Zero Sum Games — How to find sol^n to them using L.P.

Eg: Game — Prisoner's Dilemma.

Police $\rightarrow$ A, B

(who has comitted the crime)

A \ B	Silent	Other person has comitted crime
Silent	(-1, -1)	(-3, 0)
Other person comitted crime	(0, -3)	(-2, -2)

Equilibrium Stretgy.

Ques: What will be best action for (A, B) : (silent, silent)

Penalty = (-1, -1) ← Not an Equilibrium

- If A betrays = { A - other pers ---, B - silent)

  Penalty = (0, -3)

- If B betrays = { , }

  Penalty = (-3, 0)

✲ Equilibrium (Nash Equilibrium) = (other per, other pers)

(-2, -2)

Defⁿ Equilibrium A strategy pair (x, y) for players (A, B) is Nash Equilibrium if

(i) for $x' \neq x$ gain of A at (x', y) $\ngtr$ gain of A at (x, y)

(A is not better-off by choosing strategy x' instead of x)

(ii) for $y' \neq y$ gain of B at (x, y') $\ngtr$ gain of B at (x, y)

Eg. Rock-Paper-Scissor game

$R_o > S_c$

$S_c > P_a$

$P_a > R_o$

A \ B	R_o	P_a	S_c
R_o	0	-1	1
P_a	1	0	-1
S_c	-1	1	0

What A wins

A \ B	R_o	P_a	S_c
R_o	0	1	-1
P_a	-1	0	1
S_c	1	-1	0

What B wins

- Each row/column has $(+1, -1, 0)$
- (whatever A wins) + (whatever B wins) $= 0$

CLAIM: There is no (Pure / Det) Nash Eq in above game.

Proof: Try all possiblities

An example of randomized strategy (mixed strategy).

A	Ro	Pa	Sc
w.p.	.2	.3	.5

B	Ro	Pa	Sc
w.p.	1/2	1/2	0

		A \ B	Ro (1/2)	Pa (1/2)	Sc
0.2	—	Ro	0	−1	1
0.3	—	Pa	1	0	−1
0.5	—	Sc	−1	1	0

What A wins

Expected Payoff of A

$$\frac{0.2}{2}(0) + \frac{(0.3)}{2}(1) + \ldots$$

Zero Sum Games : Games in which for any choice of strategies for players (A, B)

$$(\text{Pay off of } A) + (\text{Pay-off of } B) = 0$$

To express them, only one matrix suffices

$(P) = (p_{ij})$ — pay off of A

$-P$ — pay off of B

Mixed strategy of A : $x = (x_1 \ldots x_m)$ with $\sum_{i=1}^{m} x_i = 1$, $x \geq 0$

Mixed strategy of B : $y = (y_1 \ldots y_n)$ with $\sum_{j=1}^{n} y_j = 1$, $y \geq 0$

Expected Pay-off of A :

$$= \sum_{i,j} p_{ij} \cdot \text{Prob}[\text{Alice play } i] \cdot \text{Prob}[\text{Bob plays } j]$$

$$= \sum_{ij} p_{ij} \cdot x_i \cdot y_j = x^T P y$$

Expected Pay-off of B :

$$= \sum_{i,j} (-p_{ij}) \cdot \text{Prob}[\text{Alice play } i] \cdot \text{Prob}[\text{Bob plays } j]$$

$$= \sum_{ij} (-p_{ij}) \cdot x_i \cdot y_j = (-x^T P y)$$

Qus : How to define Equilibrium when we have mixed/randomized strategies.

Def^n A mixed strategy pair $(\tilde{x}, \tilde{y})$ for players (A, B) is Nash Equilibrium if

(i) for $x' \neq \tilde{x}$ Exp Paoff of A at $(x', \tilde{y})$ $\ngtr$ Exp Payof of A at $(\tilde{x}, \tilde{y})$

(ii) for $y' \neq \tilde{y}$ Exp Payof of B at $(\tilde{x}, y')$ $\ngtr$ Exp Payof. of B at $(\tilde{x}, \tilde{y})$

H.W. 1 Suppose A has declared its mixed strategy

$x = (x_1 \cdots x_m)$ $\left(\sum x_i = 1, \quad x \geq 0\right)$.

Then, how can we find best mixed stetgy for y.

Hint: Very simple L.P.

x is known / declared

— Find best "y" for B

H.W. 2 Compute Dual of LP you get in H.W. 1