Online Algorithms. – data is coming in online fashion.

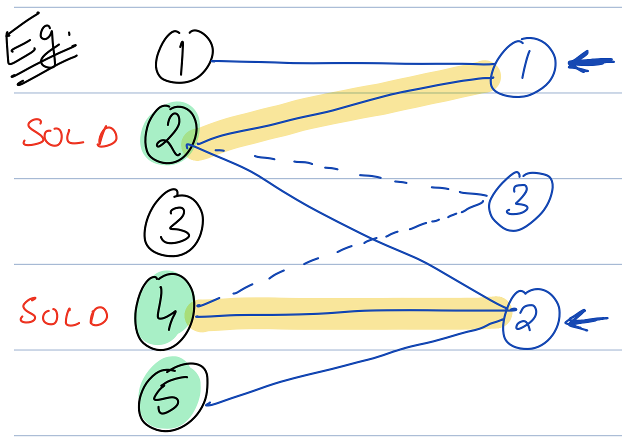
performance of our "online" algo $\underline{\text{at par}}$ with performance of best algo in <u>offline</u> setting

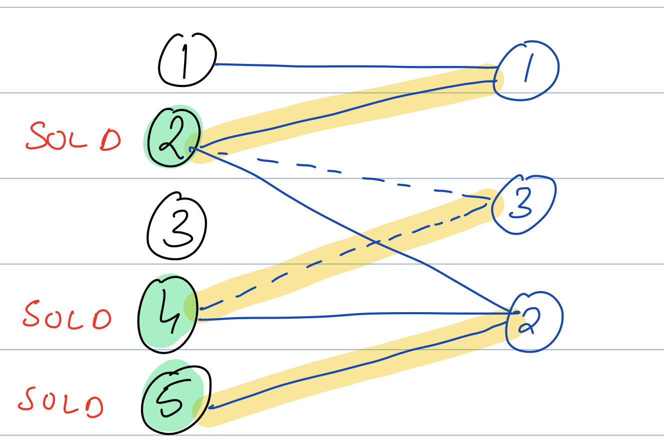
↳ access to all data

(*) On-line Matching – Maximize the size of our matching when buyers are coming in online manner.

n goods	n buyers (ONLINE)	n goods	n buyers (OFFLINE)

Eg:





$|\text{matching}| = 2$ $\qquad$ $|\text{matching}| = 3$

$\sigma = (\ 1:(1,2),\ 2:(2,4,5),\ 3:(2,4))$

Offline – How to compute maximum matching in poly-time.

"Det-Online-Algo" (G, σ)

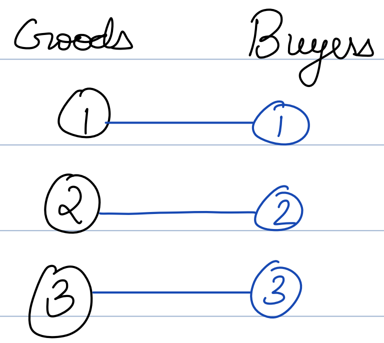
① $M = \phi$

② For $i = 1$ to $|\sigma|$:

Match Buyer i to ~~any~~ FIRST of available goods he/she wants to buy, update M

③ Return M

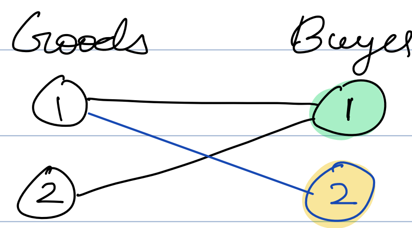
Competitive Ratio of Det-Online-Algo : $\min_{G,\sigma} \left(\frac{|\text{det-online-Match}(G,\sigma)|}{|\text{best-offline-match}(G,\sigma)|} \right)$



|online-matching obtained by det algo.| = 3

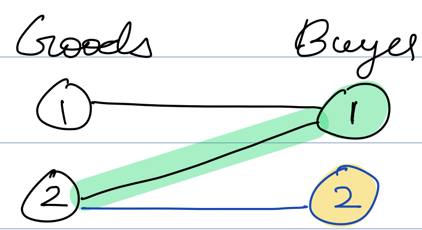
|best offline match| = 3

Case (i)



$\sigma_1 = (1:(1,2),\ 2:(1))$

Case (ii)



$\sigma_2 = (1:(1,2),\ 2:(2))$

If we take any algo "A" for online matching then $\sigma = (1:(1,2),\ \_\_)$

(i) → either A will match buyer 1 to good 1 } for σ_1 |online-match| = 1

(ii) → or A will match buyer 1 to good 2 } for σ_2 |online-matching| = 1

Proof

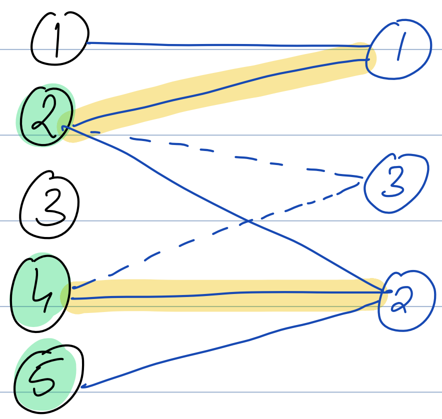
CLAIM 1 For any det algo A, $Cr(A) \le \frac{1}{2}$

CLAIM 2 : $Cr(\text{det-online-algo}) \geq 1/2$

$Cr = \frac{1}{2}$

Proof of Claim 2: Def^n (Maximal Maching) = A matching M of G in which if we add any other edge then it is no longer matching.

Eg:



$M = \{(2,1), (4,2)\}$ form a Maximal matching.

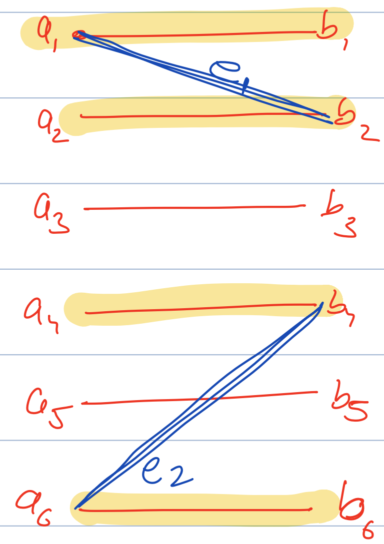
$\forall$ e in $G \setminus M$, $M \cup \{e\}$ is not a matching

FACT - our det algo computes a maximal-matching

FACT - If M_1 is a maximal matching & M_{opt} is a maximum —"—

then $|M_1| \geq \frac{|M_{opt}|}{2}$

Proof: Suppose $M_{opt} = \{ (a_1 b_1) (a_2 b_2) \dots (a_k b_k) \}$



Take any matching.

$M_1 = (e_1, e_2)$

$|M_1| \not\leq \frac{|M_{opt}|}{2}$

These are not sharing any vertices with M_{opt} so we can say

$M_1 \neq$ maximal matching

Naive-Rand-Algo $(G\ \sigma)$

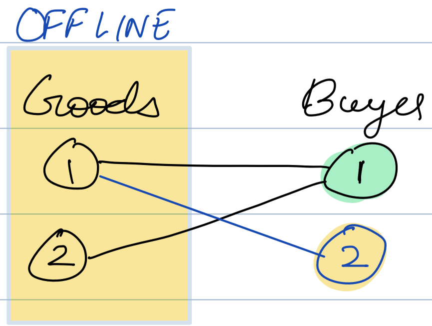
① $M = \phi$

② For $i = 1$ to $|\sigma|$:

Match Buyer i randomly to any of available goods he/she wants to buy, update M

③ Return M

Eg:



$$\text{Exp}(|\text{online matching}|) = 1.5$$

$$|\text{offline-matching}| = 2$$

$$C.R. \leq \frac{1.5}{2}$$

(H.W.) CR (above rand algo) $= 1/2$

Ex. $\forall$ det online algo A for online matching problem we have $CR(A) \leq 1/2$

and, if A computes a maximal-matching then $CR(A) = 1/2$