



# A generalized model for spontaneous imbibition in a horizontal, multi-layered porous medium

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## HIGHLIGHTS

- Fluid front does not always lead in smallest pores during spontaneous imbibition.
- Layering pattern and contrasts in flow properties result in different flow behavior.
- A generalized 1-D model is developed to predict imbibition for any arrangement.
- Breakthrough time of the porous medium strongly depends on the layering pattern.

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## ABSTRACT

Spontaneous imbibition of a wetting fluid in a homogeneous porous medium follows Washburn's diffusive law. For heterogeneous porous media, the imbibition phenomenon deviates from the diffusive dynamics. A Washburn-like model, developed in the past predicts the imbibition in a three-layered porous medium. In this work, we use experiments to establish the applicability of the model for a horizontal three-layered porous medium. The model is dependent on a scaling law to determine the relative positions of the fronts in the layers. In a multi-layered porous medium, the scaling law cannot predict the relative front advancements. We use a five layered porous medium to develop a strategy and generalize the model for multi-layered porous medium. We show that, the layering pattern, capillary pressure and permeability contrasts among the layers impacts the imbibition behavior significantly and leads to different travel times of the fronts of the invading fluid in the layers.

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## 1. Introduction

The surface tension forces responsible for driving a wetting fluid into a porous medium are important for applications in wicks of loop heat pipes (Xu et al., 2014; Liu et al., 2019; Wang et al., 2015; Ji et al., 2017), fabric for diapers (Fuisz, 1988; Neading and Cate, 2003; Lamb, 1986), microfluidic devices (Dudek et al., 2018, 2016, 2017), and oil recovery from porous reservoirs (Trivedi and Babadagli, 2009; Ersland et al., 2010; Geiger et al., 2012; Morrow and Mason, 2001; Babadagli and Boluk, 2005; Glass et al., 2001). The capillary forces drive the wetting fluid into the pore spaces displacing the non-wetting fluid. Lucas (1918) and Washburn (1921) observed in a single capillary that, the spontaneous imbibition follows diffusive dynamics, i.e., the square of the imbibed length,  $l$  is proportional to the time of imbibition,  $t$ , i.e.,  $l^2 = Dt$ . The constant of proportionality,  $D = \left(\frac{r^2 Pc}{4\mu}\right)$ , where  $r$  is the radius of the capillary

and  $\mu$  is the viscosity of the invading phase. The capillary pressure  $Pc = \frac{2\gamma \cos \theta}{r}$  is calculated according to the Young–Laplace equation (Young, 1805; de Laplace, 1806), where  $\gamma$  is the surface tension and  $\theta$  is the contact angle of the wetting liquid with the capillary wall.

Diffusive dynamics of imbibition is observed in homogeneous porous medium also (Hammecker and Jeannette, 1994; David et al., 1993; Gruener et al., 2012; Hammecker et al., 1993). The imbibition length is given by  $l^2 = \left(\frac{2K Pc}{\mu}\right)t$ , where,  $K$  is the permeability and  $Pc$  is the capillary pressure of the medium. A coarse grain porous medium imbibes fluid faster as compared to a fine grain porous medium (Li et al., 2015). This is because, permeability increases as square of grain size and capillary pressure increases as inverse of grain size. Therefore, a fine grain porous medium has high viscous resistance as compared to a coarse grain porous medium. Beyhaghi et al., Masoodi et al. (2007, 2011) and Zarandi et al., modeled using Darcy's law to see the effect of permeability and capillary pressure to optimize wick design (Beyhaghi et al., 2014; Masoodi et al., 2007; Masoodi et al., 2011; Zarandi et al., 2018).

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However, in most cases of interest, such as loop heat pipes (Wang et al., 2015; Ji et al., 2017), paper microfluidics (Adkins et al., 2016; Cate et al., 2014; Walji and MacDonald, 2016), oil reservoirs (Dawe et al., 1992; Ringrose et al., 1993), and diapers (Lamb, 1986; Hahn et al., 1994), the layered porous medium is desired or is naturally occurring. Reyssat et al. (2009) showed that Lucas-Washburn like equation is insufficient to describe the imbibition in the direction of permeability variation. They developed a reduced order model to describe the imbibition in a layered porous medium. However, the imbibition perpendicular to the direction of layering is more relevant for applications in oil reservoirs where, the porous medium is heterogeneous, essentially made of sediment layers ranging from several meters to less than a millimeter in thickness (Dawe et al., 1992; Ringrose et al., 1993; Stauffer and Dracos, 1986; Schaetzl and Thompson, 2015) and for diapers, fabric designs (Lamb, 1986; Hahn et al., 1994; Das et al., 2009; Morris, 1955). In the design of wicks of loop heat pipes, the permeability of layering perpendicular to the fluid flow can be a promising design improvement.

Bico and Quéré (2003), were the first to show that the imbibition is faster in the fine pores in a porous medium with hydrodynamic interactions among the pores. They reasoned that, the fluid flow from the large pores to the small pores due to the higher capillary suction is responsible for the faster imbibition of fluid in the small pores. Unsal et al. (2007a,b) experimentally showed the spontaneous imbibition phenomenon in capillaries permitting lateral cross flow. They used a rod in a slot arrangement to construct the geometry of three interacting capillaries. Their experimental observations show that the leading front is in the narrowest capillary which is in concurrence with the studies of Bico and Quéré.

Ashraf et al. (2018) using the analogy of pores performed detailed Volume of Fluid simulations in a two pore system. They showed that, the fluid transfer described by Bico and Quéré (2003) occurs at the fluid front locations only, which leads to a faster imbibition in the fine pore. In their studies, spontaneous imbibition is modeled by discretizing the domain and solving the flow equations. Using such simulations, for large scale porous medium like oil reservoirs requires a significant upscaling to mimic the permeability due to micro size layers. Moreover, the simulations are computationally expensive and the resources involved in such simulations are both tedious and time consuming. Developing a Darcy like model predicting the spontaneous imbibition in heterogeneous systems can provide insights in understanding the flow behavior. Tavangarrad et al. (2018) used Darcy velocity to model imbibition in a stack of hydrophilic fibrous layers. Ashraf et al. (2018) developed a reduced order model describing the imbibition in the two and three interacting pore systems.

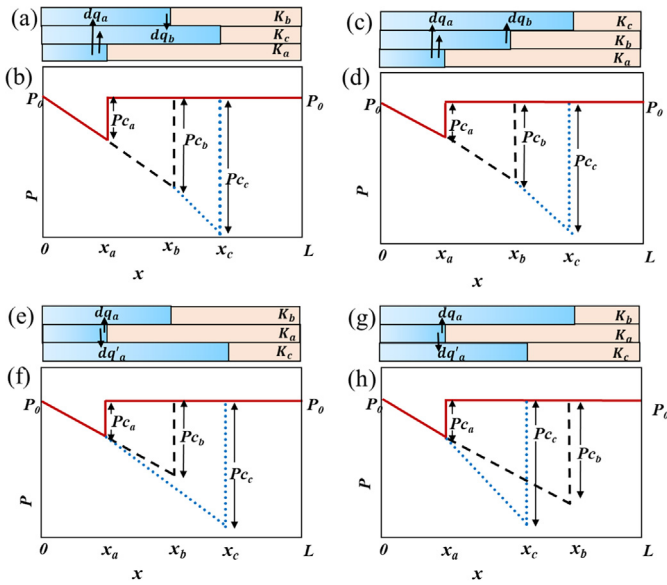
Ashraf et al. (2017, 2019) showed using experiments and the reduced order model that, in a layered porous medium, the imbibition is faster in the porous layer made of fine grains, for a two layered porous medium. They further showed that, the leading front will not always be in the fine grained porous layer for a three layered porous medium. They explained that, the grain size contrast and the arrangement of the layers determine the leading front layer. Ashraf and Phirani (2019) first determined the relative fluid front positions in the layers using a scaling law and then developed their model to predict the imbibition in each layer. However, real porous media like an oil reservoir, aquifer and soil horizons or the desired porous media like the diapers, fabrics are usually made of more than three layers. The relative fluid front locations, which are used to develop governing equations in Ashraf et al. (2017) for spontaneous imbibition are difficult to predict for such a multi-layered porous system. Moreover, developing a reduced order model to predict the complex imbibition pattern becomes cumbersome as the number of layers in a multi-layered porous medium increase. Therefore, a generalized model is needed, which predicts

the imbibition in a multi-layered porous medium made of any number of layers.

In this work, we develop a generalized, one-dimensional lubrication approximation model governing the spontaneous imbibition in a multi-layered porous medium. We first review the work done in spontaneous imbibition for three-layered porous medium. We then extend the model for a five-layered system to understand the generalization of the model and different cases required, if any. In the five-layered porous medium, we show for a given arrangement that the permeability and the capillary pressure contrasts of the layers govern the leading front layer. We also show that, if individual properties of the layers are same and only arrangement is different, then also, the imbibition behavior changes. The reduced order model is then generalized for multiple, randomly arranged layers. The generalized model is able to predict the layer having the leading front for any given arrangement of the layers. We observe that the travel time and time of breakthrough of the wetting fluid front is different for different arrangements. The previously described models (Ashraf et al., 2017; Ashraf and Phirani, 2019) were limited to only two and three layered porous media, whereas the generalized model developed in the current study can predict imbibition in a porous medium made of any number of layers. This will have implications for oil recovery prediction and in design of layered porous medium for microfluidic applications.

## 2. Imbibition in a three-layered porous medium

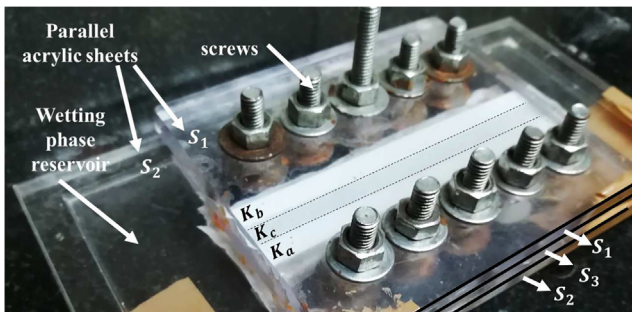
Ashraf et al. (2017) used experiments to observe the imbibition behavior in a three layered porous medium. They first used Volume of Fluid (VOF) simulations, considering interacting capillaries as a proxy to understand the spontaneous imbibition phenomenon in detail (Ashraf et al., 2018). In their study, they found that, (1) the pressure gradient in the layers is same if the layers are filled with invading fluid, which is continuous and (2) the cross-flow of the invading fluid occurs from a large permeability layer to the small permeability layer just before the fluid front in the large permeability layer. This is due to the high capillary suction of the small permeability layer. By lubrication approximation, they consider cross-flow to occur 'at' the invading front location(s). Ashraf and Phirani (2019) extended these assumptions to a layered porous medium, which are also used in the current study. Ashraf and Phirani (2019) performed experiments using various wetting and non-wetting fluids considering three layers of permeability  $K_a, K_b$  and  $K_c$  made of grains of diameters  $d_a, d_b$  and  $d_c$ , respectively. If  $d_a > d_b > d_c$ , then using Kozeny-Carman equation (Bear, 1972),  $K_a > K_b > K_c$  and the corresponding capillary pressure  $P_{ci}$  are such that,  $P_{ca} < P_{cb} < P_{cc}$ , using Young-Laplace pressure equation (Young, 1805; de Laplace, 1806). Their model gives different governing equations for different arrangements of the three layers namely (i) the small permeability ( $K_c$ ) layer between the large and the medium permeability layers as shown in the schematics of Fig. 1(a), ii) the medium permeability ( $K_b$ ) layer between the large and the small permeability layers as shown in Fig. 1(c), iii) the large permeability ( $K_a$ ) layer between the small and the medium permeability layers, as shown in Fig. 1(e). Ashraf et al. (2017) explained different leading fronts in the arrangements using anticipated pressure vs length curves as shown in Fig. 1(b), (d), (f) for the arrangements shown in Fig. 1(a), (c), (e), respectively. For the arrangements shown in Fig. 1(a), (c), the only possible pressure vs length curves are shown in Fig. 1(b), (d), respectively, such that  $P_{ca} < P_{cb} < P_{cc}$ . In these cases,  $x_a < x_b < x_c$  always, where,  $x_i$  is the imbibition length in the layer made of grains of diameter  $d_i$ . Any other front location will not be possible for  $P_{ca} < P_{cb} < P_{cc}$ . They also explained that, apart from arrangements of the layers,



**Fig. 1.** Schematics of arrangement of layers and the corresponding pressure gradients in a 3-layered porous medium. (a, b) The small permeability layer  $K_c$  is in the middle,  $x_a < x_b < x_c$ . (c, d) The medium permeability layer  $K_b$  is in the middle,  $x_a < x_b < x_c$ . (e, f) The large permeability layer  $K_a$  is in the middle,  $x_a < x_b < x_c$ . (g, h) The large permeability layer  $K_a$  is in the middle,  $x_a < x_c < x_b$ .

the contrast between the permeability and capillary pressure of the layers also governs the layer having the leading fluid front, for a particular arrangement. For example, Fig. 1(e), (g) show the schematics of the arrangement where the large permeability layer is in between the medium and the small permeability layers. The leading front is in different layers as shown in Fig. 1(e), (g). This is because, after  $x_a$ , the pressure gradient in layers  $K_b$  and  $K_c$  are different. Depending on the pressure gradients, to satisfy  $P_{c_a} < P_{c_b} < P_{c_c}$ , the leading front can be either in the small permeability layer ( $K_c$ ) or in the medium permeability layer ( $K_b$ ) as shown by pressure gradient curves in Fig. 1(f), (h). Ashraf and Phirani (2019) gave a scaling law governing the leading front for the arrangement where the large permeability layer is in the center. To prove the relation given by Ashraf and Phirani (2019), we perform experiments for arrangement (iii) where the large permeability layer is in the middle.

Fig. 2 shows the experimental setup used in our experiments. The experimental setup is made of two parallel acrylic sheets S1 and S2, which serve as the top and bottom surfaces of the porous medium as shown in Fig. 2. The acrylic sheets S1 and S2 are held by screws as shown in Fig. 2 with a spacing of 0.15 cm in between. The inlet and outlet ends of the porous medium are sealed by a thin, permeable fibrous layer of negligible resistance. The space between the acrylic plates is filled by layers of glass beads with



**Fig. 2.** Schematic of experimental setup for spontaneous imbibition in a layered porous medium.

gentle tapping at regular intervals to ensure the compact packing of the beads. The glass beads of all sizes are cleaned by rinsing with acetone to remove the surface impurities, followed by washing with distilled water. The glass beads are then dried in the oven to remove the traces of acetone. The open end of the porous medium is sealed by another acrylic sheet S3 as shown in Fig. 2, which holds the porous layers intact. A reservoir is constructed at the inlet end of the porous medium to introduce the invading fluid, which imbibes into the porous medium due to capillary suction.

The porous medium is 10 cm long, 1.8 cm in width and 0.15 cm in height. Each of the three porous layers is 0.6 cm wide. The Bond number ( $Bo$ ) is the ratio of the gravitational to the surface tension forces. It is expressed as  $Bo = \frac{\Delta \rho g h^2}{\gamma}$ , where  $\Delta \rho$  is the difference in the densities of the wetting and non-wetting phases,  $g$  is the acceleration due to gravity and  $h$  is the height of the porous medium. For the experiments performed in this study,  $Bo$  is of order 0.01, indicating that the gravitational effects are negligible. The permeability of a porous layer made of grains of diameter  $d_i$  is  $K_i = \Lambda d_i^2$ , according to the Kozeny-Carmen relation (Bear, 1972), where  $\Lambda$  is a function of the porosity and is of order  $10^{-4}$ . The capillary pressure is determined as  $P_{c_i} = \frac{c\gamma}{d_i}$  (Reyssat et al., 2009), where  $c$  is a constant and  $\gamma$  is the surface tension of the invading liquid. The constant  $c$  includes the contact angle of the invading fluid with the pore surfaces. In this study, due to the same cleaning process for all glass beads, the value of  $c$  is considered to be of same

order of magnitude for all the bead sizes. Therefore,  $\frac{K_b}{K_a} = \frac{d_b^2}{d_a^2}$ ,  $\frac{K_c}{K_a} = \frac{d_c^2}{d_a^2}$  and  $\frac{P_{c_b}}{P_{c_a}} = \frac{d_a}{d_b}$ ,  $\frac{P_{c_c}}{P_{c_a}} = \frac{d_a}{d_c}$ . In general, real porous media are usually not composed of spherical particles and in such cases, the porosity of each of the layers needs to be known. As permeability is a function of the porosity, the corresponding permeability of the layers will also change accordingly for layers made of a scatter of bead sizes. In this study, we have determined the porosity for homogeneous packs of glass beads used in the experiments. The porosity of all the layers is measured to be  $0.35 \pm 0.01$ .

The layered porous medium, which is initially filled with air is placed over an LED light source, to illuminate the experimental setup uniformly. The reservoir at the inlet is filled with dodecane of viscosity 1.35 cP, which acts as the wetting fluid. As the wetting fluid contacts the porous medium, it is spontaneously driven at different velocities in each of the porous layers, depending on the permeability and capillary pressures of the layers. The experiments for each arrangement are conducted thrice for reproducibility. The experiments are recorded by Nikon D750 camera at a resolution of  $1280 \times 720$  pixels and at 60 fps. A customized MATLAB® program tracks the advancing fluid front in the porous layers with time.

We consider two cases of arrangement (iii); for case (A), the glass beads of diameters  $d_a = 274 \pm 25 \mu\text{m}$ ,  $d_b = 220 \pm 20 \mu\text{m}$  and  $d_c = 125 \pm 25 \mu\text{m}$  are used. For case (B), the glass beads of diameters  $d_a = 274 \pm 25 \mu\text{m}$ ,  $d_b = 125 \pm 25 \mu\text{m}$  and  $d_c = 50 \pm 25 \mu\text{m}$  are used. In case (A), the  $\frac{K_b}{K_a} = 0.64$ ,  $\frac{K_c}{K_a} = 0.20$ ,  $\frac{P_{c_b}}{P_{c_a}} = 1.24$ ,  $\frac{P_{c_c}}{P_{c_a}} = 2.19$  and in case (B), the  $\frac{K_b}{K_a} = 0.20$ ,  $\frac{K_c}{K_a} = 0.03$ ,  $\frac{P_{c_b}}{P_{c_a}} = 2.19$ ,  $\frac{P_{c_c}}{P_{c_a}} = 5.48$ . Ashraf and Phirani (2019) explained that, when  $\frac{K_b}{K_a} \left( \frac{P_{c_b}}{P_{c_a}} - 1 \right) - \frac{K_c}{K_a} \left( \frac{P_{c_c}}{P_{c_a}} - 1 \right) < 0$  as in case (A), the front in the small permeability layer should lead and when  $\frac{K_b}{K_a} \left( \frac{P_{c_b}}{P_{c_a}} - 1 \right) - \frac{K_c}{K_a} \left( \frac{P_{c_c}}{P_{c_a}} - 1 \right) > 0$  as in case (B), the front in the medium permeability layer should lead. In the current work, we focus on porous media where, the imbibition length direction is much larger than the thickness of the layers. For the cases where the thickness of the layers is also important, the model needs to be modified by including thickness.

### 2.1. Comparison of experiments and one-dimensional model

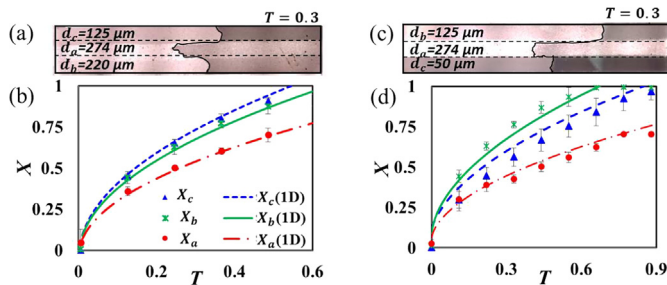
We compare the experimental observations with the one-dimensional model by Ashraf and Phirani (2019) for the cases (A) and (B) as shown in Fig. 3. The snapshots of imbibition are shown in Fig. 3(a), (c), while Fig. 3(b), (d) show the fluid front imbibition length vs time for the cases (A), (B), respectively. According to the relation given by Ashraf and Phirani (2019), in case (A), we anticipate the front in the small permeability layer to lead and in case (B), we anticipate the front in the medium permeability layer to lead, which are also observed experimentally. The dimensionless lengths are scaled as  $X_i = \frac{x_i}{L}$ , where  $x_i$  is the distance traveled by the fluid front from the inlet in layer with bead diameter  $d_i$  and  $L$  is the total length of the porous medium. The dimensionless time  $t$  is scaled as  $T = \left( \frac{P_{c0} K_a}{\phi \mu u L^2} \right) t$ . The value of  $\frac{P_{c0} K_a}{\phi \mu u L^2}$  used in all the experiments is approximately  $0.006 \text{ s}^{-1}$ . We see that, the one-dimensional model of Ashraf and Phirani (2019) compares well with the experimental observations for both the cases (A) and (B) of arrangement (iii) and is able to predict the leading front accurately.

In Fig. 3, we see that the fluid front in the large permeability layer lags behind the fronts in the other two layers. It can be observed that, due to the inherent heterogeneity in each of the layers, the imbibition front is not a straight line but is slightly curved. The length invaded by a curved imbibition front is determined by the customized MATLAB code which determines the average length of the imbibition pixel wise. Three experiments are performed for each of the cases (A) and (B). The average value of length imbibed for the three experiments is plotted with dimensionless time and the error bars represent the experimental error due to the variation in the imbibition lengths in the three experiments.

### 3. Model development for five-layered porous medium

When we have more than three layers, the governing equations for different arrangements or for permeability contrasts in the same arrangement will be different. For a  $n$ -layered porous medium, there are  $\frac{n!}{2}$  possible arrangements and each kind of arrangement can have varying degrees of permeability contrasts and will lead to a different imbibition behavior. In this section we explore a five layered arrangement for generalization of these governing equations.

We now consider five layers in a porous medium denoted as  $a, b, c, d, e$ , which can be positioned randomly in the five layered

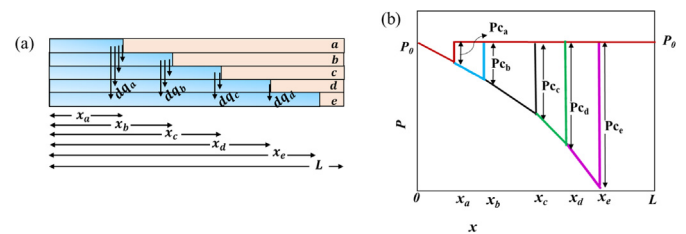


**Fig. 3.** Snapshots at dimensionless time  $T = 0.3$  and dimensionless imbibition length vs dimensionless time for the spontaneous imbibition in arrangement (iii) of a 3-layered porous medium where the large permeability layer is between the medium and the small permeability layers. (a) Experimental snapshot of case (A)  $x_a < x_b < x_c$ . (b) Comparison of experiments with the one-dimensional lubrication approximation for case (A). (c) Experimental snapshot of case (B)  $x_a < x_c < x_b$ . (d) Comparison of experiments with the one-dimensional lubrication approximation for case (B). The error bars in (b), (d) represent the standard deviation in the experimental results.

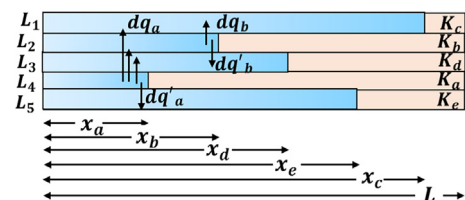
porous medium. The order of permeability of the layers is  $K_a > K_b > K_c > K_d > K_e$  and capillary pressures of the layers are in the order  $P_{c_a} < P_{c_b} < P_{c_c} < P_{c_d} < P_{c_e}$ . The total number of random arrangements are 60 for  $n = 5$ . Fig. 4(a) shows an ordered arrangement of the layers, which is a special case. The position of the fluid fronts can be explained using the only possible anticipated pressure gradient graph shown in Fig. 4(b) which shows pressure vs length curves in all the layers. We see that, for an ordered arrangement,  $x_a < x_b < x_c < x_d < x_e$ , irrespective of the contrast in the permeability and capillary pressure of the layers. This is because, the capillary pressure jump across the fluid front in each of the layers should be sufficient enough to attain the pressure of the non-wetting fluid, i.e.,  $P_0$ , as shown in Fig. 4(b). As the capillary pressure in the largest permeability layer is the least, the fluid front in it will have to be the first fluid front from the inlet such that, after the capillary pressure jump at  $x_a$ , the pressure in the largest permeability layer will be  $P_0$ . According to the assumptions of Ashraf et al. (2017, 2019) the pressure drop in all the layers is same till the first fluid front, i.e.,  $x_a$ , as shown in Fig. 4(b). After  $x_a$ , the pressure drop in layers  $b, c, d, e$  is same till the fluid front  $x_b$ , as they are filled with wetting fluid as shown in Fig. 4(b). Similarly, the next fluid front will be in layer  $c$  and the pressure gradient is same in the layers  $c, d, e$  after  $x_b$ . For the layers  $d, e$  the pressure drop is same from  $x_c$  to  $x_d$ , as shown in Fig. 4(b) and layer  $e$  will have the leading fluid front. Therefore, for an ordered arrangement, Fig. 4(b) is the only possible pressure gradient profile.

We now consider one arrangement of five layered porous medium to understand the model development for a randomly arranged multi-layered system. Fig. 5 shows the layers  $c, b, d, a, e$ , having permeability  $K_c, K_b, K_d, K_a, K_e$  in the five layered porous medium. We name this arrangement  $cbdae$ . For this arrangement, eight relative fluid front positions are possible as shown by the anticipated pressure gradient graphs in Fig. 6, depending on the capillary pressure and the permeability contrast of the layers. The leading front can be in any of the layers  $c, d$  or  $e$  as shown in Fig. 6.

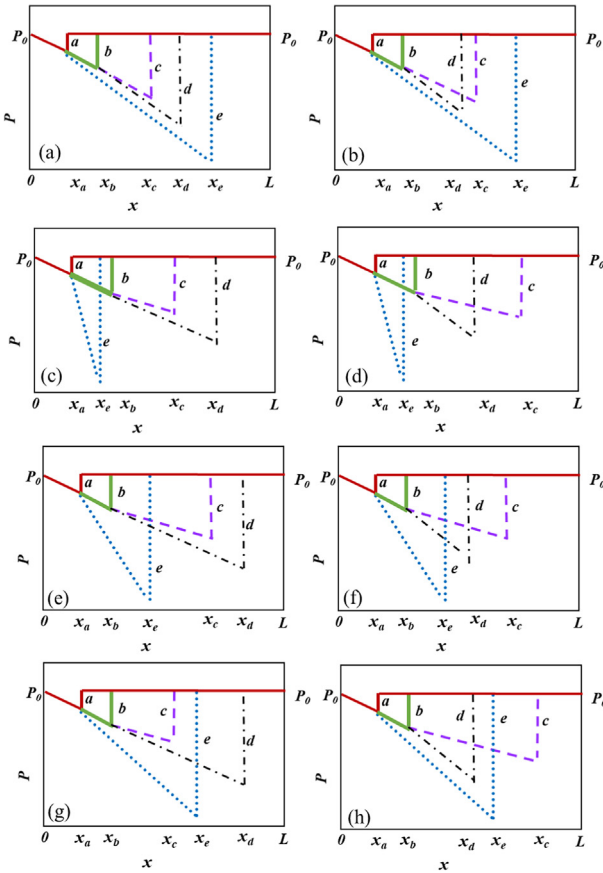
From Fig. 6, we see that the trailing front is always in the largest permeability layer. Let the distance imbibed by the fluid front be  $x_a$ , from the inlet. The Darcy pressure drop in all the layers is same from the inlet till  $x_a$ , given by,



**Fig. 4.** Spontaneous imbibition in an ordered arrangement of layers in a five layered porous medium. (a) Schematic of the fluid fronts' position. (b) Anticipated pressure gradient graph.



**Fig. 5.** Schematic of spontaneous imbibition in a random arrangement of layers in a five layered porous medium.



**Fig. 6.** All the possible anticipated pressure gradient graphs for the arrangement shown in Fig. 5.

$$P(x_a, t) - P_0 = -\frac{\mu x_a}{K_a} v_a(t), \quad (1)$$

where,  $\mu$  is the viscosity of the invading phase,  $v_a(t)$  is the velocity of fluid in the layer  $a$  from inlet to  $x_a$  and  $P(x_a, t), P_0$  are the pressures of wetting fluid at  $x_a$  and inlet, respectively. As all the five layers are filled with wetting phase till  $x_a$ , the pressure gradient in all the layers is same, which implies,

$$\frac{v_a(t)}{K_a} = \frac{v_b(t)}{K_b} = \frac{v_c(t)}{K_c} = \frac{v_d(t)}{K_d} = \frac{v_e(t)}{K_e}, \quad (2)$$

where,  $v_i(t)$  is the velocity before  $x_a$  in the layer having permeability  $K_i$ . After the first fluid front at  $x_a$ , the layer  $a$  is filled with non-wetting phase of negligible viscosity, which is air. Due to the sudden jump in pressure across the fluid front at  $x_a$ , some of the wetting fluid transfers from the layer  $a$  to the other layers, just before  $x_a$  as shown in Fig. 5. Let  $dq_a$  be the transfer of wetting phase from the large permeability layer to the layers positioned on one side of  $a$ , i.e.,  $c, b, d$ , as shown in Fig. 5. Similarly, let  $dq'_a$  be the wetting phase transfer from layer  $a$  to the layers present on the other side of layer  $a$ , i.e.,  $e$ , as shown in Fig. 5. After fluid transfer, the velocity of the fluid front in layer  $a$  becomes,

$$\phi \frac{dx_a}{dt} = v_a(t) - \frac{dq_a + dq'_a}{A}, \quad (3)$$

where  $A$  is the cross section area and  $\phi$  is the porosity of the layers. The fluid transfer  $dq'_a$  from layer  $a$  increases the fluid velocity in layer  $e$ . The velocity of the fluid front in layer  $e$  having permeability  $K_e$  is  $\frac{dx_e}{dt}$ , given by,

$$\phi \frac{dx_e}{dt} = v_e(t) + \frac{dq'_a}{A}. \quad (4)$$

Now, we see from the anticipated pressure gradient plots of Fig. 6 that, the pressure gradient in the layers  $c, b, d$  is different from the pressure gradient in the layer  $e$ . This is because, after  $x_a$ , the wetting fluid in  $c, b, d$  is separated from the wetting fluid in  $e$  by the non-wetting fluid in  $a$ , and hence is not continuous. Therefore the pressure drop in layer  $e$ , from  $x_a$  to  $x_e$  is,

$$P(x_e, t) - P(x_a, t) = -\frac{\mu(x_e - x_a)}{K_e} \left( v_e(t) + \frac{dq'_a}{A} \right), \quad (5)$$

where,  $P(x_e, t)$  is the pressure of wetting fluid at  $x_e$  and  $P(x_a, t)$  is the wetting fluid pressure at  $x_a$ .

On the other side of layer  $a$ , the fluid transfer  $dq_a$  distributes in the layers  $c, b, d$  such that, the pressure gradient is same after  $x_a$ . This causes a change in the wetting fluid velocity in the layers  $c, b, d$  after  $x_a$ , such that,

$$\frac{v_c(t) + \alpha_{a,c} \frac{dq_a}{A}}{K_c} = \frac{v_b(t) + \alpha_{a,b} \frac{dq_a}{A}}{K_b} = \frac{v_d(t) + \alpha_{a,d} \frac{dq_a}{A}}{K_d}. \quad (6)$$

Here,  $\alpha_{a,l}$  is the fraction of wetting fluid transfer from layer  $a$  to a layer having permeability  $K_l$  ( $l = c, b, d$ ). We know that,  $\sum_{l=c,b,d} \alpha_{a,l} = 1$ . From Eqs. (2) and (6), for a layer having permeability  $K_l$  ( $l = c, b, d$ ), we have,

$$\alpha_{a,l} = \frac{K_l}{\sum_{l=c,b,d} K_l}. \quad (7)$$

We see from Fig. 6 showing the possible pressure gradient graphs that, among the layers  $c, b, d$ , the largest permeability layer will have the trailing front, i.e., layer  $b$ , as shown in Fig. 6. It can also be observed from Fig. 6 that, the pressure gradient is same in layers  $c, b, d$  after  $x_a$ , which is given as,

$$P(x_b, t) - P(x_a, t) = -\frac{\mu(x_b - x_a)}{K_b} \left( v_b(t) + \frac{\alpha_{a,b} dq_a}{A} \right), \quad (8)$$

where,  $P(x_b, t)$  is the pressure of wetting fluid at  $x_b$ . At  $x_b$ , due to the sudden jump of pressure across the front in layer  $b$ , some of the wetting fluid transfers into the layers  $c$  and  $d$ . If  $dq_b$  is the fluid transfer from layer  $b$  to layer  $c$  and  $dq'_b$  is the fluid transfer from layer  $b$  to layer  $d$ , the velocity of the fluid front in layer  $b$ , i.e.,  $\frac{dx_b}{dt}$  is given as,

$$\phi \frac{dx_b}{dt} = v_b(t) + \frac{\alpha_{a,b} dq_a - dq_b - dq'_b}{A}. \quad (9)$$

As there is only one layer on either sides of layer  $b$ , the pressure drop in the layers  $c$  and  $d$  are independently found. The pressure drop of wetting fluid from  $x_b$  to  $x_c$  in layer  $c$  is,

$$P(x_c, t) - P(x_b, t) = -\frac{\mu(x_c - x_b)}{K_c} \left( v_c(t) + \frac{\alpha_{a,c} dq_a + dq_b}{A} \right), \quad (10)$$

where,  $P(x_c, t)$  is the pressure of wetting fluid at  $x_c$ . The velocity of the fluid front in layer  $c$  after the fluid transfer  $dq_b$  is,

$$\phi \frac{dx_c}{dt} = v_c(t) + \frac{\alpha_{a,c} dq_a + dq_b}{A}. \quad (11)$$

Similarly, the pressure drop of wetting fluid from  $x_b$  to  $x_d$  in layer  $d$  is,

$$P(x_d, t) - P(x_b, t) = -\frac{\mu(x_d - x_b)}{K_d} \left( v_d(t) + \frac{\alpha_{a,d} dq_a + dq'_b}{A} \right), \quad (12)$$

where,  $P(x_d, t)$  is the pressure of wetting fluid at  $x_d$ . We find the velocity of fluid front in layer  $d$  after the fluid transfer  $dq'_b$  at  $x_b$ , which is given as,

$$\phi \frac{dx_d}{dt} = v_d(t) + \frac{\alpha_{a,d} dq_a + dq'_b}{A}. \quad (13)$$

We know that the pressure jump across the fluid front in each of the layers is equal to the capillary pressure of that layer, i.e.,  $P(x_i, t) - P_0 = -P_{c_i}$  ( $i = a, b, c, d, e$ ). By using Eqs. (2), (3), (4), (9), (11), (13) to substitute for velocity of the fronts in Eqs. (1), (5), (8), (10) and (12), we obtain the equations governing the flow behavior in a 5-layered system as,

$$P_{c_a} = \frac{\phi \mu x_a}{K_a + K_b + K_c + K_d + K_e} \times \left( \frac{dx_a}{dt} + \frac{dx_b}{dt} + \frac{dx_c}{dt} + \frac{dx_d}{dt} + \frac{dx_e}{dt} \right), \quad (14)$$

$$P_{c_e} - P_{c_a} = \frac{\phi \mu (x_e - x_a)}{K_e} \left( \frac{dx_e}{dt} \right), \quad (15)$$

$$P_{c_b} - P_{c_a} = \frac{\phi \mu (x_b - x_a)}{K_b + K_c + K_d} \left( \frac{dx_b}{dt} + \frac{dx_c}{dt} + \frac{dx_d}{dt} \right), \quad (16)$$

$$P_{c_c} - P_{c_b} = \frac{\phi \mu (x_c - x_b)}{K_c} \left( \frac{dx_c}{dt} \right), \quad (17)$$

$$P_{c_d} - P_{c_b} = \frac{\phi \mu (x_d - x_b)}{K_d} \left( \frac{dx_d}{dt} \right). \quad (18)$$

The Eqs. (14)–(18) can be converted to dimensionless form by considering dimensionless length as  $X_i = \frac{x_i}{L}$ , dimensionless time as  $T = \frac{K_a P_{c_a}}{\phi \mu L^2}$ , dimensionless capillary pressure as  $\beta_i = \frac{P_{c_i}}{P_{c_a}}$  and dimensionless permeability as  $\kappa_i = \frac{K_i}{K_a}$ . Then, the Eqs. (14)–(18) become,

$$1 = \frac{X_a}{1 + \kappa_b + \kappa_c + \kappa_d + \kappa_e} \times \left( \frac{dX_a}{dT} + \frac{dX_b}{dT} + \frac{dX_c}{dT} + \frac{dX_d}{dT} + \frac{dX_e}{dT} \right), \quad (19)$$

$$\beta_e - 1 = \frac{X_e - X_a}{\kappa_e} \left( \frac{dX_e}{dT} \right), \quad (20)$$

$$\beta_b - 1 = \frac{X_b - X_a}{\kappa_b + \kappa_c + \kappa_d} \left( \frac{dX_b}{dT} + \frac{dX_c}{dT} + \frac{dX_d}{dT} \right), \quad (21)$$

$$\beta_c - \beta_b = \frac{X_c - X_b}{\kappa_c} \left( \frac{dX_c}{dT} \right), \quad (22)$$

$$\beta_d - \beta_b = \frac{X_d - X_b}{\kappa_d} \left( \frac{dX_d}{dT} \right). \quad (23)$$

Now, by rearranging the terms in Eqs. (19)–(23) and adding the governing equations, we obtain,

$$2 \left( 1 + \sum_{i=b}^e \beta_i \kappa_i \right) T = \sum_{j=a}^e X_j^2. \quad (24)$$

Eq. (24) gives the imbibition behavior for the entire layered porous medium, which is analogous to Washburn's law. However, the lengths imbibed in each of the layers cannot be predicted even if the effective permeability  $K_{eff} = \sum_{i=a}^e K_i$  and the effective capillary pressure  $P_{c_{eff}} = \frac{\sum_{i=a}^e K_i P_{c_i}}{\sum_{i=a}^e K_i}$  are known. We also see that, the pressure till  $X_a$  is same in all the layers, irrespective of the arrangement of the layers. Therefore, Eq. (19) is same for all the arrangements, which can be written as,

$$\frac{1 + \kappa_b + \kappa_c + \kappa_d + \kappa_e}{X_a} = \frac{dX_a}{dT} + \frac{dX_b}{dT} + \frac{dX_c}{dT} + \frac{dX_d}{dT} + \frac{dX_e}{dT}, \quad (25)$$

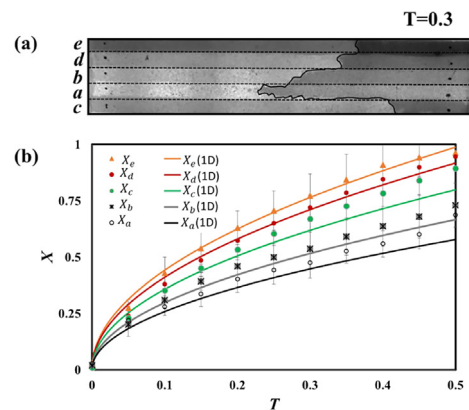
Eq. (25) predicts that the summation of lengths imbibed in the layers is same for all the arrangements of the given layers.

### 3.1. Imbibition in a five layered porous medium

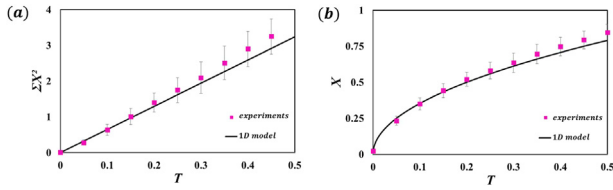
We perform three experiments for one arrangement of a five layered porous medium using the experimental setup described in Section 2. The porous medium is 10 cm in length, 1.8 cm in width, 0.15 cm in height and each layer is 3.6 mm wide. The order of the arrangement of the beads in the experiment is shown in Fig. 7, which is *edbac*. The bead sizes used are  $d_a = 275 \pm 25 \mu\text{m}$ ,  $d_b = 220 \pm 20 \mu\text{m}$ ,  $d_c = 175 \pm 25 \mu\text{m}$ ,  $d_d = 125 \pm 25 \mu\text{m}$  and  $d_e = 50 \pm 25 \mu\text{m}$ . The ratio of permeability are,  $\kappa_b = 0.64$ ,  $\kappa_c = 0.4$ ,  $\kappa_d = 0.2$  and  $\kappa_e = 0.03$ . The capillary pressure ratios are,  $\beta_b = 1.24$ ,  $\beta_c = 2$ ,  $\beta_d = 2.19$  and  $\beta_e = 5.48$ . In Fig. 7 we show the comparison of spontaneous imbibition experiment with the one-dimensional model. We see that the model predicts the leading front reasonably well. The experimental observation for the five layered porous medium also satisfies Eqs. (24) and (25). Fig. 8(a), (b) show  $X$  vs  $T$  and  $\sum X^2 \propto T$ . Therefore, our one-dimensional model is able to predict the imbibition behavior in a multi-layered porous medium. The error bars in Fig. 7(b) represent the error estimates in the experiments.

We now analyze the five layered porous medium using the model developed here. Fig. 9 shows the different cases of five layered porous medium in which the layers *c, b, d, a* and *e* are made of bead diameters  $d_c, d_b, d_d, d_a$  and  $d_e$ , respectively. We consider that for a bead diameter,  $d_i$ , the permeability scales as  $K_i = \Lambda d_i^2$  and capillary pressure is  $P_{c_i} = \frac{\gamma}{d_i}$ . Fig. 9 shows the six of the eight possible cases shown in Fig. 6. We observe from Fig. 9 that, depending upon the contrast in permeability and capillary pressure, the imbibition pattern and the leading front in the layers changes for the same arrangement. The bead diameters in microns for each of the layers is mentioned in the length vs time curves in Fig. 9 and we see that the leading front can be in any of the layers *c, d* or *e*.

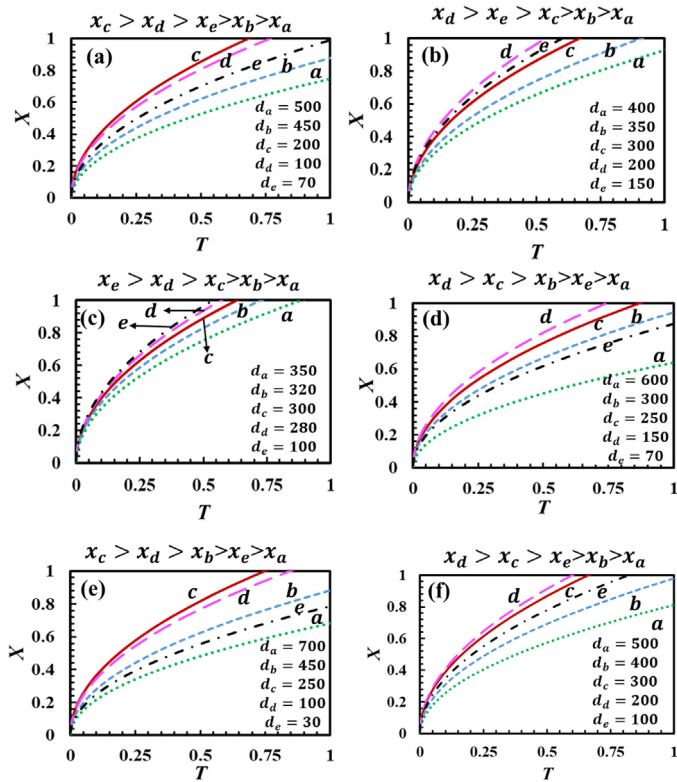
Now, we consider bead diameters  $d_a = 500 \mu\text{m}$ ,  $d_b = 400 \mu\text{m}$ ,  $d_c = 300 \mu\text{m}$ ,  $d_d = 200 \mu\text{m}$ ,  $d_e = 100 \mu\text{m}$  and use the generalized one-dimensional model to explore imbibition in different arrangements. We observe from Fig. 10 that, for different arrangements of layers having the same physical properties, the imbibition behavior also changes. In Fig. 10(a), we show the length vs time plot for the ordered arrangement, i.e., *abcde* shown in Fig. 4 where,  $X_a < X_b < X_c < X_d < X_e$  as explained in Section 3. The plots (b), (c), (d), (e) and (f) of Fig. 10 show the imbibition length with time for arrangements having layers in the order *cbdae, bedac, bdace, dceab* and *ebcad*, respectively, having the same



**Fig. 7.** (a) Snapshot of the imbibition in a five layered porous medium at  $T = 0.3$ . (b) Comparison of the experiment with the one-dimensional model for the dimensionless lengths imbibed in the layers with time for a five layered porous medium. The error bars represent the standard deviation from the average imbibition lengths in the experiments.



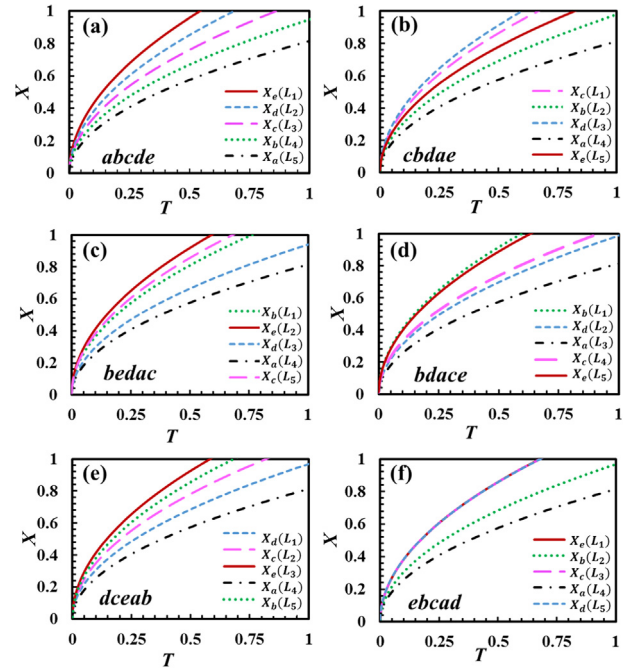
**Fig. 8.** Comparison of the experimental and the one-dimensional model for, (a) average length imbibed, (b) sum of squares of dimensionless lengths imbibed vs dimensionless time.



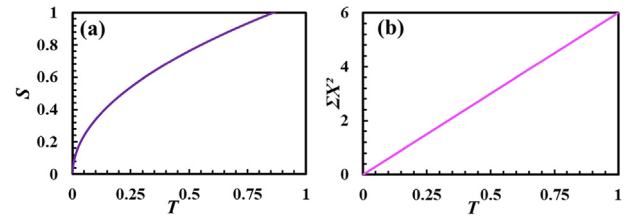
**Fig. 9.** A plot of the dimensionless lengths imbibed in the layers with time for different bead diameters of the arrangement shown in Fig. 5. The bead diameters are indicated in each of the plots (a), (b), (c), (d), (e) and (f) in microns and the curves are annotated with the subscript of the respective bead diameters  $d_a, d_b, d_c, d_d$  and  $d_e$ .

bead sizes as in the case shown in Fig. 10(a). Fig. 11(a) shows that the total average length imbibed in the entire porous medium is same for all the arrangements shown in Fig. 10. Fig. 11(b) shows that, the sum of squares of length imbibed with time is also same and is proportional to time for all the arrangements as given by Eq. (25).

We see from the model development of the five layered porous medium that, we first need to divide the layers into two regions  $c, b, d$ , and  $e$ , one on either side of the largest permeability layer  $a$ . All the layers till the fluid front in the largest permeability layer, i.e.,  $x_a$ , will have the same pressure gradient. Each of the two regions  $c, b, d$  and  $e$  are separated by the non-wetting fluid in the largest permeability layer. As the pressure gradient in all the layers of a region is same if they are filled with the same fluid, the pressure gradient in  $c, b, d$  is same after  $x_a$ . Further, the region  $c, b, d$  is divided into two regions  $c$  and  $d$ , which are separated by the layer  $b$  having the largest permeability among the layers of the region. There is only one layer  $e$  on the other side of layer  $a$ , which terminates the process of dividing the layers into two regions. Similarly,



**Fig. 10.** A plot of the dimensionless lengths imbibed in the layers with time for bead diameters  $d_a = 500 \mu\text{m}$ ,  $d_b = 400 \mu\text{m}$ ,  $d_c = 300 \mu\text{m}$ ,  $d_d = 200 \mu\text{m}$ ,  $d_e = 100 \mu\text{m}$  for different arrangements. The order of arrangement of the layers are indicated in each of the plots (a), (b), (c), (d), (e) and (f).



**Fig. 11.** (a) The average length imbibed in the porous medium with time and (b) the sum of squares of length imbibed with time for the bead diameters  $d_a = 500 \mu\text{m}$ ,  $d_b = 400 \mu\text{m}$ ,  $d_c = 300 \mu\text{m}$ ,  $d_d = 200 \mu\text{m}$ ,  $d_e = 100 \mu\text{m}$  for different arrangements, shown in Fig. 10.

as the layers  $c$  and  $d$  cannot be divided further, any of the layers  $e, c, d$  can have the leading fluid front, which is shown in Fig. 6. Therefore, we show that, the imbibition in a multi-layered porous medium strongly depends on the arrangement of layers, the contrast in permeability and capillary pressure. We now develop the generalized model for a multi-layered porous medium.

#### 4. Generalized model

We consider a multi-layered porous medium made of  $n$ -layers, where each layer has a specific permeability and associated capillary pressure. The permeability of the layers is  $K_a > K_b > K_c > \dots > K_n$  in the order of grain sizes  $d_a > d_b > d_c > \dots > d_n$ , for which the corresponding capillary pressure of the layers will be  $P_{c_a} < P_{c_b} < P_{c_c} < \dots < P_{c_n}$ . The layers in one arrangement are numbered by  $L_1, L_2, \dots, L_{(n-1)}, L_n$ , in addition to the representation of the layers by their permeability as shown in Fig. 12. Now,  $\frac{n!}{2}$  such arrangements are possible. By considering the assumptions of Ashraf et al. (2017, 2019), we develop a one-dimensional model to predict the fluid front positions for any given arrangement of the layers in a  $n$ -layered porous medium.

As explained in Sections 2 and 3, if there is a multi-layered porous medium having  $n$ -layers, the pressure gradient is different only when the invading fluid is present in sections of the layers that do not interact with each other. We know that, the first fluid front from the inlet is in the layer having the largest permeability ( $K_a$ ) corresponding to the lowest capillary pressure jump ( $Pc_a$ ), at  $x_a$ , as shown in Fig. 12. We use the following is a step-by-step procedure to predict the imbibition phenomenon in the porous layers.

1. We find the largest permeability layer,  $L_i$  (having permeability  $K_a$ ) and divide the porous medium into two regions,  $L_1$  to  $L_{(i-1)}$  and  $L_{(i+1)}$  to  $L_n$ , as shown in Fig. 12.
2. The pressure drop from inlet till the first fluid front, i.e., the fluid front at  $x_a$  is same for all the layers and is determined by Darcy's law.
3. Again the region  $L_1$  to  $L_{(i-1)}$  is further divided by finding the large permeability layer among these layers. The fluid front succeeding  $x_a$  among the layers  $L_1$  to  $L_{(i-1)}$  will be in the largest permeability layer corresponding to the least pressure jump. Let the layer  $L_j$  be the largest permeability layer among the layers  $L_1$  to  $L_{(i-1)}$  with a permeability  $K_b$ . We then divide these layers into two subsequent regions  $L_1$  to  $L_{(j-1)}$  and  $L_{(j+1)}$  to  $L_{(i-1)}$ , as shown in Fig. 12.  
The pressure drop is same in layers  $L_1$  to  $L_{(i-1)}$  from  $x_a$  to  $x_b$ , which is found using Darcy's law. After  $x_b$ , the pressure drop is different for layers  $L_1$  to  $L_{(j-1)}$  and  $L_{(j+1)}$  to  $L_{(i-1)}$ , although these subsequent regions are filled with invading fluid. This is due to the resident fluid in layer  $K_b$  from  $x_a$  to  $x_b$  separating the two regions.
4. Similarly, we find the large permeability layer among layers  $L_{(i+1)}$  to  $L_n$ , which is  $K_c$  (of layer  $L_k$ ), as shown in Fig. 12, and further divide the layers into two regions. The subsequent regions are  $L_{(i+1)}$  to  $L_{(k-1)}$  and  $L_{(k+1)}$  to  $L_n$ , as shown in Fig. 12. The pressure drop in  $L_{(i+1)}$  to  $L_n$  is same from  $x_a$  to  $x_c$ , which is determined using Darcy's law. After  $x_c$ , the pressure drop due to the invading fluid is different for layers  $L_{(i+1)}$  to  $L_{(k-1)}$  and  $L_{(k+1)}$  to  $L_n$ , as they are separated by the resident phase in layer  $L_k$ .
5. The four regions  $L_1$  to  $L_{(j-1)}$ ,  $L_{(j+1)}$  to  $L_{(i-1)}$ ,  $L_{(i+1)}$  to  $L_{(k-1)}$  and  $L_{(k+1)}$  to  $L_n$  are further divided into eight subsequent regions by finding the large permeability layer in each of these regions as explained in steps 3 and 4. This division of layers is continued till there is only one layer left in the subsequent region. The pressure drop in each of the regions is determined using Darcy's law. For a  $n$ -layered porous medium,  $n$  such regions are formed by dividing the layers according to the fluid fronts' positions.

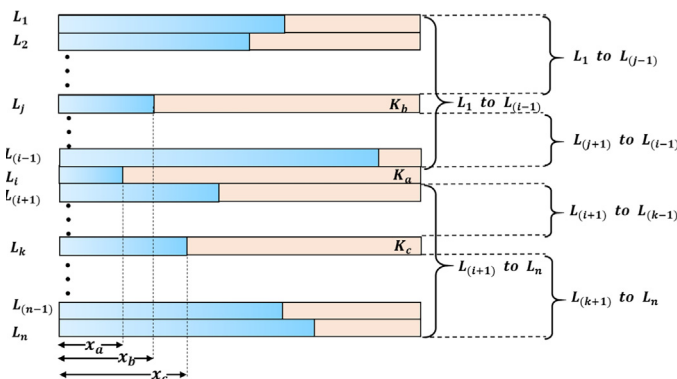


Fig. 12. Spontaneous imbibition in a  $n$ -layered porous medium.

6. As the imbibition is spontaneous, the total pressure drop across the fluid front in each layer is equal to the capillary pressure of that layer, i.e., the pressure jump across the fluid fronts located at  $x_i = Pc_i$  where  $i = a, b, \dots, n$ . The  $n$  pressure drop equations for the  $n$ -layers are solved for the fluid front imbibition in the multi-layered porous medium with time.

#### 4.1. Imbibition in a multi-layered porous medium

We consider a  $n$ -layered porous medium and the governing equations are developed as described in Section 4. A MATLAB program is used to execute the algorithm and to solve the governing equations of a randomly arranged layered porous medium. We consider  $n = 20$ , which has  $1.216 \times 10^{18}$  possible random arrangements of layers and solve for 1000 such random cases. The bead size has a uniform distribution of diameters with the smallest diameter as  $10 \mu\text{m}$  and the largest diameter as  $200 \mu\text{m}$ . A random number generator is used to obtain the random positioning of the layers in the porous medium, which then proceeds to develop equations governing the flow behavior, as explained for five layered porous medium in Section 3.

Fig. 13 shows the bead diameters in microns vs dimensionless imbibition lengths at  $T = 0.5$  for 6 different arrangements of a twenty layered porous medium. For the case of ordered arrangement, in which the layers are placed in the order of capillary pressure, the imbibition fronts are also in the order of the corresponding capillary pressures of the layers. This can be observed from Fig. 13 that the imbibition front in the smallest permeability layer leads for an ordered arrangement as shown by the green circles. For other arrangements, we see from Fig. 13 that, the leading front is strongly dependent on the arrangement of the layers. Therefore, the breakthrough time or the time at which the leading front reaches the outlet end of the porous medium also varies significantly. We show in Fig. 14, the dimensionless grain diameter vs the corresponding breakthrough time for 1000 arrangements. Dimensionless grain diameter is the ratio of the grain diameter of the layer having the leading front to the grain diameter of the large permeability layer ( $d_a$ ). We see that, for all the arrangements, the dimensionless breakthrough time lies between 0.55 and 0.7, whereas the dimensionless grain diameter of the leading front is between 0.05 and 0.55, i.e., the breakthrough can occur in any of the layers having grain sizes from  $10 \mu\text{m}$  to  $110 \mu\text{m}$ . Therefore, by choosing a suitable arrangement of the layers, an application based porous medium can be designed with desired flow properties.

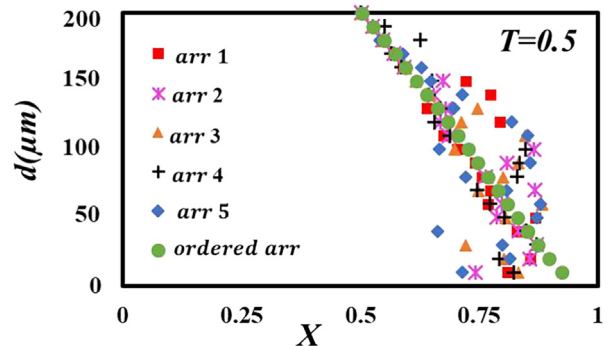
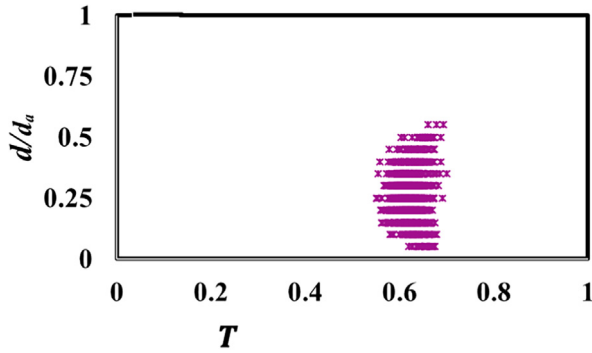


Fig. 13. Bead diameter  $d$  of the layer (in microns) vs dimensionless imbibition length in the layers for 6 arrangements of a 20 layered porous medium. The green circles indicate the ordered arrangement of layers where the fluid fronts are in the order of the capillary pressures of the layers. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



**Fig. 14.** The ratio of the leading front's layer grain diameter to the grain diameter of the large permeability layer vs the dimensionless breakthrough time for spontaneous imbibition in 1000 random arrangements of a twenty layered porous medium.

## 5. Conclusions

In summary, we developed a generalized one-dimensional, lubrication approximation model to predict the spontaneous imbibition in a multi-layered porous medium. First, we perform experiments on a three layered porous medium having the large permeability layer placed between the small and the medium permeability layer. We find that the leading fluid front is not always in the fine pores and strongly depends on the contrasts in the permeability and capillary pressure. This is in concurrence with the explanation given by Ashraf and Phirani (2019), and excellently matches with the one-dimensional model. We then develop a generalized model using the assumptions of Ashraf et al. (2017, 2018, 2019), which predicts the position of the fluid fronts for any arrangement of a multi-layered porous medium. We describe the generalized model development in one arrangement of a five layered porous medium. Using the pressure vs length curves, we show that depending on the permeability and capillary pressure contrasts, the leading fluid front can be in different layers for a given arrangement. We also show that, the layering arrangement in the porous medium leads to different travel times of the fronts of the invading fluid in the layers. This changes the breakthrough time and location of the wetting fluid on the outlet side of the porous medium. In a twenty layered porous medium, we use 1000 arrangements to show that the leading front layer can vary significantly. In the twenty layered case used in this study where the maximum and minimum grain diameters are 200  $\mu\text{m}$  and 10  $\mu\text{m}$ , the leading front can vary in the layers made of grains 10  $\mu\text{m}$  to 110  $\mu\text{m}$ , only based on the arrangement. This study is useful in predicting the imbibition behavior in the soil layers, aquifer and oil reservoir. The one-dimensional model can be used in designing microfluidic and diagnostic devices.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix A

The pressure across the fluid front in each layer is the capillary pressure of the layer, i.e.,  $P_0 - P_w(x_i, t) = P_{c_i}$  for a layer made of beads of diameter  $d_i$ . Therefore, for the layer having permeability  $K_a$ , the pressure drop from inlet till  $x_a$  can be expressed from Eq. (1) as,

$$P_0 - P(x_a, t) = P_{c_a}. \quad (26)$$

Similarly from the inlet till the front in the layer having permeability  $K_e$ , using Eqs. (1) and (5), we have,

$$[P_0 - P(x_a, t)] + [P(x_a, t) - P(x_e, t)] = P_{c_e}. \quad (27)$$

For the layer having permeability  $K_b$ , using the Eqs. (1) and (8), the pressure drop from inlet till the front  $x_b$  is written as,

$$[P_0 - P(x_a, t)] + [P(x_a, t) - P(x_b, t)] = P_{c_b}. \quad (28)$$

The pressure drop in layer having permeability  $K_c$  from the inlet till  $x_c$  is obtained from Eqs. (1), (8) and (10) as,

$$[P_0 - P(x_a, t)] + [P(x_a, t) - P(x_b, t)] + [P(x_b, t) - P(x_c, t)] = P_{c_c}. \quad (29)$$

Similarly, the pressure drop till the fluid front in layer having permeability  $K_d$  can be written using Eqs. (1), (8) and (12) as,

$$[P_0 - P(x_a, t)] + [P(x_a, t) - P(x_b, t)] + [P(x_b, t) - P(x_d, t)] = P_{c_d}. \quad (30)$$

Substituting Eq. (7) in Eqs. (9), (11), (13) and adding the velocity of the fronts in all the layers given by Eqs. (3), (4), (9), (11), (13), we obtain,

$$\phi \sum_{i=a}^e \frac{dx_i}{dt} = \sum_{i=a}^e v_i(t). \quad (31)$$

We know from Eq. (2) that the velocity in the fronts before the first fluid front are related as,

$$v_l(t) = \frac{v_a(t)}{K_a} K_l \quad l = b, c, d, e. \quad (32)$$

By substituting Eq. (32) in the right hand side of Eq. (31), the sum of velocity of fluid in the layers before  $x_a$  is expressed as,

$$\sum_{i=a}^e v_i(t) = \frac{v_a(t)}{K_a} \sum_{i=a}^e K_i. \quad (33)$$

From Eqs. (31) and (33), the velocity of fluid in layer having permeability  $K_a$  is expressed as,

$$v_a(t) = \frac{K_a}{K_a + K_b + K_c + K_d + K_e} \times \phi \sum_{i=a}^e \frac{dx_i}{dt}. \quad (34)$$

The velocity  $v_a(t)$  from Eq. (34) is substituted in Eq. (1), which gives us the governing Eq. (14), i.e.,

$$P_{c_a} = \frac{\phi \mu x_a}{K_a + K_b + K_c + K_d + K_e} \times \left( \frac{dx_a}{dt} + \frac{dx_b}{dt} + \frac{dx_c}{dt} + \frac{dx_d}{dt} + \frac{dx_e}{dt} \right), \quad (35)$$

Now, subtracting Eq. (26) from Eq. (27), we have,

$$P(x_a, t) - P(x_e, t) = P_{c_e} - P_{c_a}. \quad (36)$$

By substituting Eq. (5) in Eq. (36), we have,

$$\frac{\mu(x_e - x_a)}{K_e} \left( V_e(t) + \frac{dq_a}{A} \right) = P_{c_e} - P_{c_a}. \quad (37)$$

By substituting  $\left( V_e(t) + \frac{dq_a}{A} \right) = \phi \frac{dx_e}{dt}$  from Eq. (4), we obtain,

$$Pc_e - Pc_a = \frac{\mu(x_e - x_a)}{K_e} \phi \frac{dx_e}{dt}, \quad (38)$$

which is the governing Eq. (15).

Similarly, by subtracting Eq. (26) from Eq. (28), we have,

$$P(x_a, t) - P(x_b, t) = Pc_b - Pc_a. \quad (39)$$

Substituting Eq. (8) in Eq. (39) gives us,

$$\frac{\mu(x_b - x_a)}{K_b} \left( V_b(t) + \alpha_{a,b} \frac{dq_a}{A} \right) = Pc_b - Pc_a. \quad (40)$$

By substituting Eq. (32) in Eqs. (9), (11) and (13) adding them, we obtain,

$$\phi \left( \frac{dx_b}{dt} + \frac{dx_c}{dt} + \frac{dx_d}{dt} \right) = \frac{v_a(t)}{K_a} (K_b + K_c + K_d) + \frac{dq_a}{A}. \quad (41)$$

From Eq. (41), the fluid transfer is determined as,

$$\frac{dq_a}{A} = \phi \left( \frac{dx_b}{dt} + \frac{dx_c}{dt} + \frac{dx_d}{dt} \right) - \frac{v_a(t)}{K_a} (K_b + K_c + K_d). \quad (42)$$

By substituting Eq. (42) in Eq. (40), we obtain the governing Eq. (16), which is,

$$Pc_b - Pc_a = \frac{\phi \mu(x_b - x_a)}{K_b + K_c + K_d} \left( \frac{dx_b}{dt} + \frac{dx_c}{dt} + \frac{dx_d}{dt} \right), \quad (43)$$

Now, subtracting Eq. (28) from Eq. (29), gives,

$$P(x_b, t) - P(x_c, t) = Pc_c - Pc_b. \quad (44)$$

Substituting Eq. (10) in Eq. (44), we obtain,

$$\frac{\mu(x_c - x_b)}{K_c} \left( V_c(t) + \alpha_{a,c} \frac{dq_a}{A} + \frac{dq_b}{A} \right) = Pc_c - Pc_b. \quad (45)$$

By substituting Eq. (11) in Eq. (45), we obtain,

$$Pc_c - Pc_b = \frac{\phi \mu(x_c - x_b)}{K_c} \left( \frac{dx_c}{dt} \right), \quad (46)$$

which is the governing Eq. (17).

Similarly, to obtain the governing Eq. (18), we subtract Eq. (28) from Eq. (30), which gives us,

$$P(x_b, t) - P(x_d, t) = Pc_d - Pc_b. \quad (47)$$

Substituting Eq. (12) in Eq. (47), we obtain,

$$\frac{\mu(x_d - x_b)}{K_d} \left( V_d(t) + \alpha_{a,c} \frac{dq_a}{A} + \frac{dq'_b}{A} \right) = Pc_d - Pc_b. \quad (48)$$

By substituting Eq. (13) in Eq. (48), we obtain the governing Eq. (18)

$$Pc_d - Pc_b = \frac{\phi \mu(x_d - x_b)}{K_d} \left( \frac{dx_d}{dt} \right). \quad (49)$$

The governing equations can be non-dimensionalized by defining dimensionless length as  $X_i = \frac{x_i}{L}$ , where  $L$  is the total length of the porous medium. Similarly, the permeability and the capillary pressure are made dimensionless by considering  $\kappa_i = \frac{K_i}{K_a}$  and  $\beta_i = \frac{Pc_i}{Pc_a}$ , respectively, where  $i = a, b, c, d, e$ .

Modifying Eq. (35) by the dimensionless parameters gives,

$$1 = \left( \frac{\phi \mu L^2}{Pc_a K_a} \right) \times \frac{X_a}{1 + \kappa_b + \kappa_c + \kappa_d + \kappa_e} \times \left( \frac{dX_a}{dt} + \frac{dX_b}{dt} + \frac{dX_c}{dt} + \frac{dX_d}{dt} + \frac{dX_e}{dt} \right). \quad (50)$$

The term  $\left( \frac{\phi \mu L^2}{Pc_a K_a} \right)$  on the right hand side has dimensions of time, i.e. s, which is used to non-dimensionalize time,  $T$ . Therefore, the dimensionless time is expressed as,

$$T = \frac{t}{\left( \frac{\phi \mu L^2}{Pc_a K_a} \right)}. \quad (51)$$

Similarly the governing Eqs. (38), (43), (46) and (49) are also made dimensionless. The equations governing the imbibition behavior in a five layered porous medium for the arrangement shown in Fig. 5 become,

$$1 = \frac{X_a}{1 + \kappa_b + \kappa_c + \kappa_d + \kappa_e} \times \left( \frac{dX_a}{dT} + \frac{dX_b}{dT} + \frac{dX_c}{dT} + \frac{dX_d}{dT} + \frac{dX_e}{dT} \right), \quad (52)$$

$$\beta_e - 1 = \frac{X_e - X_a}{\kappa_e} \left( \frac{dX_e}{dT} \right), \quad (53)$$

$$\beta_b - 1 = \frac{X_b - X_a}{\kappa_b + \kappa_c + \kappa_d} \left( \frac{dX_b}{dT} + \frac{dX_c}{dT} + \frac{dX_d}{dT} \right), \quad (54)$$

$$\beta_c - \beta_b = \frac{X_c - X_b}{\kappa_c} \left( \frac{dX_c}{dT} \right), \quad (55)$$

$$\beta_d - \beta_b = \frac{X_d - X_b}{\kappa_d} \left( \frac{dX_d}{dT} \right). \quad (56)$$

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