

COL106: Data Structures and Algorithms

Lecture 27

Keerti Choudhary
Department of Computer Science, IIT Delhi

Today's Lecture

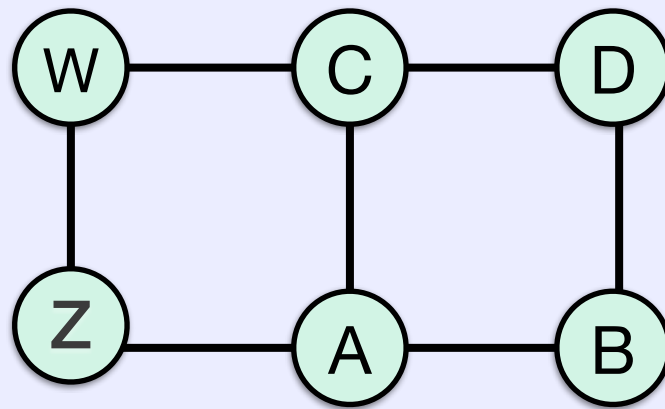
1. Application of BFS:

Computing smallest cycle in an undirected graph

2. Quiz on 2-3-4 Trees and Heaps

Application of BFS: Finding Smallest cycle

Goal: Given an undirected graph $G = (V, E)$, find a cycle of smallest length in G .



Smallest cycle in G
has length 4.

We will show that using BFS tree the smallest length cycle can be computed in

$O(|V| \cdot |E|)$ time

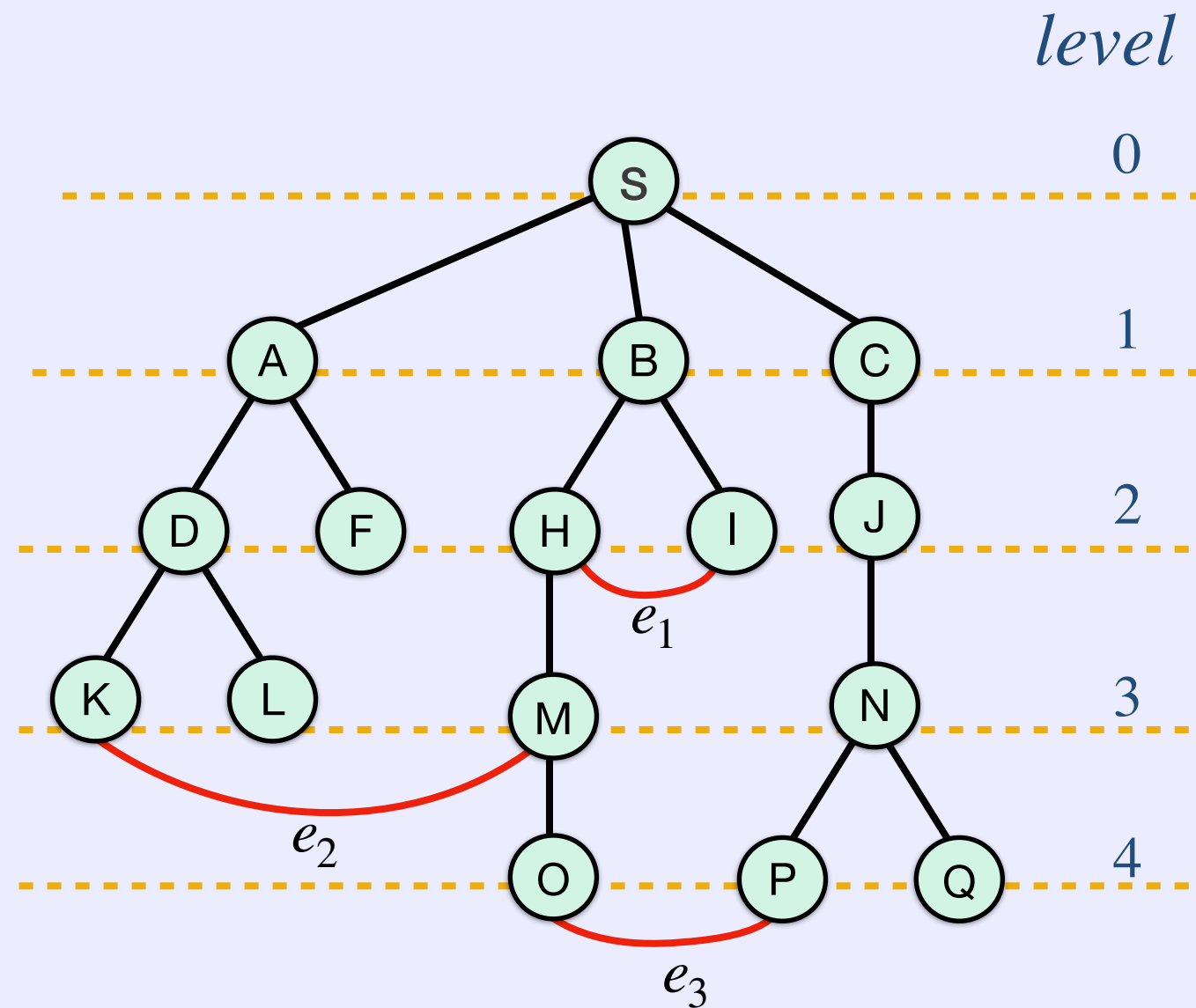
Smallest Cycle containing s

Question:

Let R be the set of all non-tree edges $e = (x, y)$ for which $\text{LCA}(x, y) = s$.

How can you find set R ?

Why are we studying set R ?



BFS tree, T
along with non-tree edges.

$$R = \{e_2, e_3\}$$

Smallest Cycle containing s

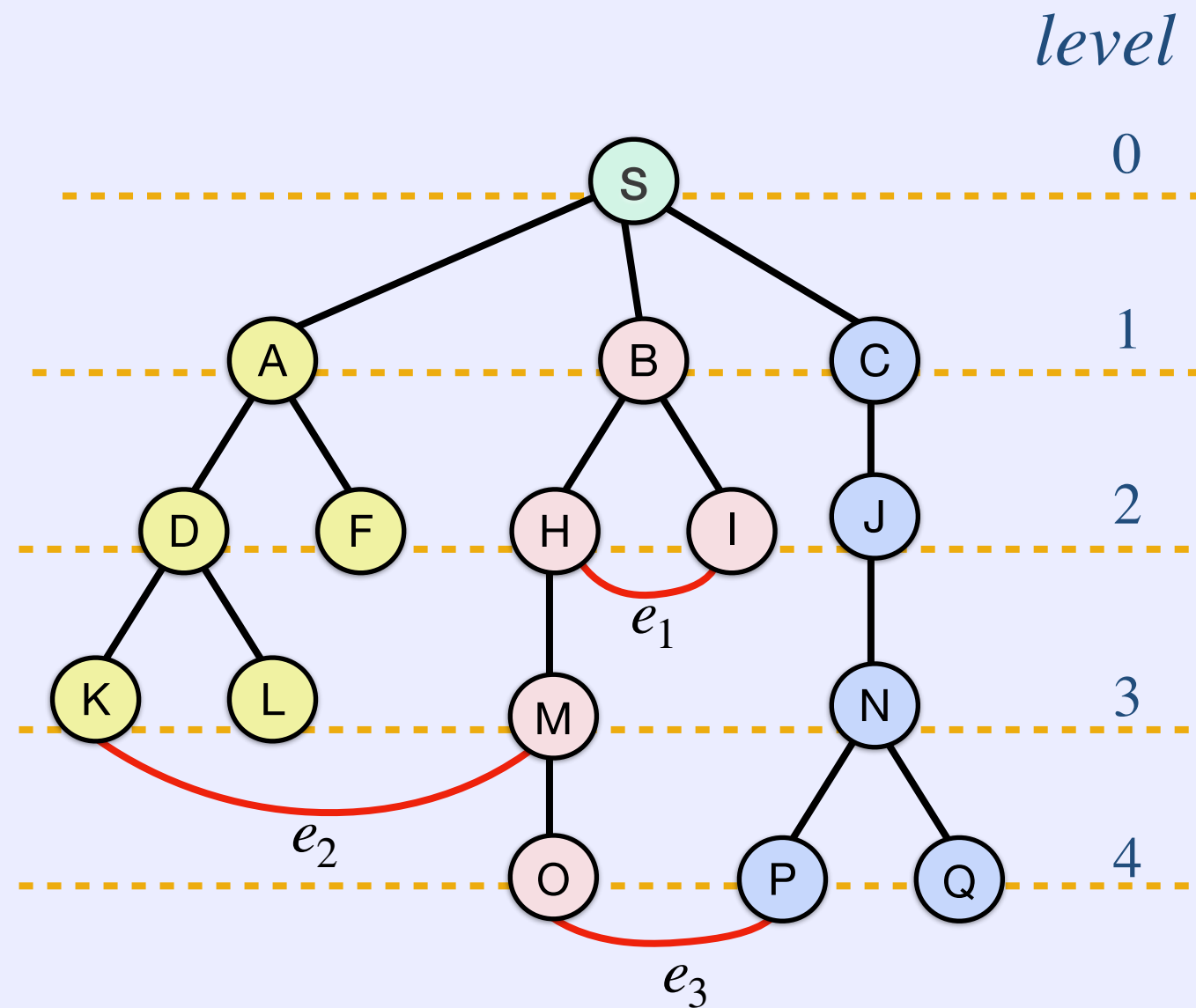
Steps to find R :

1. For each vertex x in $T - \{s\}$,
set **Label**(x) = Highest ancestor
of x in $T - \{s\}$.

Time to compute labels is $O(m + n)$.

2. For each non-tree edge (x, y) , add
 (x, y) to R iff $\text{Label}(x) \neq \text{Label}(y)$

Time to scan non-tree edges is
again $O(m + n)$.



BFS tree, T
along with non-tree edges.

$$R = \{e_2, e_3\}$$

Smallest Cycle containing s

Claim:

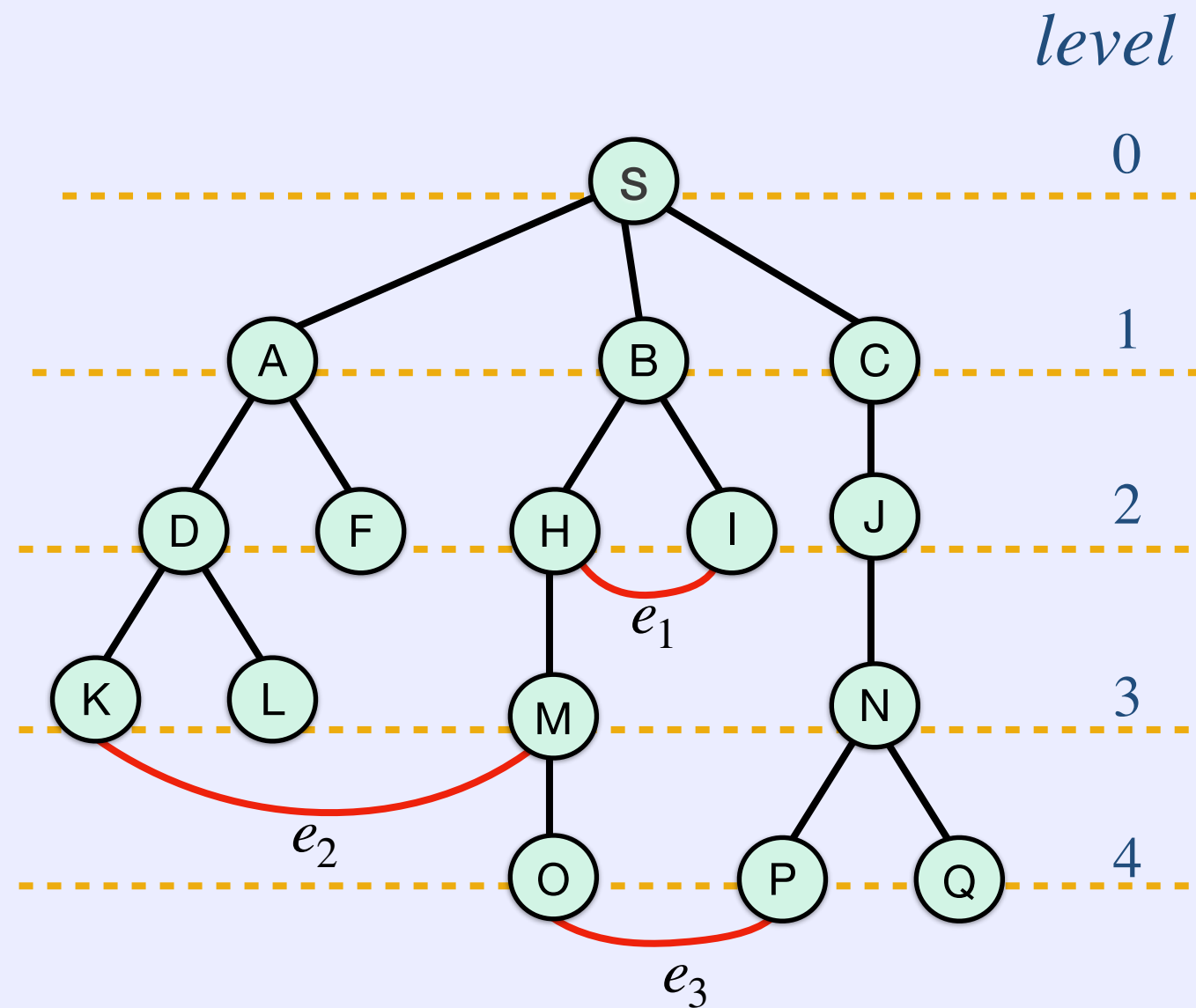
Let $e = (x, y)$ be non-tree edge in set R for which the following is minimum:

$$\text{"Level}(x) + \text{Level}(y)\text{"}$$

Then,

$C = \text{treepath}(s, x) \circ (x, y) \circ \text{treepath}(y, s)$
is a shortest cycle containing s .

Proof: Homework



BFS tree, T
along with non-tree edges.

$$R = \{e_2, e_3\}$$

Homework Question

Prove the claim in the previous slide using the following hint.

Let $e = (x, y)$ be non-tree edge in set R for which $\text{Level}(x) + \text{Level}(y)$ is minimum.
Then, each cycle in G containing s must have length at least

$$1 + \text{Level}(x) + \text{Level}(y)$$