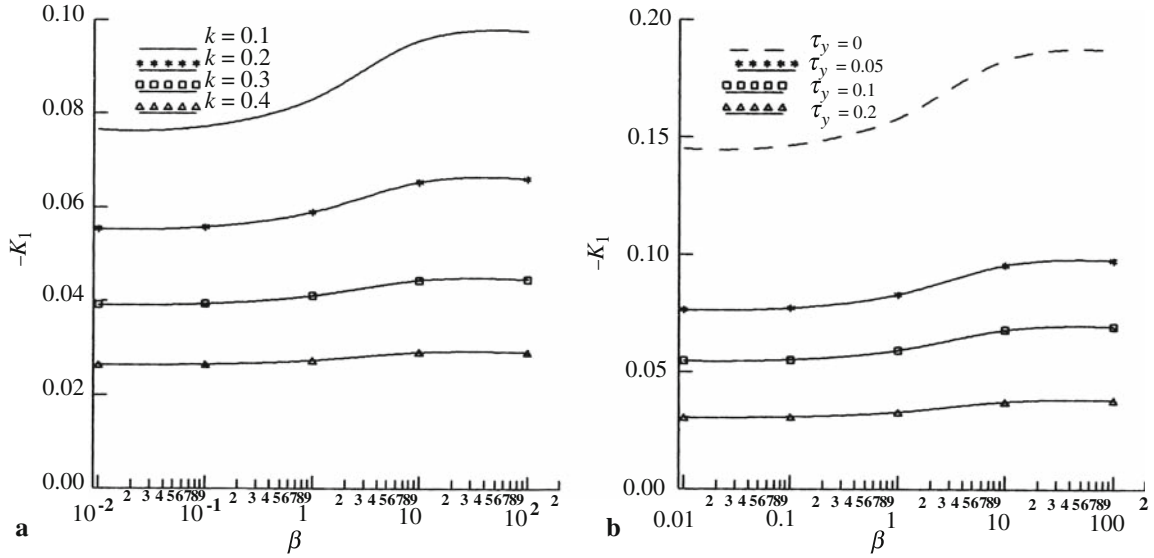


Fig. 4 Variation of the negative asymptotic convection coefficient $-K_1$ with wall absorption parameter β **a** for $\tau_y = 0.05$, **b** for $k = 0.1$

$\beta = 0$, $-K_1$ is equal to $1/2$ [13], which implies that the injected solute is convected along the annulus with an average velocity of the flow. In the case of pipe flow of a Newtonian fluid, when wall reaction is present [12], the solute moves with a velocity greater than the average velocity. However, in an annulus it is observed that the solute is convected along with a velocity lower than the average velocity of the flow when the yield stress is as small as 0.05. When the annular gap increases, it is seen that $-K_1$ decreases. The effect of yield stress on $-K_1$ is plotted in Fig. 4b for a fixed value of k ($=0.1$). When the yield stress τ_y is as small as 0.05, the value of $-K_1$ reduces to 0.0977 from 0.1872, the corresponding value in the case of a Newtonian fluid, for $\beta = 100$.

4.3.1 Asymptotic dispersion coefficient K_2

Figure 5a shows the variation of the asymptotic dispersion coefficient K_2 (from which $1/Pe^2$ has been deduced) versus the wall absorption parameter, β , for different values of k . In contrast to the effect of β on $-K_0$

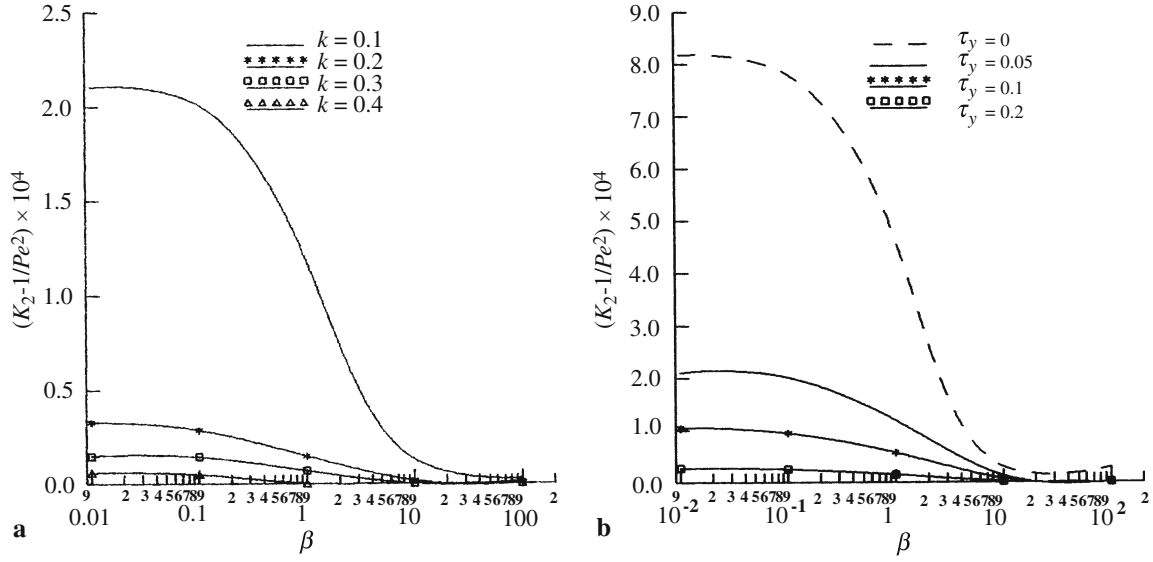


Fig. 5 Variation of the asymptotic dispersion coefficient $(K_2 - 1/Pe^2)$ with wall absorption parameter β **a** for $\tau_y = 0.05$, **b** for $k = 0.1$

and $-K_1$ it is seen that K_2 is effectively decreased by interfacial mass transfer. As $\beta \rightarrow \infty$ the dispersion coefficient $K_2 \rightarrow 2.953 \times 10^{-6}$, which is a 100-fold decrease from its value for no wall reaction, which is 2.104×10^{-4} when $k = 0.1$ and $\tau_y = 0.05$. Thus, one would expect that an increase in β leads to an increase in the number of moles of reactive solute undergoing absorption, and hence there is a significant decrease in dispersion coefficient. The absorption of solutes is observed to be more as the annular gap reduces. Due to the wall reaction, the transverse diffusion inhibits the axial dispersion and as a result the axial dispersion coefficient decreases as was noticed by Sankarasubramanian and Gill [12]. It is observed that K_2 decreases from (1.19×10^{-4}) when k is as small as 0.1 to 6.028×10^{-8} when $k = 0.4$ for $\beta = 1.0$ and $\tau_y = 0.05$, as in the case of dispersion of solute in a Newtonian fluid in an annulus without wall absorption [13]. The effect of yield stress on the asymptotic dispersion coefficient against wall absorption parameter is illustrated in Fig. 5b. It is found that the yield stress of the fluid decreases the dispersion coefficient significantly. When $\tau_y = 0.05$, the value of $K_2(1.19 \times 10^{-4})$ is reduced four times the value 4.66×10^{-4} , corresponding to the Newtonian case for $\beta = 1.0$. With increase in the yield stress, the values of K_2 are further decreased.

In Fig. 6 the variation in relative dispersion coefficient [i.e. $(K_2 - 1/Pe^2)/(K_2^N - 1/Pe^2)$, the superscript “N” refers to the value corresponding to the Newtonian fluid] for different values of yield stress is plotted. The relative dispersion coefficient decreases with increase in the yield stress. It is seen that it approaches zero as the yield stress takes the values 0.4.

4.4 Mean concentration

The variation of mean concentration $(C_m \times Pe)$ as a function of time for the variation of β , τ_y , k when $z = 0.5$ is shown in Fig. 7a–c. These profiles obtained from the Eq. (49). The transport coefficients involved are approximated by the asymptotic equivalents of the same. Figure 7a shows the variation of mean concentration with yield stress τ_y when $\beta (=0.01)$ and $k (=0.1)$ are fixed. It is observed from Fig. 7a that the peak of the mean concentration increases with τ_y . From Fig. 7a (when $\tau_y = 0.05$) and 7b it is seen that an increase in the absorption parameter β leads to a depletion in the amount of reactive material, and therefore the peak of the mean concentration decreases as was noticed by Sankarasubramanian and Gill [12] for a Newtonian case in a pipe flow. It decreases from 6.72 when $\beta = 0.01$ to 0.0089 as β increases to 1. From Fig 7a, c, it is observed that as k increases from 0.1 to 0.2 (when $\tau_y = 0.05$, $\beta = 0.01$), the peak of the mean concentration distribution increases. A similar pattern is observed in the case of Newtonian fluids when $\beta = 0.01$.

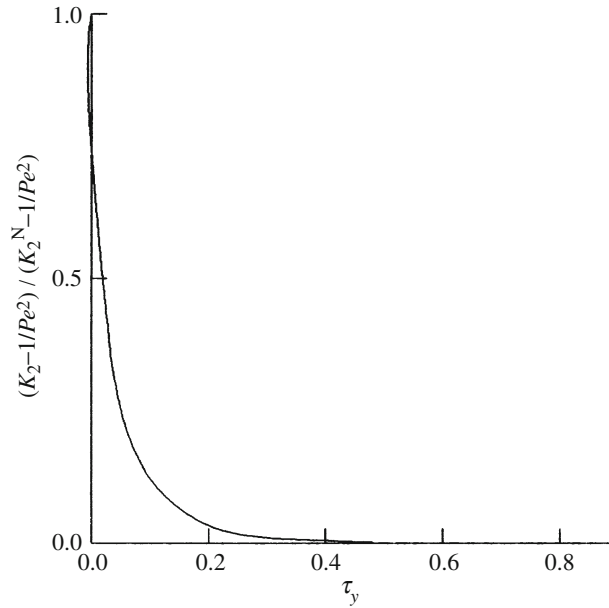


Fig. 6 Variation of the relative dispersion coefficient $(K_2 - 1/Pe^2)/(K_2^N - 1/Pe^2)$ with τ_y for $k = 0.1$, $\beta = 1.0$

5 Application to catheterized artery

The analysis of the present mathematical model can be used to understand the indicator dilution technique in a catheterized artery. It is important to study the dispersion mechanism considering the wall characteristics due to the conductive nature of artery walls. Catheters are used to inject the dye and to withdraw blood samples for the purpose of measurements [30,31]. The introduction of a catheter in the blood vessels can be a potent cause of eddy formation and mixing of blood. The sampling system always introduces some distortion in the time–concentration curve, so that the recorded curve is not of the same shape as the in situ concentration–time relation at the withdrawal site [32]. Experimental studies with calibrations based on mathematical models for the removal of the catheter distortion are reported by Parrish et al. [33] and Goresky and Silverman [34]. Results obtained from the mathematical model for the removal of catheter induced distortion of indicator dilution curves of Sparacino et al. [35] show sharper and higher peaks that were less smooth than the measured one.

The aim is to provide the correction to the values measured when a catheter is inserted in the artery, based on the concept of longitudinal dispersion of materials, due to the combined action of absorption, convection and diffusion. In the mathematical model described in Sect. 2, the outer cylinder represents the artery wall with conductance. The inner coaxial cylinder can correspond to a catheter. The value k (<1), ratio of the radius of the catheter to the artery, is varied in the range 0.1–0.4. The wall absorption parameter β quantifies the permeability of the arterial wall. Its values are varied from 0.01 to 1, so that the results can be related to physiological situations.

The variations of the asymmetric absorption coefficient $-K_0$, convection coefficient $-K_1$ and the dispersion coefficient $K_2 - 1/Pe^2$ are shown in Table 1 with the wall absorption parameter β and catheter size k . From Table 1, it can be seen that the presence of the catheter and increase in the size of the catheter help the dyes or other substances to get out of the blood vessels for all values of the absorption parameter β .

It is found that the presence of the catheter inhibits the convection coefficient [decreases from 0.0765 ($k = 0.1$) to 0.0264 ($k = 0.4$), when $\beta = 0.01$] and dispersion coefficient [decreases from 0.21×10^{-3} ($k = 0.1$) to 0.14×10^{-7} ($k = 0.4$), when $\beta = 0.01$].

Table 2 shows the values of the convective and dispersion coefficients for different values of yield stress and catheter radii. In the Newtonian case when the radius of the catheter is 0.4, it is observed that the value of the convective coefficient is reduced by four times of the corresponding value when $k = 0.1$ and the reduction factor in the dispersion coefficient is 1,000. When the radius of the catheter is 0.1 and the yield stress is 0.2, the value of the convective dispersion coefficient is decreased by 5 times the value corresponding to the Newtonian

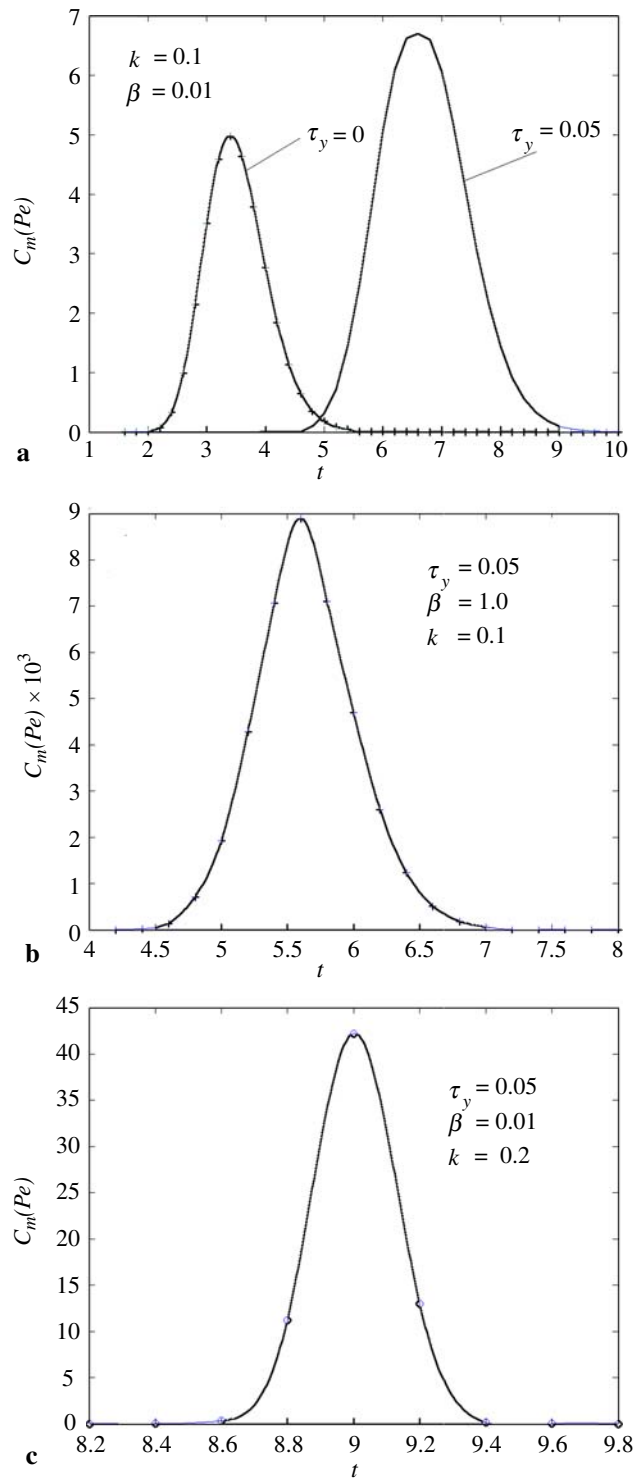


Fig. 7 Variation in the mean concentration with time

case, and the dispersion coefficient by 30 times. Further reductions in the coefficients are seen with an increase in the radius of the catheter. The combined effect of the presence or increase in the radius of the catheter and yield stress is to inhibit the dispersion coefficients.

Table 1 Variation of K_0 , K_1 and K_2 with β and k for $\tau_y = 0.05$

k	$\beta = 0.01$			$\beta = 0.1$			$\beta = 1.0$		
	$-K_0$	$-K_1$	$K_2 - 1/Pe^2$	$-K_0$	$-K_1$	$K_2 - 1/Pe^2$	$-K_0$	$-K_1$	$K_2 - 1/Pe^2$
0.1	0.02015	0.0765	0.21×10^{-3}	0.1971	0.0772	0.20×10^{-3}	1.6003	0.0831	0.11×10^{-5}
0.2	0.02078	0.0554	0.32×10^{-5}	0.2036	0.0558	0.29×10^{-5}	1.6701	0.0589	0.14×10^{-5}
0.3	0.02193	0.0394	0.15×10^{-6}	0.2152	0.0394	0.14×10^{-6}	1.7953	0.0411	0.69×10^{-7}
0.4	0.02376	0.0264	0.14×10^{-7}	0.2336	0.0265	0.12×10^{-7}	1.9893	0.0272	0.60×10^{-8}

Table 2 Variation of K_1 and K_2 with k and τ_y for $\beta = 1.0$

k	$\tau_y = 0.0$		$\tau_y = 1.0$		$\tau_y = 0.2$	
	$-K_1$	$K_2 - 1/Pe^2$	$-K_1$	$K_2 - 1/Pe^2$	$-K_1$	$K_2 - 1/Pe^2$
0.1	0.1583	0.46×10^{-3}	0.0593	0.57×10^{-4}	0.0330	0.15×10^{-4}
0.2	0.1185	0.66×10^{-5}	0.04064	0.71×10^{-6}	0.0210	0.16×10^{-6}
0.3	0.0879	0.34×10^{-6}	0.0271	0.29×10^{-7}	0.0127	0.50×10^{-8}
0.4	0.0632	0.31×10^{-7}	0.0171	0.23×10^{-8}	0.0070	0.24×10^{-9}

6 Conclusions

The dispersion of a solute in an annulus flowing through a Casson fluid is analysed. Considering the irreversible wall absorption characteristics, the entire dispersion process is characterized by the three transport coefficients viz. absorption coefficient, convection coefficient and dispersion coefficient. The absorption coefficient increases from 1.577 in the case of a pipe flow to 2.336 in an annular flow for $k = 0.4$ (when $\beta = 1$). Hence, absorption is favoured in an annular flow. The absorption and convection coefficients are seen to increase with β , while the dispersion coefficient is seen to decrease with β . The yield stress is found to have no effect on the absorption coefficient. When $\beta = 1.0$ and $k = 0.1$, the presence of yield stress decreases the values of the convection coefficient from 0.158 in the case of Newtonian fluid to 0.083, and the dispersion coefficient from 0.466×10^{-3} to 0.119×10^{-3} for $\tau_y = 0.05$. The results of the mathematical model are analysed with regard to the phenomenon of dispersion in a catheterized artery. It is seen that the presence of a catheter and increase in the catheter size help the dyes or other solutes to get off the blood vessels for all values of the absorption parameter. The coefficients of convection and dispersion are found to be affected significantly by the presence of a catheter and the non-Newtonian character of the fluid. Although the present study brings out the effect of wall absorption and non-Newtonian rheology on dispersion in a catheterized artery, the model has to be further redefined for the analysis of the dispersion process when the catheter is eccentrically positioned in the artery, which will be more realistic.

Acknowledgments The authors are grateful to the referees for their constructive suggestions and critical comments, which led to improve the quality of the paper.

References

1. Kuehn, T.H., Goldstein, R.J.: An experimental and theoretical study of natural convection in the annulus between horizontal concentric cylinders. *J. Fluid Mech.* **4**, 695–719 (1976)
2. Tsui, Y.T., Tremblay, B.: On transient natural convection heat transfer in the annulus between concentric, horizontal cylinders with isothermal surfaces. *Int. J. Heat Mass Trans.* **27**, 103–111 (1984)
3. Carta, G., Decarli, J.P., Byers, C.H., Sisson, W.G.: Separation of metals by continuous annular chromatography with step elution. *Chem. Eng. Commun.* **79**, 207–227 (1989)
4. DeCarli, J.P.: Displacement separations by continuous annular chromatography. *A I Ch. E J.* **36**, 1220–1228 (1990)
5. Bloomingburg, G.F., Carta, G.: Separation of protein mixtures by continuous annular chromatography with step elution. *Chem. Eng. J.* **55**, B19–B27 (1994)
6. Sarkar, A., Jayaraman, G.: Correction to flow rate-pressure drop relation in coronary angioplasty-steady streaming effect. *J. Biomech.* **31**, 781–791 (1998)
7. Sarkar, A., Jayaraman, G.: Nonlinear analysis of oscillatory flow in the annulus of an elastic tube—application to catheterized artery. *Phys. Fluids* **13**, 2901–2912 (2001)
8. Dash, R.K., Jayaraman, G., Mehta, K.N.: Flow in a catheterized artery with stenosis. *J. Biomech.* **32**, 49–61 (1999)

9. Back, L.H., Kwack, E.Y., Back, M.R.: Flow rate pressure drop relation in coronary angioplasty: catheter obstruction effect. *J. Biomech. Eng. Trans. ASME* **118**, 83–89 (1996)
10. Taylor, G.I.: Dispersion of soluble matter in solvent flowing slowly through a tube. *Proc. R. Soc. Lond. A* **219**, 186–203 (1953)
11. Gill, W.N., Sankarasubramanian, R.: Exact analysis of unsteady convective diffusion. *Proc. R. Soc. Lond. A* **316**, 341–350 (1970)
12. Sankarasubramanian, R., Gill, W.N.: Unsteady convective diffusion with interphase mass transfer. *Proc. R. Soc. Lond. A* **333**, 115–132 (1973)
13. Rao, A.R., Deshikachar, K.S.: An exact analysis of unsteady convective diffusion in an annular pipe. *ZAMM* **67**, 189–195 (1987)
14. Davidson, M.R., Schroter, R.C.: A theoretical model of absorption of gases by the bronchial wall. *J. Fluid Mech.* **129**, 313–335 (1983)
15. Phillips, C.G., Kaye, S.R., Robinson, C.D.: Time dependent transport by convection and diffusion with exchange between two phases. *J. Fluid Mech.* **297**, 373–401 (1995)
16. Jayaraman, G., Pedley, T.J., Goyal, A.: Dispersion of solute in a fluid flowing through a curved tube with absorbing walls. *QJMAM* **51**, 577–598 (1998)
17. Balasubramanian, V., Jayaraman, G., Iyengar, S.R.K.: Boundary retention effects upon contaminant dispersion in secondary flows. *ZAMM* **5**, 335–352 (1999)
18. Balasubramanian, V., Jayaraman, G., Iyengar, S.R.K.: Influence of secondary flow on diffusion with heterogeneous reaction. *Chem. Eng. J.* **92**, 15–26 (2003)
19. Sarkar, A., Jayaraman, G.: The effect of wall absorption in annular flows. *Acta Mech.* **158**, 105–119 (2001)
20. Sarkar, A., Jayaraman, G.: The effect of wall absorption on dispersion in oscillatory flow in an annulus: application to a catheterized artery. *Acta Mech.* **172**, 151–167 (2004)
21. Agarwal, S., Jayaraman, G.: Numerical simulation of dispersion in the flow of power-law fluids in curved tubes. *Appl. Math. Model.* **18**, 504–512 (1994)
22. Sharp, M.K.: Shear augmented dispersion in non-Newtonian fluids. *Ann. Biomed. Eng.* **21**, 407–415 (1993)
23. Dash, R.K., Jayaraman, G., Mehta, K.N.: Shear augmented dispersion of a solute in a Casson fluid flowing in a conduit. *Ann. Biomed. Eng.* **28**, 373–385 (2000)
24. Nagarani, P., Sarojamma, G., Jayaraman, G.: Effect of boundary absorption in dispersion in Casson fluid flow in a tube. *Ann. Biomed. Eng.* **32**, 706–719 (2004)
25. Nagarani, P., Sarojamma, G., Jayaraman, G.: Exact analysis of unsteady convective diffusion in Casson fluid flow in an annulus—Application to catheterized artery. *Acta Mech.* **187**, 189–202 (2006)
26. Bird, R.B., Stewart, W.E., Lightfoot, E.N.: *Transport Phenomena*. Wiley, New York (1960)
27. Aroesty, J., Gross, J.F.: The mathematics of pulsatile flow in small blood vessels I. Casson theory. *Microvasc. Res.* **4**, 1–12 (1972)
28. Aroesty, J., Gross, J.F.: Pulsatile flow in small blood vessels I. Casson theory. *Biorheology* **9**, 33–42 (1972)
29. Dash, R.K., Jayaraman, G., Mehta, K.N.: Estimation of increased flow resistance in a narrow catheterized artery—a theoretical model. *J. Biomech.* **29**, 917–930 (1996)
30. McDonald, D.A.: *Blood Flow in Arteries*. Edward Arnold, London (1974)
31. Mathie, R.T.: *Blood Flow Measurement in Man*. Castlehouse Publications Turnbridge Wells, England (1982)
32. Milnor, W.R., Jose, A.D.: Distortion of indicator-dilution curves by sampling systems. *J. Appl. Physiol.* **15**, 177–180 (1960)
33. Parrish, D., Gibbons, G.E., Bell, J.W.: A method for reducing the distortion produced by catheter sampling systems. *J. Appl. Physiol.* **17**, 369–371 (1962)
34. Goresky, C.A., Silverman, M.: Effect of correction of catheter distortion on calculated liver sinusoidal volumes. *Am. J. Physiol.* **207**, 883–892 (1963)
35. Sparacino, G., Vicini, P., Bonadona, R., Marraccini, P., Lehtovirta, M., Ferrannini, E., Cobelli, C.: Removal of catheter distortion in multiple indicator dilution studies: A deconvolution-based method and case studies on glucose blood-tissue exchange. *Med. Biol. Eng. Comput.* **35**, 337–343 (1997)