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POLYNOMIAL CHAOS BASED SOLUTION TO INVERSE PROBLEMS IN PETROLEUM RESERVOIR ENGINEERING

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ABSTRACT

In reservoir simulations, model parameters such as porosity and permeability are often uncertain and therefore better estimates of these parameters are obtained by matching the simulation predictions with the production history. Bayesian inference provides a convenient way of estimating parameters of a mathematical model, starting from a probable range of parameter values and knowing the production history. Bayesian inference techniques for history matching require computationally expensive Monte Carlo simulations, which limit their use in petroleum reservoir engineering. To overcome this limitation, we perform accelerated Bayesian inference based history matching by employing polynomial chaos (PC) expansions to represent random variables and stochastic processes. As a substitute to computationally expensive Monte Carlo simulations, we use a stochastic technique based on PC expansions for propagation of uncertainty from model parameters to model predictions. The PC expansions of the stochastic variables are obtained using relatively few deterministic simulations, which are then used to calculate the probability density of the model predictions. These results are used along with the measured data to obtain a better estimate (posterior distribution) of the model parameters using the Bayes rule. We demonstrate this method for history matching using an example case of SPE1CASE2 problem of SPEs Comparative Solution Projects. We estimate the porosity and permeability of the reservoir from limited and noisy production data.

INTRODUCTION

Petroleum resources are the primary sources of energy worldwide and their demand is expected to increase in the coming years [1]. The ever-increasing demand of petroleum necessitates efficient management of existing oil and gas fields and development of new fields. Production forecasts for development and management of oil reservoirs are done with the aid of reservoir simulations, which simulate flow of oil, water, and gas through porous media [2–4]. Reservoir simulation is an invaluable tool which helps in minimizing economic risks for capital investment in oil and gas reservoirs by predicting oil recovery. Therefore, a reservoir model should reliably predict the production from a petroleum reservoir [5].

Accurate production forecasts using reservoir simulations require precise knowledge of the geological properties of the reservoirs [6]. The subsurface of a petroleum reservoir is invariably complex and heterogeneous [7]. The geological properties and initial conditions of the reservoir, such as porosity, permeability, saturations, are obtained by analyzing rock core samples at well locations. While core analysis yields accurate petrophysical data, such data are available only at discrete well locations. Other techniques such as seismic imaging provide a remote view of the subsurface reservoir and do not yield exact geological properties of the reservoir rocks [8]. Based on limited geological data, geologists develop a reservoir model, which takes into account the most probable geological properties of the reservoir rocks [9, 10]. These geological properties are often determined by solving inverse problems using the history data. The model parameters are estimated to match the production history.

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The traditional industrial method to perform history matching is manual which requires experience [11]. The traditional method sometimes becomes infeasible for complex models where larger number of uncertain model parameters are involved. Nowadays, the process has been automated to a large extent where the problems are approached by two methodologies: (i) gradient-based methods which predict single values of model parameters such that the mismatch between predictions and observation data is minimized, and (ii) Bayesian inference which provides probability distribution of model parameters for which reservoir simulations are able to predict mean and standard deviation in observed data. The Bayesian inference based method requires minimization of an objective function to reduce the difference between history data and production forecasts.

Among various stochastic methods to obtain the production forecasts using uncertainty quantification in forward modeling, the Monte Carlo (MC) method is the most common and easiest to implement [12]. In the MC method, a large number of deterministic simulations are performed for different realizations of model parameters, sampled from their probability densities. Thereafter, the simulation outputs are analysed to obtain statistical information such as mean, variance, and probability density of production forecasts. While MC simulations yield accurate results, the need for large number of deterministic simulations for convergence precludes their application for realistic reservoir studies. In both these methods the set of realizations are sampled using Monte Carlo which require huge data set and prove to be computationally expensive.

The large computational cost of the MC method can be overcome using the experimental design (ED) method, a widely used technique in the petroleum industry [13, 14]. In the ED method, a response surface is fit to the production forecast from a smaller set of deterministic simulations and this response surface acts as a proxy for the reservoir model. Thereafter, MC simulations are performed using the response surface leading to significant computational efficiency. The drawback of the ED method is that it does not take into account the probability distribution of model parameters but gives equal weightage to all deterministic simulations, making it less reliable for uncertainty quantification [13]. Moreover, Lawal [15] has shown that use of ED methods for modeling uncertainty in petroleum industry can violate basic physics and do not preserve correlation between otherwise related variables.

The aforementioned limitations of MC and ED methods can be overcome using the polynomial chaos (PC) expansion method, in which a stochastic process is represented by a series of orthogonal polynomials in terms of random variables [16]. The coefficients of PC expansion can be obtained by projecting multiple solutions of the deterministic simulations onto the PC basis [13, 17]; this method is referred to as the probabilistic collocation method (PCM) [18]. The PC method exhibits fast convergence and requires significantly smaller number of determin-

istic evaluations, compared with MC methods to achieve statistically relevant results. In addition, the coefficients of PC expansions provide information about sensitivity of model predictions on model parameters [19]. Marzouk et al [20] have assessed the utility of PC method to solve inverse problem on a transient diffusion problem. Due to these advantages, the PC method has been applied for uncertainty quantification in a variety of fields including structural mechanics [21, 22], fluid flow [23], reacting flows [19, 24] and petroleum engineering [18, 25].

Previous studies on use of PC approach for reservoir studies have primarily focused on demonstrating the accuracy and computational efficiency of PC based techniques over MC and ED methods [13, 26, 27]. In contrast, the aim of the current work is to leverage the PC method to solve inverse problems in reservoir simulations and predict the values of model parameters. In this work, we elucidate the effects of permeability and porosity in petroleum reservoirs on production forecasts in a realistic petroleum reservoir simulation. As a model problem, we analyze the standard SPE1CASE2 problem of SPE's Comparative Solution Projects [28] by considering porosity and permeability as random parameters. Using the PC method, that we have validated with MC simulations, we show the effect of uncertainty in porosity and permeability on the production parameters and predict a better posterior value of porosity and permeability by solving an inverse problem using a synthetic history data. Our simulations also provide interesting insights into the individual and coupled contribution of the uncertain reservoir parameters on the overall uncertainty in predictions of reservoir simulations. The aim of the current work is also to leverage the computational efficiency of the PC method for history matching the reservoir model.

METHODOLOGY

Reservoir model

In the present work, we perform history matching on a petroleum reservoir model to predict the values of geological model properties such as porosity and permeability. As a model problem, we consider the standard SPE1CASE2 problem of SPE's Comparative Solution Projects [28] which is a three-dimensional black oil reservoir simulation problem. The geometry of the reservoir and the stratification and reservoir properties are illustrated in Figure 1. The reservoir is 10,000 feet in x and y directions and 100 feet deep in z -direction. The reservoir is discretized into 10×10 grid blocks with each grid block of 1000×1000 feet in x and y directions and 3 grid blocks in z -direction. The top, middle and bottom layers are 20, 30 and 50 feet deep respectively. The porosity of each layer is 0.3. The permeabilities of top, middle, and bottom layers are 500 mD, 50 mD, and 200 mD, respectively. There is one injection well located at grid block (1, 1, 1) and one production well at (10, 10, 3) as shown in Figure 1.

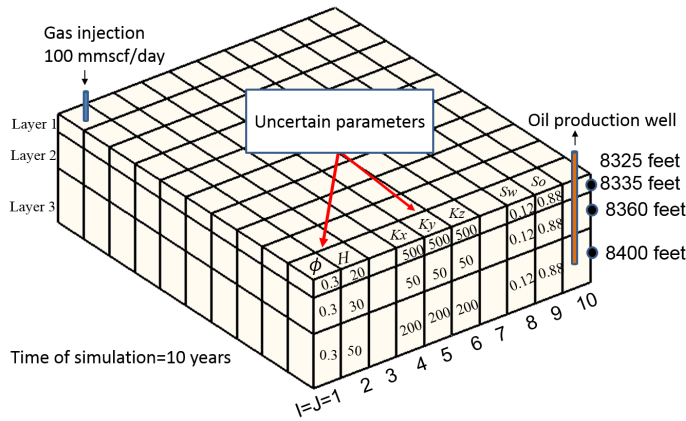


FIGURE 1. RESERVOIR GEOMETRY AND GRID SYSTEMS SHOWING THE STRATIFICATION AND RESERVOIR PROPERTIES. SHOWN HERE IS THE GEOMETRY OF STANDARD SPE1CASE2 PROBLEM WHICH CONTAINS PREDEFINED VALUES OF POROSITY AND PERMEABILITY AT EACH GRID BLOCK. IN THE ORIGINAL SPE1CASE2 PROBLEM, THE POROSITY AND PERMEABILITY OF EACH LAYER IS UNIFORM. IN THE CURRENT WORK WE HAVE REPLACED THE POROSITY OF THE FIELD AND PERMEABILITY OF THE TOP LAYER WITH UNCERTAIN VARIABLES.

Initially, the reservoir is undersaturated with gas. The initial oil and water saturation are 0.88 and 0.12, respectively. Gas is injected at the injection well at a constant injection rate of 100 mmscf/day. Oil is produced at the production well at a constraint of maximum production rate of 20,000 stb/day or a minimum flowing bottom hole pressure of 1000 psia. All reservoir boundaries, except the wells, are impermeable. The initial reservoir pressure is 4800 psia at a depth of 8400 feet. The simulations are run for a period of 10 years. The simulation data is saved at an interval of one month. The phase behaviour properties are taken from the standard problem [28].

Governing equations for flow

The SPE1CASE2 problem is based on the black oil simulation, which is a simplified three-phase two-component system. In the black oil model, three phases are modeled, namely, oil, gas and water. Gas miscibility in oil is taken as a function of pressure according to a specified phase behaviour. The governing equations of black oil model consist of the following mass conservation equations for gas, oil, and water phase [29],

$$\frac{\partial(\phi \rho_g S_g + \phi \rho_{go} S_o)}{\partial t} + \nabla \cdot (\rho_g v_g + \rho_{go} v_o) = q_g, \quad (1)$$

$$\frac{\partial(\phi \rho_o S_o)}{\partial t} + \nabla \cdot (\rho_o v_o) = q_o, \quad (2)$$

$$\frac{\partial(\phi \rho_w S_w)}{\partial t} + \nabla \cdot (\rho_w v_w) = q_w. \quad (3)$$

Here ρ represents the density, v is the velocity, q is the source or sink term which represents the well terms, ϕ is the porosity of the reservoir and S represents the saturation of different phases. In the above equations, $\nabla \cdot$ represents the divergence operator in three spatial dimensions (x, y, z) and t refers to time. In these equations, the subscripts g, o and w refer to the gas, oil and water phases respectively and go refers to the gas dissolved in oil. The saturations of gas, oil and water phases are related by,

$$S_g + S_o + S_w = 1. \quad (4)$$

The velocities in equations (1)-(3) are given by the Darcy's law,

$$v_i = -\frac{K k_{ri}}{\mu_i} (\nabla p_i - \rho_i g \nabla z), \quad (5)$$

where, $i = g, o, w$. Here, K is the absolute permeability of reservoir, k_{ri} is the relative permeability of the i -th phase, p is the pressure, g is the gravity term, z is the depth of reservoir and μ is the viscosity of the different phases. The capillary pressure is neglected in the standard problem, hence, $p_o = p_w$ and $p_g = p_o$.

Substituting equation (5) in equations (1), (2) and (3) for the different phases and solving with equations (4) along with the prescribed initial and boundary conditions, we obtain the fluid saturations and pressure at different points and time. For our simulations, we used 'Flow' simulator of Open Porous Media (OPM) [30] to solve the deterministic reservoir simulation equations. 'Flow' is an open source reservoir simulator which is a fully implicit, black oil simulator, capable of running industry standard simulation models.

Stochastic model formulation

In the current work, the geological properties such as porosity of the reservoir and the permeability of the top layer are considered uncertain random variables. To quantify the uncertainty in production fields due to uncertainty in these model parameters, we use a non-intrusive stochastic method based on PC expansions. The non-intrusive uncertainty quantification method using PC expansions along with history matching is summarized in Figure 2 and specific details are provided in subsequent subsections. First, we express the random variables of porosity and

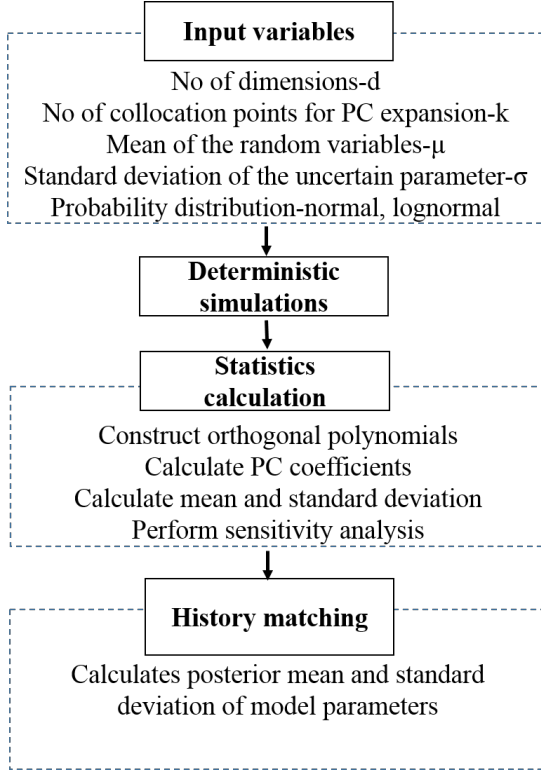


FIGURE 2. ILLUSTRATION OF THE PC METHODOLOGY. THE UNCERTAIN PARAMETERS ARE GIVEN AS INPUT AND RANDOM VARIABLES ARE GENERATED BASED ON THE PROBABILITY DISTRIBUTION OF INPUT PARAMETERS. DETERMINISTIC SIMULATIONS ARE PERFORMED ON THE RANDOM DATA AND STATISTICAL CALCULATIONS ARE PERFORMED ON THE OUTPUT DATA USING PC EXPANSION. LATER HISTORY MATCHING IS PERFORMED TO PREDICT THE POSTERIOR ESTIMATES OF THE MODEL PARAMETERS.

permeability in terms of an additional stochastic dimension ξ . Next, deterministic simulations based on the black oil model are performed for different realizations of the random model parameters. The results of these deterministic simulations are used to construct the PC expansions of model predictions such as oil production rate and gas production rate. Then, these PC expansions are used to calculate mean and standard deviation in model predictions. Using these PC expansions and with the help of synthetically generated history data, we predict the posterior values of the mean and standard deviation of the uncertain model parameters. We now explain each of these steps in more detail.

PC expansions. In the PC method, the simulation outputs have an additional dependence of stochastic dimensions $\xi = [\xi_1, \xi_2, \xi_3, \dots, \xi_d]$, corresponding to d uncertain variables. For normally distributed model parameters, $\xi \sim N(0, 1)$ is chosen as

a random variable with standard normal probability density function. Hence a log-normally distributed model parameter β can be described in terms of ξ as

$$\beta = \exp(\mu_\beta + \sigma_\beta \xi) \quad (6)$$

where μ_β is the mean value of parameter β and σ_β is the standard deviation. In the present problem, we have $d = 2$ for the two uncertain variables. The simulation outputs such as oil production rate and gas production rate, denoted by f_i are expressed using spectral PC expansions [17] as

$$f_i(x, y, z, t, \xi) = \sum_{k=0}^{\infty} f_{i,k}(x, y, z, t) \Psi_k(\xi), \quad (7)$$

where $f_{i,k}$ represents the coefficients of the i -th production parameters at that location and time and Ψ_k are the basis functions of the PC expansion.

For one uncertain parameter, the basis functions $\Psi_k = \psi_k$ are one-dimensional Hermite polynomials.

$$\psi_0 = 1, \psi_1 = \xi, \psi_2 = \xi^2 - 1, \psi_3 = \xi^3 - 3\xi, \psi_4 = \xi^4 - 6\xi^2 + 3. \quad (8)$$

For practical purpose, the infinite series in equation (7) is truncated to a specific order P which is the order of the highest polynomial used in the PC expansion. Therefore, for the case of one uncertain parameter, the total number of terms in PC expansion is $P + 1$. In general, for d dimensions and highest order of polynomial as p , the total number of terms in PC expansion in equation (7) is $P + 1$ given as

$$P + 1 = \frac{(p + d)!}{p!d!}. \quad (9)$$

The PC basis functions $\Psi_k(\xi)$ are d -dimensional Hermite polynomials which are the product of one dimensional Hermite polynomials involving multi-index m_k^i .

$$\Psi_k(\xi) = \prod_{i=1}^n \psi_{m_k^i}(\xi_i). \quad (10)$$

For example, in our problem for two uncertain parameters ($d = 2$), the first few two-dimensional Hermite polynomials are given

by

$$\begin{aligned}
\Psi_o(\xi) &= \psi_o(\xi_1) \psi_o(\xi_2) = 1, \\
\Psi_1(\xi) &= \psi_1(\xi_1) \psi_o(\xi_2) = \xi_1, \\
\Psi_2(\xi) &= \psi_o(\xi_1) \psi_1(\xi_2) = \xi_2, \\
\Psi_3(\xi) &= \psi_2(\xi_1) \psi_o(\xi_2) = \xi_1^2 - 1, \\
\Psi_4(\xi) &= \psi_1(\xi_1) \psi_1(\xi_2) = \xi_1 \xi_2, \\
\Psi_5(\xi) &= \psi_o(\xi_1) \psi_2(\xi_2) = \xi_2^2 - 1.
\end{aligned} \tag{11}$$

We compute the coefficients $f_{i,k}$ of the corresponding PC expansion to describe the effect of uncertain model parameters on the production forecasts. To this end, we truncate equation (7) to P terms and estimate the coefficients $f_{i,k}$ using least squares regression. In this method deterministic calculations are performed for N realizations of random model parameters corresponding to $\xi^{(j)}, j = 1, \dots, N$. The stochastic dimensions $\xi^{(j)}$ are chosen using Smolyak quadrature [31]. Thereafter the coefficients $f_{i,k}$ are determined from

$$f_i(x, y, z, t, \xi^{(j)}) = \sum_{k=0}^P f_{i,k}(x, y, z) \Psi_k(\xi^{(j)}), \tag{12}$$

where $j = 1, \dots, N$, to minimize the sum of residuals. Denoting the terms in equation (12) as,

$$f = \begin{bmatrix} f_i(\xi^{(1)}) \\ \vdots \\ f_i(\xi^{(N)}) \end{bmatrix}, \hat{f} = \begin{bmatrix} f_{io} \\ \vdots \\ f_{iP} \end{bmatrix} \text{ and } Z = \begin{bmatrix} \psi_o(\xi^{(1)}) & \dots & \psi_P(\xi^{(1)}) \\ \vdots & \ddots & \vdots \\ \psi_o(\xi^{(N)}) & \dots & \psi_P(\xi^{(N)}) \end{bmatrix}, \tag{13}$$

equation (12) can be written in compact matrix notation as

$$f = Z \hat{f}. \tag{14}$$

The coefficients are computed using

$$\hat{f} = (Z^T Z)^{-1} Z^T f. \tag{15}$$

Once we obtain the coefficients of the PC expansion, we compute the various statistical moments such as mean and standard deviation in the production forecasts. The mean is given by the zeroth-order coefficient $\hat{f}_{i,0}$ because the expectation $\langle \Psi_k \rangle = 0$ for $k > 0$ and the standard deviation is calculated using the higher order coefficients. The standard deviation of i -th production parameter is calculated by,

$$\sigma_i^2 = \langle (\hat{f}_i - \langle \hat{f}_i \rangle)^2 \rangle = \sum_{k=1}^P \hat{f}_{i,k}^2 \langle \Psi_k^2 \rangle. \tag{16}$$

We get the contribution of each uncertain model parameter to the overall uncertainty in production forecasts by grouping the terms in equation (16) which correspond to the same stochastic dimension.

History matching using Bayesian Inference

History matching is an inverse problem wherein model parameters of reservoir simulation are determined from the observed production data. Bayesian inference for history matching has an advantage over gradient based methods as they are able to satisfy geological constraints and predict true values of model parameters. Moreover, casting the history matching problem in a Bayesian setting enables quantitative assessment of uncertainties in model parameters. To illustrate the use of Bayesian inference for history matching we here consider a forward problem,

$$\mathbf{d} = G(\mathbf{Z}). \tag{17}$$

Here $\mathbf{d}$ denotes the predicted value of observable data such as well-pressures and flow rates corresponding to model parameters $\mathbf{Z}$ such as porosity and permeability. In Bayesian framework, the model parameters $\mathbf{Z}$ are random variables assumed to have a prior probability density $\pi(\mathbf{Z})$. Bayesian inference entails prediction of posterior probability density of model parameters $\pi(\mathbf{Z}|\mathbf{d}_{obs})$ taking into account the observed data $\mathbf{d}_{obs}$. The observed data $\mathbf{d}_{obs}$ differs from the predicted data $\mathbf{d}$ due to measurement uncertainties. The measurement uncertainties are assumed to be additive,

$$\mathbf{d}_{obs} = \mathbf{d} + \mathbf{e} = G(\mathbf{Z}) + \mathbf{e}, \tag{18}$$

where $\mathbf{e}$ denotes measurement errors. Therefore, the probability density of observed data given the model parameters is $\pi(\mathbf{d}_{obs}|\mathbf{Z}) = \pi_e(\mathbf{d}_{obs} - G(\mathbf{Z}))$, where π_e denotes probability density of measurement error. $\pi(\mathbf{d}_{obs}|\mathbf{Z})$ is called the likelihood function. The measurement errors for different observations can be assumed to be uncorrelated and hence the likelihood function can be taken as

$$\pi(\mathbf{d}_{obs}|\mathbf{Z}) = \prod_i p_i(\mathbf{d}_{obs,i} - \mathbf{d}_i). \tag{19}$$

Here, p_i denotes the probability density of i -th component of $\mathbf{e}$, and $\mathbf{d}_i$ and $\mathbf{d}_{obs,i}$ respectively the i -th component of predicted and observed data.

The inverse problem of history matching deals with calculation of the probability density of model parameters given observed data $\pi(\mathbf{Z}|\mathbf{d}_{obs})$. To this end, we employ the Bayes' rule

$$\pi(\mathbf{Z}|\mathbf{d}_{obs}) = \frac{\pi(\mathbf{d}_{obs}|\mathbf{Z})\pi(\mathbf{Z})}{\int \pi(\mathbf{Z}|\mathbf{d}_{obs})\pi(\mathbf{Z})d\mathbf{Z}}. \tag{20}$$

We use the Metropolis-Hastings algorithm [32], which is a Markov chain Monte Carlo (MCMC) method, to obtain samples from the posterior density $\pi(\mathbf{Z}|\mathbf{d}_{obs})$. The MCMC method involves large number of computations of the forward model Eq. (17), which makes this method computationally expensive. To accelerate the MCMC method, we first obtain the PC expansion of $G(\mathbf{Z})$ for a prior distribution of model parameters $\mathbf{Z}$, which in this work is the distribution of porosity and permeability. Thereafter, the PC expansion of $G(\mathbf{Z})$ is used in MCMC method, instead of solving the exact forward problem while sampling from the posterior distribution. The use of PC expansion of the forward model enables sampling from the posterior distribution in significantly less time.

RESULTS AND DISCUSSION

Before we discuss quantification of uncertainty in SPE1CASE2 problem due to uncertainty in porosity and permeability, we present a brief discussion of the physics of the problem. We performed simulations using OPM Flow simulator [30] and verified our results with the results of Eclipse simulator. As gas is injected at a constant rate of 100 mmscf/day in the injection well located in the top layer, it displaces the oil horizontally towards the production well. Oil is produced at a maximum oil production rate of 20,000 stb/day for a period of 52 months. Thereafter, the pressure inside the reservoir reaches the minimum value and oil production rate decreases continuously. Figure 3(a) depicts the deterministic oil production rate for a time period of 10 years where $\phi = 0.3$ and $k = 500$ mD. When the gas reaches the other end of the top layer, due to continuous injection it moves towards the bottom layer and reaches the production well at about $t = 43$ months. Figure 3(c) shows the gas production rate for the deterministic simulation. Along with the production of oil, concurrently, gas is also produced from the production well. Initially the gas which was dissolved in oil in the reservoir is produced but once the injected gas reaches the production well, the gas production rate increases suddenly as shown in Figure 3(c) at $t = 43$ months.

Validation of PC method with Monte Carlo simulations

After getting an insight into the physics of the reservoir model, we now present quantification of uncertainty in production parameters due to uncertainty in porosity and permeability. To start with forward modeling, first we retain $\pi(\mathbf{d}|\mathbf{Z})$. For that we solve the forward problem with prior distribution of $\pi(\mathbf{Z})$. Here, $\mathbf{Z}$ represents the geological properties like porosity and permeability which are random variables. The mean values of porosity of each layer is 0.3 and permeabilities of the top, middle and bottom layers are 500 mD, 50 mD and 200 mD respectively as mentioned in Figure 1. We assumed a 20% standard deviation around the mean values of porosity in the reservoir and

permeability of the top layer. First we performed Monte Carlo simulations taking porosity of the reservoir and permeability of the top layer as random variables. Next, we use PC method to quantify the uncertainty, as discussed in the next section.

To validate the method for uncertainty quantification using PC expansion method, we compared the mean and standard deviation in production parameters using MC simulations with the mean and standard deviation in production parameters using PC based method. Computation of the coefficients for a fourth-order PC expansion, for two uncertain variables required 289 simulations using Smolyak [31] quadrature based stochastic collocation. The results obtained from fourth-order PC expansion for 289 deterministic simulations match well with 10,000 MC simulations. For all our computations, we have represented the production forecasts with fourth-order PC expansions. The fourth-order PC expansions were sufficient in describing the uncertainty as the spectral mode strengths of higher order is significantly smaller than lower order mode strengths. The results show that the computational cost of PC method is much less than MC method.

Now, we will discuss the effects of introducing uncertainty on the production parameters. Figure 3(a) and 3(b) show the expected values of oil production rate and the standard deviation associated with them as a function of time. Figure 3(c) and 3(d) show the expected values of gas production rate and the standard deviation associated with them as a function of time. We see in Figure 3(a) and 3(c) that the mean values of production show more dispersion in contrast to the sudden change in rates in the deterministic simulation. Due to the effect of uncertainty, the discontinuities disappear in the mean production profiles. This is in agreement with the results of Zhang and Tchelepi [33] where they have solved for 1D and 2D Buckley-Leverett problem. The dispersive behaviour in a model reservoir problem can be explained in a similar way that the mean profiles are average of many shock waves.

The peak standard deviation in oil production rate is obtained when there is a sudden drop in the maximum oil production rate as shown in Figure 3(b). The peak standard deviation for gas production rate is obtained when the breakthrough of gas takes place from the production well as shown in Figure 3(d). The mean profiles of the production parameters and the associated standard deviations can be used to construct confidence intervals for them and help in economic forecasts. It is clear from these observations that oil recovery will take longer and gas breakthrough may occur earlier in reservoirs when uncertainty is taken into account.

Forward sensitivity analysis: Contribution of individual uncertainties and coupled contribution

In Figure 3, we have shown the predicted temporal evolution of overall uncertainty in production profile. However, the

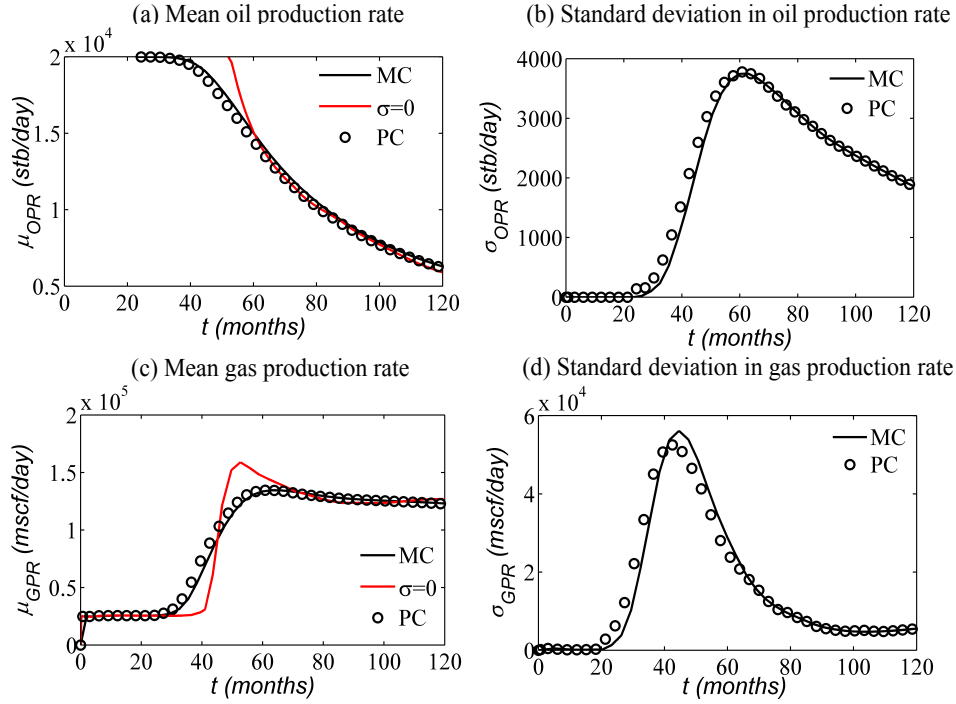


FIGURE 3. COMPARISON OF 10,000 MC SIMULATIONS WITH 289 PC SIMULATIONS AND THEIR DEVIATION FROM THE DETERMINISTIC SIMULATIONS. MEAN AND STANDARD DEVIATION OF OIL PRODUCTION RATE AND GAS PRODUCTION RATE ARE SHOWN HERE. THE MEAN AND STANDARD DEVIATIONS CONVERGE WELL FOR FOURTH-ORDER PC SIMULATIONS WITH 289 COLLOCATION POINTS FOR TWO UNCERTAIN PARAMETERS. (A) MEAN OIL PRODUCTION RATE VS TIME. (B) STANDARD DEVIATION IN OIL PRODUCTION RATE VS TIME. (C) MEAN GAS PRODUCTION RATE VS TIME (D) STANDARD DEVIATION IN GAS PRODUCTION RATE VS TIME.

PC methodology also provides the contribution of individual uncertain parameters or coupled contributions to the overall uncertainty. Figure 4(a) shows the temporal variation of contribution of uncertainty in porosity and permeability to the overall uncertainty in oil production rate. In the present problem, the porosity of the reservoir is the major contributor to the overall uncertainty at all times. The void space which are determined by the porosity could be filled with water, oil or gas. Hence, the production potential directly depends on the porosity. Therefore, porosity is a crucial parameter while exploring a reservoir. The uncertainty attains a maximum value where the oil production rate drops at $t = 52$ months from the maximum set constraint. Thereafter, the permeability contributes to the overall uncertainty. The contribution of uncertainty in the gas production rate follows a similar trend as shown in Figure 4(b).

Figure 4 also shows the contribution of coupled effect of porosity and permeability to the overall uncertainty in oil production rate and gas production rate. Although the coupled contributions are small when compared with the individual contributions of the model parameters, the results shown in Figure 4 show that stochastically independent uncertain parameters can interact

and affect the uncertainty in model predictions. The sensitivity analysis of individual contributions to overall uncertainty helps in deciding the properties that need to be measured with more accuracy. Such simulations also help in finding more sensitive parameters for history matching. Hence, it is important to note that MC and ED methods do not enable such individual and coupled sensitivity analysis.

History matching

Once the PC expansion is known for the model predictions in forward modeling, we can solve inverse problems to predict the model parameters. History matching helps to find a better estimate of the model parameters and thus helps to reduce the predictive uncertainty in the reservoir model. We use the Bayesian inference along with PCE to perform history matching. Our results from forward modeling show that gas production rate has a higher peak value of standard deviation than oil production rate when divided with their respective mean values. This means that gas production rate is more sensitive to the variation in model parameters. Hence we have taken gas production rate as the measured data to predict the posterior values of porosity and per-

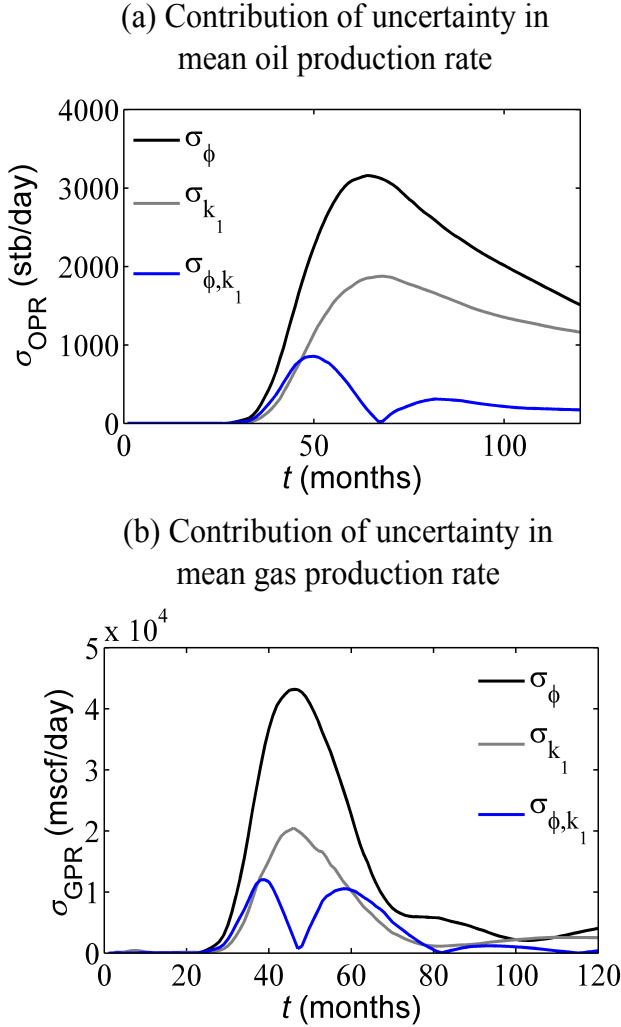


FIGURE 4. CONTRIBUTION OF INDIVIDUAL AND COUPLED UNCERTAINTIES OF THE MODEL PARAMETERS ON THE MODEL PREDICTIONS. THE EFFECT OF POROSITY AND PERMEABILITY IS SHOWN HERE, WHERE, ϕ IS THE POROSITY, k_1 IS THE PERMEABILITY OF THE TOP LAYER. (A) INDIVIDUAL AND COUPLED EFFECTS OF UNCERTAINTY IN POROSITY AND PERMEABILITY ON THE UNCERTAINTY IN OIL PRODUCTION RATE (B) INDIVIDUAL AND COUPLED EFFECTS OF UNCERTAINTY IN POROSITY AND PERMEABILITY ON THE UNCERTAINTY IN GAS PRODUCTION RATE.

meability. To get the synthetic observed data to perform history matching, we add white Gaussian random noise with $\sigma = 9\%$ to the measured data of gas production rate at all time steps. In petroleum industry measured data is not available at all the time steps. So to limit the data, we have taken the measured data at time intervals of five months. We took the prior probability distribution of porosity and permeability to be log-normally dis-

TABLE 1. THE PARAMETERS USED FOR HISTORY MATCHING.

Parameters	Layer	True value	Prior	Posterior
Porosity	1-3	0.27	$0.3(\pm 0.015)$	$0.26(\pm 0.01)$
Permeability	1	450 mD	$500(\pm 50)$	$436.28(\pm 21.08)$

tributed with mean value of 0.3 and 500 mD respectively with standard deviation of 20% around the mean values. Based on this prior probability distribution, we first obtained the PC expansion for the forward problem. Thereafter, we used the Metropolis-Hastings algorithm to obtain 10,000 samples from the posterior distribution. The use of PC expansion for forward problem significantly accelerated the sampling from posterior distribution. Finally, we calculated the mean and standard deviation from the posterior distribution.

The model parameter values used in the synthetic reference case and the results obtained after history matching are listed in Table 1. Figure 5(a) shows the prior distribution of porosity and permeability, and Figure 5(b) shows samples obtained from the posterior distribution. We observe that the posterior distribution is significantly narrow compared with the prior distribution and is centered around the true values of porosity and permeability. The posterior distribution also gives us information about how the porosity and permeability must be tweaked so as to get the history matched data. For a constant porosity, when the permeability is increased breakthrough of gas occurs earlier hence more gas is produced and production of oil drops. However, for a constant permeability if porosity is increased, it means more oil is initially present in the reservoir, hence maximum oil production takes place from the reservoir and breakthrough of the gas is delayed. To produce the same amount of gas, earlier breakthrough must take place for which the porosity of the reservoir must also decrease.

CONCLUSION

The overall basis of the current work has been to solve inverse problem to predict the value of porosity and permeability in a reservoir model using Bayesian inference. These studies would help in better estimate of the model parameters and improve the reservoir model. These studies also help in improving production forecast from a reservoir and minimize the risks. PC method is fast for inverse problems as compared to MC method. It also gives useful information about sensitivity of the model parameters by providing contribution of different geological uncertain parameters to overall uncertainty in production forecasts. This sensitivity analysis helps in taking precautions while measuring

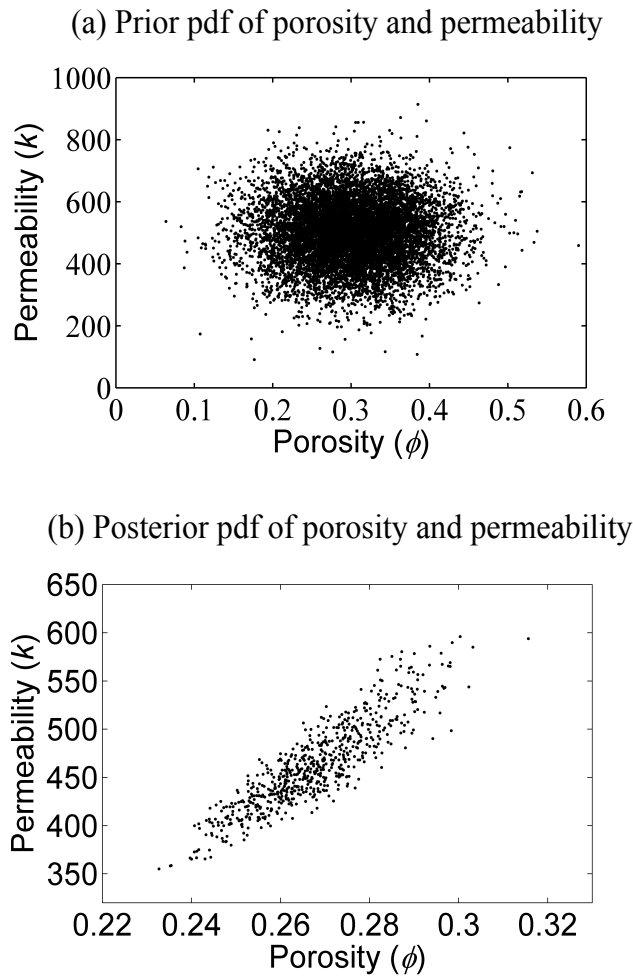


FIGURE 5. SAMPLES OBTAINED FROM THE POSTERIOR PROBABILITY DENSITIES OF POROSITY AND PERMEABILITY. AFTER HISTORY MATCHING THE MODEL PARAMETER REALIZATION VALUES CONVERGE TO THE TRUE VALUES MUCH BETTER.

the geological properties which are more sensitive to production parameters.

We present the results where we first validate our methodology for uncertainty quantification using PC expansion by comparing them with Monte Carlo simulations. Here, we show the mean values of time varying production forecasts like oil production rate, gas production rate and their standard deviation. Next we studied the effects of individual and coupled contributions of the uncertain parameters to overall uncertainty of production forecasts. At last, with the help of predictions from forward modeling, we solved the inverse problem to predict a better estimate of the model parameters.

The PC method shows faster convergence as compared with MC method and require 1000 times less number of deterministic simulations. The most significant new finding of the current work is that the uncertainty in porosity and permeability leads to early drop in oil production rate and early breakthrough of injected gas from the production well. This can be attributed to the dispersion effect induced by uncertainty. Moreover, among the two uncertain parameters, the uncertainty in porosity contributes more to the overall uncertainty in the simulation forecast. Also, using history matching the accuracy of the prediction model can be greatly improved and the uncertainty in predicted parameters can be reduced. Moreover, history matching using Bayesian inference not only gives the better estimate of predicted model but also the most probable range in which the model parameters lie.

REFERENCES

- [1] Holditch, S. A., and Chianelli, R. R., 2008. "Factors that will influence oil and gas supply and demand in the 21st century". *MRS bulletin*, **33**(4), pp. 317–323.
- [2] Fanchi, J. R., 2005. *Principles of applied reservoir simulation*. Elsevier.
- [3] Aziz, K., and Settari, A., 1979. *Petroleum reservoir simulation*. Chapman & Hall.
- [4] Smith, R. G., and Maitland, G. C., 1998. "The road ahead to real-time oil and gas reservoir management". *Chemical Engineering Research and Design*, **76**(5), pp. 539–552.
- [5] Costa, L. A. N., Maschio, C., and Schiozer, D. J., 2014. "Application of artificial neural networks in a history matching process". *Journal of Petroleum Science and Engineering*, **123**, pp. 30–45.
- [6] Zhang, D., Li, L., Tchalepi, H., et al., 2000. "Stochastic formulation for uncertainty analysis of two-phase flow in heterogeneous reservoirs". *Spe Journal*, **5**(01), pp. 60–70.
- [7] Haldorsen, H., Brand, P., and Macdonald, C., 1987. "Review of the stochastic nature of reservoirs". In *Mathematics in oil production*.
- [8] Heidemann, J., Ye, W., Wills, J., Syed, A., and Li, Y., 2006. "Research challenges and applications for underwater sensor networking". In *Wireless Communications and Networking Conference, 2006. WCNC 2006. IEEE, Vol. 1*, IEEE, pp. 228–235.
- [9] Pyrcz, M. J., and Deutsch, C. V., 2014. *Geostatistical reservoir modeling*. Oxford university press.
- [10] Dubrule, O., 2003. *Geostatistics for seismic data integration in earth models*. Society of Exploration Geophysicists.
- [11] Gu, Y., Oliver, D. S., et al., 2005. "History matching of the punq-s3 reservoir model using the ensemble kalman filter". *SPE journal*, **10**(02), pp. 217–224.
- [12] Zhang, D., 2002. "Stochastic methods for flow in porous media: coping with uncertainties academic press san diego". *CA Google Scholar*.

- [13] Heng, L., Sarma, P., and Dongxiao, Z., 2011. "A comparative study of the probabilistic-collocation and experimental-design methods for petroleum-reservoir uncertainty quantification". *SPE Journal*, **16**(2), pp. 429–439.
- [14] Van Elk, J. F., Guerrero, L., et al., 2000. "Improved uncertainty management in field development studies through the application of the experimental design method to the multiple realisations approach". In SPE Asia Pacific Oil and Gas Conference and Exhibition, Society of Petroleum Engineers.
- [15] Lawal, K. A., et al., 2009. "Modelling subsurface uncertainties with experimental design: Some arguments of non-conformists". In Nigeria Annual International Conference and Exhibition, Society of Petroleum Engineers.
- [16] Wiener, N., 1938. "The homogeneous chaos". *American Journal of Mathematics*, **60**(4), pp. 897–936.
- [17] Le Maître, O., and Knio, O. M., 2010. *Spectral methods for uncertainty quantification: with applications to computational fluid dynamics*. Springer Science & Business Media.
- [18] Li, H., Zhang, D., et al., 2009. "Efficient and accurate quantification of uncertainty for multiphase flow with the probabilistic collocation method". *Spe Journal*, **14**(04), pp. 665–679.
- [19] Jha, A. K., and Bahga, S. S., 2016. "Uncertainty quantification in modeling of microfluidic t-sensor based diffusion immunoassay". *Biomicrofluidics*, **10**(1), p. 014105.
- [20] Marzouk, Y. M., Najm, H. N., and Rahn, L. A., 2007. "Stochastic spectral methods for efficient bayesian solution of inverse problems". *Journal of Computational Physics*, **224**(2), pp. 560–586.
- [21] Ghanem, R. G., and Spanos, P. D., 1991. "Stochastic finite element method: Response statistics". In *Stochastic finite elements: a spectral approach*. Springer, pp. 101–119.
- [22] Ghanem, R. G., and Spanos, P. D., 1991. "Spectral stochastic finite-element formulation for reliability analysis". *Journal of Engineering Mechanics*, **117**(10), pp. 2351–2372.
- [23] Le Maître, O. P., Reagan, M. T., Najm, H. N., Ghanem, R. G., and Knio, O. M., 2002. "A stochastic projection method for fluid flow: Ii. random process". *Journal of computational Physics*, **181**(1), pp. 9–44.
- [24] Reagan, M. T., Najm, H. N., Ghanem, R. G., and Knio, O. M., 2003. "Uncertainty quantification in reacting-flow simulations through non-intrusive spectral projection". *Combustion and Flame*, **132**(3), pp. 545–555.
- [25] Gao, G., Zafari, M., Reynolds, A. C., et al., 2005. "Quantifying uncertainty for the punq-s3 problem in a bayesian setting with rml and enkf". In SPE reservoir simulation symposium, Society of Petroleum Engineers.
- [26] Li, H., and Zhang, D., 2007. "Probabilistic collocation method for flow in porous media: Comparisons with other stochastic methods". *Water Resources Research*, **43**(9).
- [27] Sarma, P., Durlofsky, L. J., and Aziz, K., 2005. "Efficient closed-loop production optimization under uncertainty (spe94241)". In 67th EAGE Conference & Exhibition.
- [28] Odeh, A. S., et al., 1981. "Comparison of solutions to a three-dimensional black-oil reservoir simulation problem (includes associated paper 9741)". *Journal of Petroleum Technology*, **33**(01), pp. 13–25.
- [29] Peaceman, D. W., 2000. *Fundamentals of numerical reservoir simulation*, Vol. 6. Elsevier.
- [30] , 2015. Open Porous Media Initiative.
- [31] Smolyak, S. A., 1963. "Quadrature and interpolation formulas for tensor products of certain classes of functions". In *Doklady Akademii Nauk*, Vol. 148, Russian Academy of Sciences, pp. 1042–1045.
- [32] Chib, S., and Greenberg, E., 1995. "Understanding the metropolis-hastings algorithm". *The american statistician*, **49**(4), pp. 327–335.
- [33] Dongxiao, Z., Tchelepi, H., et al., 1999. "Stochastic analysis of immiscible two-phase flow in heterogeneous media". *SPE journal*, **4**(04), pp. 380–388.