

ICNMM2018-7737

DRAFT: SPONTANEOUS DISPLACEMENT OF RESIDENT FLUID IN HETEROGENEOUS POROUS MEDIUM

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ABSTRACT

Surface tension driven flow in which one fluid displaces another is of importance in microfluidic devices for diagnostics, lab on chip devices and flow in oil reservoirs. Spontaneous impregnation of a preferentially wetting phase displacing an existing non-wetting phase in a homogeneous porous medium is known to follow diffusive dynamics. However, in a heterogeneous porous medium the hydrodynamic interaction between the narrow and the wide pores significantly alters the impregnation behavior. Previous studies have shown that the imbibing fluid interface leads in the narrow pores contrary to the predictions from the diffusive dynamics of homogeneous porous medium. This is due to the higher suction pressure in the narrow pores which draw fluid from the wide pores. The effect of fluid properties and relative flow properties of the pores with respect to other pores on the non-wetting fluid displacement in the heterogeneous porous medium is still unknown. In the current work, we develop a quasi one-dimensional, lubrication approximation model, which predicts the spontaneous imbibition in a heterogeneous porous medium. We explore all the possible relative fluid properties and flow properties of the layers in the heterogeneous porous medium and show that our model is able to predict the flow behavior in all the cases. We also present the results of the spontaneous imbibition experiments, which agree with our model. The experiments show that the two phase interface progresses faster in the narrow pores as predicted by the one-dimensional model. The result is

important for predicting and controlling the flow behavior in a heterogeneous porous medium.

NOMENCLATURE

- L Total length of the hydrodynamically interacting porous medium
- μ_w Viscosity of the invading fluid
- μ_{nw} Viscosity of the resident fluid
- ϕ Porosity of the medium
- γ Interfacial tension
- A Cross-section area of each layer
- d_i Diameter of glass beads in the layer $i (= s, m, l)$
- P_{c_i} Capillary pressure in layer filled with beads of diameter d_i
- c Proportionality constant in determining the capillary pressure
- η Ratio of the small and large bead diameters in a two layered porous medium
- x_i Length of impregnation in layer $i (= s, m, l)$
- X_i Dimensionless length of impregnation in layer i
- v_i Velocity of fluid in layer i at the inlet and outlet
- K_i Permeability of layer i
- Λ Proportionality constant in determining the permeability
- $\frac{dx_i}{dt}$ Interstitial velocity of layer i
- $\left(\frac{dP}{dx}\right)_k|_{x_i}^{x_j}$ Pressure gradient of $\mu_k (k = w, nw)$ in the length $(x_j - x_i)$
- $\frac{dX_i}{dT}$ Dimensionless interstitial velocity of layer i

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- dq Volumetric fluid exchange between the layers at the interfaces
- P_0 Atmospheric pressure at the inlet and outlet
- $P_w(x_i, t)$ Pressure of wetting phase at a distance x_i from the inlet at time t
- $P_{nw}(x_i, t)$ Pressure of resident phase at a distance x_i from the inlet at time t

Introduction

Surface tension driven flows can be found in varied systems of nature which include root water uptake [1], blood flow in human capillaries [2, 3], seepage of rain water through the sedimentary layers of earth [4], microfluidic devices [5], diagnostics [6] and recovery from oil reservoirs [7, 8]. The early studies of surface tension driven flow in an empty capillary was done by Lucas [9] and Washburn [10] using Hagen-Poiseuille's law. By analogy, the spontaneous imbibition of a wetting fluid into a homogeneous porous medium follows diffusive dynamics similar to the Lucas-Washburn law. The diffusive law states that $l^2 = Dt$, where l is the length invaded by the wetting fluid in time t , and the constant of proportionality $D = \frac{2KP_c}{\mu}$, where K is the permeability, P_c is the capillary pressure of the medium and the viscosity of the invading liquid is μ . The permeability $K \propto d^2$ by Kozeny-Carman relation and the capillary pressure $P_c \propto \frac{1}{d}$, where d is the diameter of the grains in the homogeneous porous medium [11]. This indicates that the progression of the invading fluid front in a porous medium increases with the increase in permeability although the capillary pressure decreases. If the porous medium is initially filled with a viscous liquid, the flow behavior is altered, as shown by an analogous equation for capillaries given by Hultmark et al. [12].

Most of the naturally occurring flow phenomena are not in homogeneous porous media, like the dermatological layers of the skin [13] or the geological strata in the oil reservoirs [14]. These heterogeneities can be encompassed by describing such a porous medium as consisting of separate homogeneous layers, which hydrodynamically interact with each other. The experimental studies on such interacting nature of porous media [8, 15–18] suggest that the flow behavior in the layered porous medium is significantly different from the Lucas-Washburn law. Numerical models developed to predict the position of fluid fronts in the interacting system of pores by Bico and Qu  r   [17], Dong et al. [19–21], Ruth and Bartley [22], Unsal et al. [15, 16], Ashraf et al. [18] show that the interface in the small pore(s) leads, which is contrary to the Lucas-Washburn's law. Ashraf et al. [18] also showed that in a multilayered system, the arrangement of layers also impacts the flow behavior. The interface advance in the surface tension driven flow into a multilayered porous medium initially filled with a viscous liquid is still unknown.

In this work, we consider capillary driven flow of a wetting fluid in a layered porous medium initially filled with a viscous,

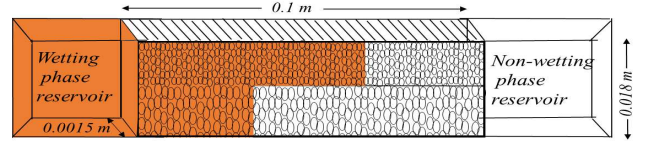


FIGURE 1. SCHEMATIC OF THE EXPERIMENTAL SETUP.

non-wetting fluid. We focus on two-layered porous medium in the present study. We develop a quasi one-dimensional lubrication approximation model to predict the interface advance in the interacting layers of the two layered porous medium. We use the one-dimensional model and the anticipated pressure gradients in the layers to establish the validity of the predictions. This analysis helps us in arriving at the final predictive model for capillary driven flow in the two layered porous medium. The model assumes quasi-static front and does not account for the dynamic nature of the menisci at the front. Therefore, we supplement our model by experiments, which show the similar invasion pattern in the two layered porous medium as predicted by the one-dimensional model.

Mathematical Model

To develop a quasi one-dimensional lubrication approximation model, we consider beads of two diameters d_l and d_s such that $d_l > d_s$ (subscript l = large, s = small), to make a two layered porous medium of length L as shown in fig. 1. The permeabilities of the layers are $K_i = \Lambda d_i^2$ ($i = l, s$) and the capillary pressure is $P_{ci} = \frac{c\gamma}{d_i}$, where c is a constant and γ is the interfacial tension [11]. The porous medium is filled with a non-wetting fluid of viscosity μ_{nw} initially. One end of the porous medium is brought in contact with a wetting fluid of viscosity μ_w . According to Ashraf et al., [18], in a two layered porous medium initially filled with a zero viscosity fluid, the interface in the small permeability layer leads.

In the current work, we develop a mathematical model for the invasion pattern in the two layered porous medium which are later verified by the anticipated pressure gradient graphs. Let the interface advance in the large and the small permeability layers be x_l and x_s in time t . The two possibilities are $x_s > x_l$ and $x_l > x_s$. First, we assume that $x_s > x_l$ similar to Ashraf et al., [18] and develop a suitable numerical model. This is followed by developing a numerical model for $x_l > x_s$.

Consider the case where the interface in the small permeability layer leads, i.e., $x_s > x_l$, as shown in fig. 2. Consider that the surface tension driven flow causes a superficial velocity of $v_l(t)$ in the large permeability layer and $v_s(t)$ in the small permeability layer, in the length preceding x_l . According to Darcy's

law, the pressure drop from the inlet to the interface at x_l is,

$$P_w(x_l, t) - P_0 = -\frac{\mu_w x_l}{K_l} v_l(t), \quad (1)$$

where, P_0 and $P_w(x_l, t)$ are the invading phase pressures at the inlet and at x_l , respectively, as shown in fig. 2. In developing the quasi one-dimensional model, we assume that the pressure gradient from the inlet till x_l is the same for both the layers, which implies $\frac{v_l(t)}{K_l} = \frac{v_s(t)}{K_s}$. At x_l , the pressure jump across the interface is $(P_{nw}(x_l, t) - P_w(x_l, t)) = P_{C_l}$ as shown in fig. 2, where $P_{nw}(x_l, t)$ is the non-wetting phase pressure at x_l . The sudden surge in pressure causes some liquid to flow into the small permeability layer at the first interface. The fluid transfer occurs due to the high capillary force in the fine pores. Same as Ashraf et al., [18], we assume that the fluid transfer occurs just before the interface at x_l and by lubrication approximation, we consider it to occur at the interface. After the fluid transfer, the velocity of the interface is,

$$\phi \frac{dx_l}{dt} = v_l(t) - \frac{dq}{A}. \quad (2)$$

In Eqn. (2), dq is the volumetric transfer of fluid from the large pores to the fine pores, A is the cross-sectional area of the layer, $\frac{dx_l}{dt}$ is the interstitial velocity of the interface and ϕ is the porosity of the layer. The fluid transfer causes the interstitial velocity in fine pores $\frac{dx_s}{dt}$ to increase, which can be written as,

$$\phi \frac{dx_s}{dt} = v_s(t) + \frac{dq}{A}. \quad (3)$$

The fine pores are filled with invading liquid till x_s . The pressure drop in the small permeability layer from x_l till the interface at x_s is,

$$P_w(x_s, t) - P_w(x_l, t) = -\frac{\mu_w(x_s - x_l)}{K_s} \left(v_s(t) + \frac{dq}{A} \right). \quad (4)$$

In Eqn. (4), $P_w(x_s, t)$ and $P_w(x_l, t)$ are the pressures of wetting phase at x_s and x_l , respectively, as shown in fig. 2. The large permeability layer is filled with the resident, non-wetting fluid from x_l to x_s . The pressures at x_s and x_l are different in the small and the large permeability layers as they are occupied with different fluids as shown in fig. 2. The pressure drop of resident fluid in the large permeability layer is given as,

$$P_{nw}(x_s, t) - P_{nw}(x_l, t) = -\frac{\mu_{nw}(x_s - x_l)}{K_l} \left(v_l(t) - \frac{dq}{A} \right), \quad (5)$$

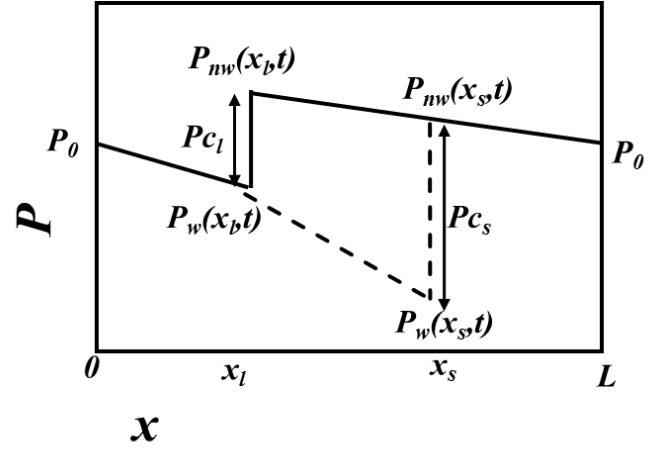


FIGURE 2. PRESSURE VS DISTANCE GRAPH FOR A TWO LAYERED POROUS MEDIUM FOR $x_s > x_l$. THE SOLID LINE INDICATES THE INTERFACE POSITION IN THE LARGE PERMEABILITY LAYER WHILE THE DASHED LINE SHOWS THE INTERFACE IN THE SMALL PERMEABILITY LAYER AT A TIME t .

where, $P_{nw}(x_s, t)$ and $P_{nw}(x_l, t)$ are the non-wetting phase pressures at x_s and x_l respectively. The pressure jump across interface in the small permeability layer at x_s is equal to its capillary pressure which is the difference between the pressures of resident and wetting phases, i.e., $(P_{nw}(x_s, t) - P_w(x_s, t)) = P_{C_s}$ as shown in fig. 2. The sudden jump in pressure across the interface causes some resident fluid to escape into the large permeability layer immediately after x_s . Considering lubrication approximation, the resident fluid transfer occurs at the interface x_s . The pressure drop in the large and the small permeability layers is the same after the resident fluid transfer, which is given by,

$$P_0 - P_{nw}(x_s, t) = -\frac{\mu_{nw}(L - x_s)}{K_l} v_l(t), \quad (6)$$

where, L is the total length of the porous medium and P_0 is the atmospheric pressure at the exit. By combining Eqns. (1) to (6) and substituting the capillary pressures as $P_{C_l} = \frac{c\gamma}{d_l}$, $P_{C_s} = \frac{c\gamma}{d_s}$, we obtain,

$$\frac{c\gamma}{d_l} = \frac{\mu_w x_l + \mu_{nw}(L - x_s)}{K_l + K_s} \phi \left(\frac{dx_l}{dt} + \frac{dx_s}{dt} \right) + \frac{\mu_{nw}(x_s - x_l)}{K_l} \phi \left(\frac{dx_l}{dt} \right), \quad (7)$$

$$c\gamma \left(\frac{1}{d_s} - \frac{1}{d_l} \right) = (x_s - x_l) \phi \left(\frac{\mu_w}{K_s} \left(\frac{dx_s}{dt} \right) - \frac{\mu_{nw}}{K_l} \left(\frac{dx_l}{dt} \right) \right). \quad (8)$$

The Eqns. (7) and (8) are reduced to dimensionless forms as,

$$1 = \frac{X_l + M(1 - X_s)}{1 + \eta^2} \left(\frac{dX_l}{dT} + \frac{dX_s}{dT} \right) + M(X_s - X_l) \left(\frac{dX_l}{dT} \right), \quad (9)$$

$$\frac{1}{\eta} - 1 = (X_s - X_l) \left(\frac{1}{\eta^2} \left(\frac{dX_s}{dT} \right) - M \left(\frac{dX_l}{dT} \right) \right). \quad (10)$$

In Eqns. (9) and (10), the permeabilities are $K_l = \Lambda d_l^2$, $K_s = \Lambda d_s^2$ and the dimensionless parameters are $X_l = \frac{x_l}{L}$, $X_s = \frac{x_s}{L}$, $\eta = \frac{d_s}{d_l}$, $M = \frac{\mu_{nw}}{\mu_w}$ and $T = t \frac{\Lambda c \gamma d_l}{\phi \mu_w L^2}$. Eqns. (9) and (10) are the governing equations for the dimensionless interface positions with time.

Now, we develop the governing equations if $x_l > x_s$ as shown in fig. 3, however in earlier studies, this case is never observed, therefore validation of this case is needed. In this, we consider that the pressure gradient in both the layers is the same till the first interface at x_s as shown in fig. 3. The pressure drop from inlet to the interface at x_s is,

$$P_w(x_s, t) - P_0 = -\frac{\mu_w x_s}{K_s} v_s(t). \quad (11)$$

In Eqn. (11), $P_w(x_s, t)$ is the pressure of invading phase at x_s and $v_s(t)$ is the superficial velocity in the small permeability layer till x_s . As the pressure gradient in both the layers is same till x_s as shown in fig. 3, $\frac{v_s(t)}{K_s} = \frac{v_l(t)}{K_l}$, implying $\frac{v_l(t)}{v_s(t)} = \frac{K_l}{K_s} > 1$. The pressure jump across the interface at x_s is equal to the capillary pressure of the small permeability layer as shown in fig. 3. At the interface, the fluid may transfers from the small permeability layer to the large permeability layer which will increase the velocity in the large permeability layer. However, the capillary suction in the small permeability layer is larger for the wetting fluid, therefore, fluid can transfer from the large permeability layer to the small permeability layer as long as the velocity of interface in the large permeability layer is greater than the velocity of the interface in the small permeability layer. As the volumetric fluid transfer from the small permeability layer to the large permeability layer is considered dq which may be positive or negative. The velocity of the interfaces in the small and in the large permeability layers can be given as,

$$\phi \frac{dx_s}{dt} = v_s(t) - \frac{dq}{A}. \quad (12)$$

$$\phi \frac{dx_l}{dt} = v_l(t) + \frac{dq}{A}. \quad (13)$$

The pressure drop of resident fluid in the small permeability layer after the fluid transfer till the second interface can be given as,

$$P_{nw}(x_l, t) - P_{nw}(x_s, t) = -\frac{\mu_{nw}(x_l - x_s)}{K_s} \left(v_s(t) - \frac{dq}{A} \right). \quad (14)$$

The pressure drop in the large permeability layer after the fluid transfer from x_s to x_l is,

$$P_w(x_l, t) - P_w(x_s, t) = -\frac{\mu_w(x_l - x_s)}{K_l} \left(v_l(t) + \frac{dq}{A} \right). \quad (15)$$

At the interface in the large permeability layer (x_l), the pressure jump equals capillary pressure in the large permeability layer, $P_{cl} < P_{cs}$. For this, the pressure gradient of the resident phase in the small permeability layer after x_s should be sufficiently high as shown in fig. 3. From x_l to L , the pressure gradient of non-wetting fluid in both the layers is same. Now, the pressure drop of the resident phase from x_l to L in both the layers is,

$$P_0 - P_{nw}(x_l, t) = -\frac{\mu_{nw}(L - x_l)}{K_s} v_s(t). \quad (16)$$

From Eqns. (11) to (16), we obtain the governing equations by eliminating the unknown fluid transfer and by substituting the velocities from Eqns. (12) and (13),

$$\frac{c\gamma}{d_l} = \frac{\mu_w x_s + \mu_{nw}(L - x_l)}{K_l + K_s} \phi \left(\frac{dx_l}{dt} + \frac{dx_s}{dt} \right) + \frac{\mu_w(x_l - x_s)}{K_l} \phi \left(\frac{dx_l}{dt} \right), \quad (17)$$

$$c\gamma \left(\frac{1}{d_s} - \frac{1}{d_l} \right) = (x_l - x_s) \phi \left(\frac{\mu_w}{K_l} \left(\frac{dx_l}{dt} \right) - \frac{\mu_{nw}}{K_s} \left(\frac{dx_s}{dt} \right) \right). \quad (18)$$

The Eqns. (17) and (18) are reduced to dimensionless forms as,

$$1 = \frac{X_s + M(1 - X_l)}{1 + \eta^2} \left(\frac{dX_l}{dT} + \frac{dX_s}{dT} \right) + (X_l - X_s) \left(\frac{dX_l}{dT} \right), \quad (19)$$

$$\frac{1}{\eta} - 1 = (X_l - X_s) \left(\left(\frac{dX_l}{dT} \right) - \frac{M}{\eta^2} \left(\frac{dX_s}{dT} \right) \right). \quad (20)$$

Validation of Mathematical Model

Now, we validate the occurrence of each of the cases $x_s > x_l$ and $x_l > x_s$ by verifying the equations with the anticipated

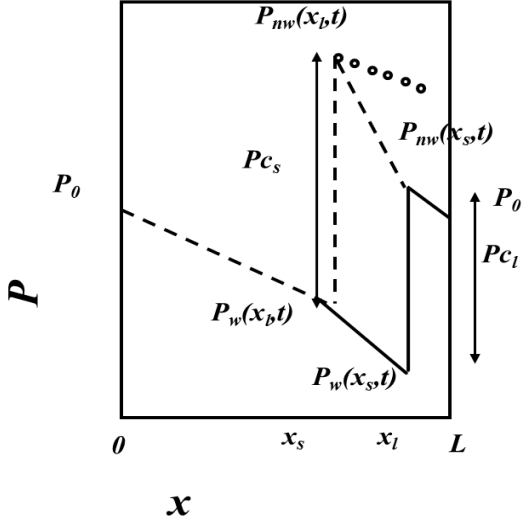


FIGURE 3. PRESSURE VS DISTANCE GRAPH FOR A TWO LAYERED POROUS MEDIUM FOR $x_l > x_s$. THE SOLID LINE INDICATES THE INTERFACE POSITION IN THE LARGE PERMEABILITY LAYER WHILE THE DASHED LINE SHOWS THE INTERFACE IN THE SMALL PERMEABILITY LAYER AT A TIME t .

pressure vs distance graphs and by obtaining suitable conditions that predict the flow behavior in a two layered porous medium. First, we consider the case $x_s > x_l$ and in Eqn. (10), we see that the left hand side of the equation is positive, indicating that the right hand side should always be greater than zero.

To verify if the right hand side of Eqn. (10) is greater than zero, we explore the relative pressure gradients in the two layers using fig. 4. From the inlet, till x_l the pressure gradient in both the layers is the same given as $\left(\frac{dP}{dx}\right)_w|_0^{x_l}$. After the invading fluid transfer at x_l , the flow rate in the large permeability layer decreases, implying for the large permeability layer

$$\frac{1}{\mu_w} \left(\frac{dP}{dx}\right)_w|_0^{x_l} > \frac{1}{\mu_{nw}} \left(\frac{dP}{dx}\right)_{nw}|_{x_l}^{x_s}. \quad (21)$$

We note from Eqn. (21) that depending on the viscosity ratio, the pressure gradient in the large permeability layer from x_l to x_s can be greater than, less than or equal to the pressure gradient from inlet to x_l as shown in fig. 4. We know that after the fluid transfer at x_l , the pressure gradient in the small permeability layer increases, i.e., $\left(\frac{dP}{dx}\right)_w|_{x_l}^{x_s} > \left(\frac{dP}{dx}\right)_w|_0^{x_l}$. Comparing the wetting and non-wetting phase pressure gradients from x_l to x_s , we find the following conditions.

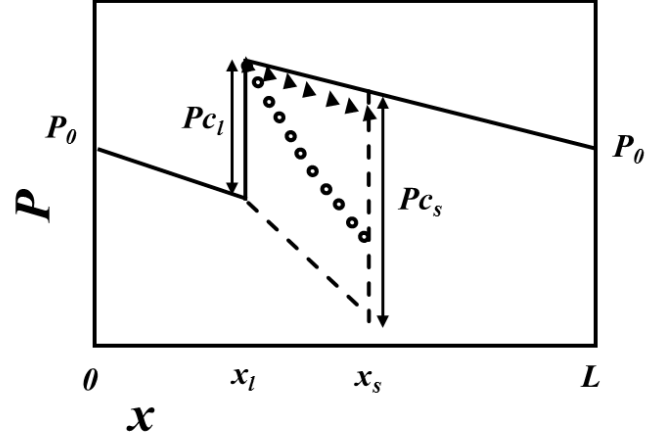


FIGURE 4. PRESSURE VS DISTANCE GRAPH FOR A TWO LAYERED POROUS MEDIUM SHOWING THAT THE PRESSURE GRADIENT OF THE INVADING PHASE IN THE SMALL PERMEABILITY LAYER IS ALWAYS GREATER THAN THE PRESSURE GRADIENT OF THE RESIDENT PHASE OF THE LARGE PERMEABILITY LAYER FROM x_l TO x_s FOR $x_s > x_l$. THE CIRCLES SHOW THAT THE PRESSURE GRADIENT OF THE NON-WETTING PHASE IS MORE THAN THE WETTING PHASE PRESSURE GRADIENT OF THE LARGE PERMEABILITY LAYER, WHILE THE TRIANGLES REPRESENT THAT THE PRESSURE GRADIENT IN THE LARGE PERMEABILITY LAYER FILLED WITH NON WETTING FLUID IS SAME AS THAT FILLED WITH WETTING FLUID.

(i) If $\mu_{nw} \leq \mu_w$, then, from Eqn. 21,

$$\left(\frac{dP}{dx}\right)_w|_{x_l}^{x_s} > \left(\frac{dP}{dx}\right)_w|_0^{x_l} > \left(\frac{dP}{dx}\right)_{nw}|_{x_l}^{x_s}. \quad (22)$$

(ii) If $\mu_{nw} > \mu_w$, then, $\left(\frac{dP}{dx}\right)_w|_{x_l}^{x_s} \leq \left(\frac{dP}{dx}\right)_{nw}|_{x_l}^{x_s}$ is also possible. However, if it were to be true, then from fig. 4, the capillary pressure jump in the small permeability layer will be less than or equal to the capillary pressure jump in the large permeability layer, which is a contradiction. Therefore, the only possible condition for pressure gradients in the layers from x_l to x_s is,

$$\left(\frac{dP}{dx}\right)_w|_{x_l}^{x_s} > \left(\frac{dP}{dx}\right)_{nw}|_{x_l}^{x_s}. \quad (23)$$

By Darcy's law, Eqn. (23) can be written as,

$$\frac{\mu_w}{K_s} \left(\frac{dx_s}{dt}\right) > \frac{\mu_{nw}}{K_l} \left(\frac{dx_l}{dt}\right), \quad (24)$$

which can be written in dimensionless form as,

$$\left(\frac{dX_s}{dT}\right) - M\eta^2 \left(\frac{dX_l}{dT}\right) > 0. \quad (25)$$

We know that $X_s > X_l$ and from Eqn. (25), it can be observed that the right hand side of Eqn. (10) is positive at all times irrespective of the viscosities of the fluids and the bead diameters used. As the velocities of the interfaces are always positive the right hand side of Eqn. (9) is always positive. This concludes that the case $x_s > x_l$ is possible.

Now, we consider the case $x_l > x_s$ and we observe that both the left hand and right hand side of Eqn. (19) are positive as the velocities of the interfaces are positive. The right hand side of Eqn. (20) should be greater than zero, as the left hand side is positive. In developing the quasi one-dimensional lubrication approximation model for the case where the interface in the large permeability layer leads, therefore, $(X_l - X_s) > 0$, implying that $\frac{dX_l}{dt} > \frac{dX_s}{dt}$. The different cases of fluid transfer are,

(i) $dq > 0$, i.e., fluid transfers from the small permeability layer to the large permeability layer. The flow rate in the small permeability layer decreases after the fluid transfer at x_s , which can be written as $\frac{1}{\mu_w} \left(\frac{dP}{dx}\right)_w|_0^{x_s} > \frac{1}{\mu_{nw}} \left(\frac{dP}{dx}\right)_{nw}|_{x_s}^{x_l}$ from Darcy's law. If $\mu_{nw} \leq \mu_w$, then $\left(\frac{dP}{dx}\right)_w|_0^{x_s} > \left(\frac{dP}{dx}\right)_{nw}|_{x_s}^{x_l}$ as shown by circles in fig.

3. We see from fig. 3 that for $P_{c_l} < P_{c_s}$, $\left(\frac{dP}{dx}\right)_w|_0^{x_s} > \left(\frac{dP}{dx}\right)_{nw}|_{x_s}^{x_l}$ is not possible. Therefore $\mu_{nw} > \mu_w$ for $dq > 0$ as shown in fig. 3.

(ii) For $dq < 0$, the fluid transfers from the large permeability layer to the small permeability layer such that, $\frac{v_l(t)+dq}{v_s(t)-dq} > 1$. Thus, the flow rate in the small permeability layer increases after the fluid transfer which can be given by Darcy's law as $\frac{1}{\mu_w} \left(\frac{dP}{dx}\right)_w|_0^{x_s} < \frac{1}{\mu_{nw}} \left(\frac{dP}{dx}\right)_{nw}|_{x_s}^{x_l}$. If $\mu_{nw} > \mu_w$, $\left(\frac{dP}{dx}\right)_w|_0^{x_s} < \left(\frac{dP}{dx}\right)_{nw}|_{x_s}^{x_l}$ always. For $\mu_{nw} \leq \mu_w$, then both the cases $\left(\frac{dP}{dx}\right)_w|_0^{x_s} < \left(\frac{dP}{dx}\right)_{nw}|_{x_s}^{x_l}$ and $\left(\frac{dP}{dx}\right)_w|_0^{x_s} > \left(\frac{dP}{dx}\right)_{nw}|_{x_s}^{x_l}$ are possible. However, we see from fig. 5 that $\left(\frac{dP}{dx}\right)_w|_{x_s}^{x_l} < \left(\frac{dP}{dx}\right)_{nw}|_{x_s}^{x_l}$ must hold, so that $P_{c_s} > P_{c_l}$.

Therefore, in both the above cases, $\left(\frac{dP}{dx}\right)_w|_{x_s}^{x_l} < \left(\frac{dP}{dx}\right)_{nw}|_{x_s}^{x_l}$. This can be written as, From Darcy's equation, we get,

$$\frac{\mu_{nw}}{K_s} \left(v_s(t) - \frac{dq}{A}\right) > \frac{\mu_w}{K_l} \left(v_s(t) + \frac{dq}{A}\right). \quad (26)$$

By substituting Eqns. (12) and (13) in Eqn. (26), we obtain,

$$\frac{\mu_{nw}}{K_s} \left(\frac{dX_s}{dt}\right) > \frac{\mu_w}{K_l} \left(\frac{dX_l}{dt}\right). \quad (27)$$

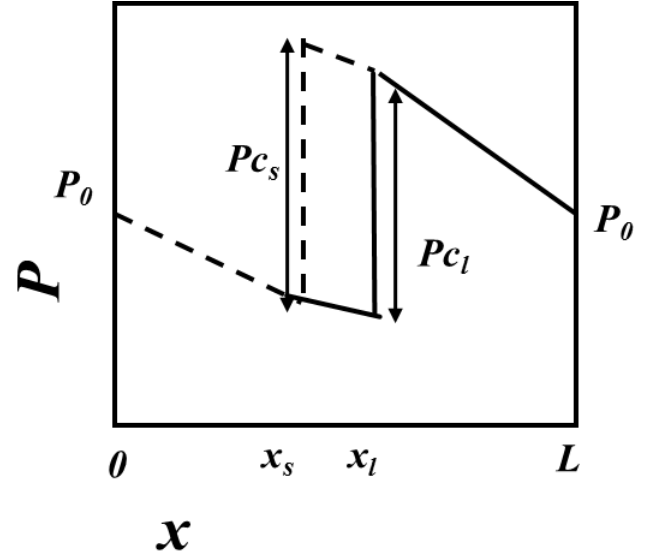


FIGURE 5. PRESSURE VS DISTANCE GRAPH FOR A TWO LAYERED POROUS MEDIUM SHOWING THAT THE PRESSURE GRADIENT OF THE RESIDING PHASE IN THE SMALL PERMEABILITY LAYER CAN BE LESS THAN THE PRESSURE GRADIENT OF THE INVADING PHASE OF THE SMALL PERMEABILITY LAYER DUE TO TRANSFER OF WETTING PHASE FROM THE LARGE PERMEABILITY LAYER FOR $x_l > x_s$

Equation (27) can be written as,

$$\left(\frac{dX_l}{dT}\right) - \frac{M}{\eta^2} \left(\frac{dX_s}{dT}\right) < 0. \quad (28)$$

As the interface in the large permeability layer is considered to be ahead of the small permeability layer interface, Eqn. (28) contradicts the Eqn. (20) in which $\left(\frac{dX_l}{dT}\right) - \frac{M}{\eta^2} \left(\frac{dX_s}{dT}\right)$ should necessarily be greater than zero. Therefore, the one-dimensional model developed for $x_l > x_s$ contradicts the condition arrived from the anticipated pressure gradients in the layers.

Experimental Methodology

To find the invasion pattern in a two-layered porous medium, we use parallel layers of glass beads which hydrodynamically interact with each other. The beads are packed into layers between two parallel acrylic surfaces. The dimensions of the porous media are $0.1 \text{ m} \times 0.018 \text{ m} \times 0.0015 \text{ m}$ as shown in the schematic of the layered porous medium in fig. 1. Three different diameters of glass beads, i.e., $170 \times 10^{-6} \text{ m}$, $220 \times 10^{-6} \text{ m}$ and $350 \times 10^{-6} \text{ m}$ termed as the small(d_s), medium(d_m) and large(d_l) beads respectively are used to make the layers. In the two layered porous

medium, two bead sizes are used for the adjacent layers. For the three bead sizes used in this study as mentioned, the two layered porous media that can be formed are (i) d_s and d_m , (ii) d_m and d_l , (iii) d_s and d_l . Each layer has a width of 0.009 m and the porosity of all the layers is checked separately, which comes out to be 0.35 approximately. Wetting and non-wetting phase reservoirs are designed at either ends of the porous medium as shown in fig. 1. In all the experiments, dodecane of viscosity 1.36×10^{-3} Pa.s is the non-wetting phase which occupies the porous medium initially and colored water of viscosity 1×10^{-3} Pa.s is the invading phase. The interfacial tension of dodecane-water is found to be 41.2×10^{-3} N.m $^{-1}$. The preferential wetting of glass to water is responsible for the spontaneous displacement of the resident fluid initially present in the porous medium. To counter the capillary back pressure, the non-wetting phase reservoir is filled with dodecane. The invasion of the water begins as the porous medium gets in contact with the wetting phase.

Results and Discussions

In all the three experiments, we observe that the invasion in narrow pores is faster than the invasion in larger pore spaces, as proved by the mathematical model. A customized image analysis is performed in MATLAB to find the pixels filled with the invading fluid at different times. The imbibition patterns in the layers as explained by the one-dimensional model for $x_s > x_l$ holds true as observed from the experiments which can be seen from the snapshots of invasion in fig. 6. In the one-dimensional model, the two-dimensional variations are not seen, therefore, in the experiments, we assume that the volume imbibed is proportional to the invasion length for the wetting fluid.

The results of one-dimensional model show that invasion into the large permeability layer begins only after the interface in the small permeability region has reached a certain distance. The experimental studies include a counter-current flow of resident fluid from the inlet that alter the invasion dynamics in the large permeability layer which is not accounted in the numerical model. Further, as the value of η is different for all the three porous media, suitable values of the constant c are considered. It should also be noted that the diameter of the large beads for case (a) shown in fig. 6 is 220×10^{-6} m while it is 350×10^{-6} m for the other two cases. For the cases (a), (b), (c) shown in fig. 6, the values of c are 5, 1.5, 2.2 respectively; which are all of the order 1. The experiment results match with the one-dimensional model prediction of interface. Thus the one-dimensional model sufficiently explains the positioning of fluid front in a two layered porous medium and the interface in the small permeability layer always leads.

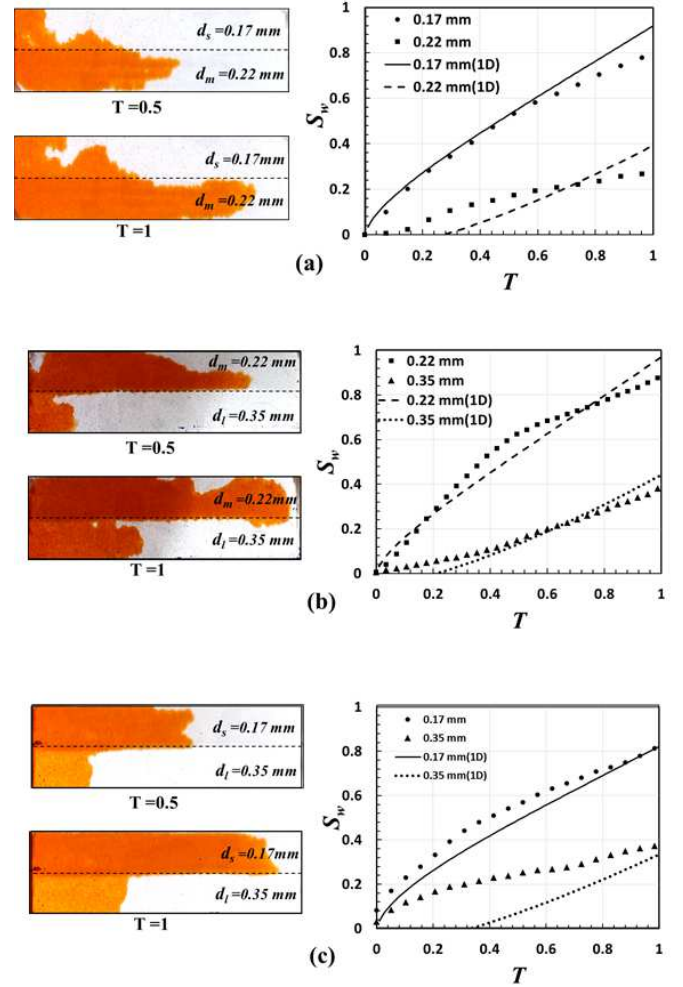


FIGURE 6. SURFACE TENSION DRIVEN FLOW OF COLORED WATER IN A TWO LAYERED POROUS MEDIUM INITIALLY FILLED WITH DODECANE. THE DIAMETERS OF THE LAYERS ARE (a) d_s , d_m , (b) m , d_l , (c) d_s , d_l , WHERE $d_s = 170 \times 10^{-6}$ m, $d_m = 220 \times 10^{-6}$ m, AND $d_l = 350 \times 10^{-6}$ m. THE IMAGES SHOW THE INVASION AT DIMENSIONLESS TIME $T = 0.5, 1$ AND THE GRAPH OF FRACTION OF THE POROUS LAYER INVADDED BY THE WETTING FLUID(S_w) VS DIMENSIONLESS TIME(T) SHOWS THAT THE FRACTION VOLUME OCCUPIED BY THE WETTING PHASE IS MORE IN THE FINE PORES AS COMPARED TO THE LARGER PORES AT ALL TIMES.

CONCLUSIONS

A wetting fluid is known to invade fine pores in an empty heterogeneous porous medium. In this work, we explore the possible imbibition patterns if the porous medium is initially filled with a viscous fluid. We first develop a quasi one-dimensional lubrication approximation model for all the possible cases of spon-

taneous displacement of the non-wetting viscous fluid by a wetting viscous fluid in a two layered porous medium by capillary action. We use anticipated pressure gradients in different parts of the layers to validate the one-dimensional model. We find that for all permeability ratios of the layers and all the viscosity ratios of the invading and the resident fluids, the invasion of the wetting fluid in the finer pores will always be more. We conducted experiments for different permeability ratios which validate the predictive nature of our one-dimensional model. The results from this study will be useful for application in the fields of diagnostics and microfluidic devices.

ACKNOWLEDGMENT

We would like to acknowledge Science and Engineering Research Board of Department of Science and Technology, Government of India for funding of this project(SERB/F/1297/2017-18), with file number ECR/2017/000257. We would also like to thank Pranjalya Singh for assistance in data acquisition.

REFERENCES

- [1] Roose, T., and Fowler, A., 2004. "A model for water uptake by plant roots". *Journal of Theoretical Biology*, **228**(2), pp. 155–171.
- [2] Huang, W., Bhullar, R. S., Fung, Y. C., et al., 2001. "The surface-tension-driven flow of blood from a droplet into a capillary tube". *Transactions-American Society of Mechanical Engineers Journal of Biomechanical Engineering*, **123**(5), pp. 446–454.
- [3] Secomb, T., Hsu, R., and Pries, A., 2001. "Motion of red blood cells in a capillary with an endothelial surface layer: effect of flow velocity". *American Journal of Physiology-Heart and Circulatory Physiology*, **281**(2), pp. H629–H636.
- [4] Gasmol, J., Rahardjo, H., and Leong, E. C., 2000. "Infiltration effects on stability of a residual soil slope". *Computers and Geotechnics*, **26**(2), pp. 145–165.
- [5] Cate, D. M., Adkins, J. A., Mettakoonpitak, J., and Henry, C. S., 2014. "Recent developments in paper-based microfluidic devices". *Analytical chemistry*, **87**(1), pp. 19–41.
- [6] Gunda, N. S. K., Singh, M., Norman, L., Kaur, K., and Mitra, S. K., 2014. "Optimization and characterization of biomolecule immobilization on silicon substrates using (3-aminopropyl) triethoxysilane (aptes) and glutaraldehyde linker". *Applied Surface Science*, **305**, pp. 522–530.
- [7] Nicolaides, C., Jha, B., Cueto-Felgueroso, L., and Juanes, R., 2015. "Impact of viscous fingering and permeability heterogeneity on fluid mixing in porous media". *Water Resources Research*, **51**(4), pp. 2634–2647.
- [8] Chaudhari, L., Hadia, N., Mitra, S., and Vinjamur, M., 2011. "Flow visualization of two-phase flow through layered porous media". *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*, **33**(10), pp. 948–958.
- [9] Lucas, R., 1918. "Ueber das zeitgesetz des kapillaren aufstiegs von flüssigkeiten". *Colloid & Polymer Science*, **23**(1), pp. 15–22.
- [10] Washburn, E., 1921. "The dynamics of capillary flow". *Physical review*, **17**(3), p. 273.
- [11] Reyssat, M., Sangne, L. Y., Van Nierop, E. A., and Stone, H. A., 2009. "Imbibition in layered systems of packed beads". *EPL (Europhysics Letters)*, **86**(5), pp. 56002–p1 – 560002–p6.
- [12] Hultmark, M., Aristoff, J. M., and Stone, H. A., 2011. "The influence of the gas phase on liquid imbibition in capillary tubes". *Journal of Fluid Mechanics*, **678**, pp. 600–606.
- [13] Balois, T., and Amar, M. B., 2014. "Morphology of melanocytic lesions in situ". *Scientific reports*, **4**, p. 3622.
- [14] Tiab, D., and Donaldson, E. C., 2015. *Petrophysics: theory and practice of measuring reservoir rock and fluid transport properties*. Gulf professional publishing.
- [15] Unsal, E., Mason, G., Ruth, D., and Morrow, N., 2007. "Co-and counter-current spontaneous imbibition into groups of capillary tubes with lateral connections permitting cross-flow". *Journal of colloid and interface science*, **315**(1), pp. 200–209.
- [16] Unsal, E., Mason, G., Morrow, N. R., and Ruth, D. W., 2009. "Bubble snap-off and capillary-back pressure during counter-current spontaneous imbibition into model pores". *Langmuir*, **25**(6), pp. 3387–3395.
- [17] Bico, J., and Quéré, D., 2003. "Precursors of impregnation". *EPL (Europhysics Letters)*, **61**(3), pp. 348–353.
- [18] Ashraf, S., Visavale, G., Bahga, S. S., and Phirani, J., 2017. "Spontaneous imbibition in parallel layers of packed beads". *The European Physical Journal E*, **40**(4), p. 39.
- [19] Dong, M., Dullien, F., and Zhou, J., 1998. "Characterization of waterflood saturation profile histories by the 'complete' capillary number". *Transport in porous media*, **31**(2), pp. 213–237.
- [20] Dong, M., Dullien, F., and Dai, L., 2005. "D. li, immiscible displacement in the interacting capillary bundle model, part1, development of interacting capillary bundle model". *Transport in Porous Media*, **59**, pp. 1–18.
- [21] Dong, M., Dullien, F. A., Dai, L., and Li, D., 2006. "Immiscible displacement in the interacting capillary bundle model part ii. applications of model and comparison of interacting and non-interacting capillary bundle models". *Transport in Porous media*, **63**(2), pp. 289–304.
- [22] Ruth, D., and Bartley, J., 2011. "Capillary tube models with interaction between the tubes". *Transport in porous media*, **86**(2), pp. 479–482.