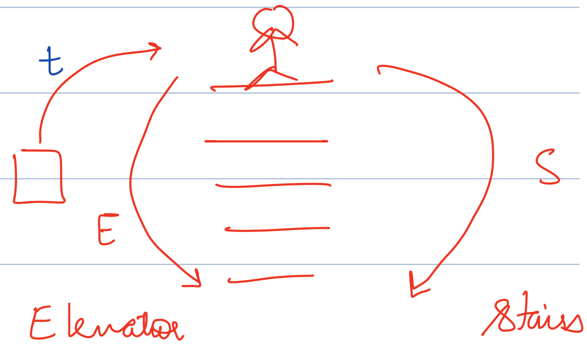
Online Algo - Elevator Puzzle



Optimize:

Time to reach Ground floor

Best Offline Algo (when t was known)

If $t + E \le S$: Use elevator
else : use stairs

$\Big\}$ time = $MIN(t+E, S)$

t = Time taken by elevator to reach top floor (Unknown)
S = Time by steps to reach ground floor.

Online setting

Unknow: Time "t" lift will take to reach Top floor

Best online strategy = ?

Natural Algo

Wait for "α" seconds,
if lift arrives, then take
it, else take Stairs

$$\boxed{\text{Time} = \begin{cases} t+E & t \le \alpha \\ \alpha + S & t > \alpha \end{cases}}$$

Algo 1 $\alpha = S$

Wait for "S" seconds.

If elevator comes : Take elevator

else : Take stairs

Best Offline Algo.

$$\boxed{\text{Time} = \begin{cases} \underline{t+E} & t \le S \\ S+S & t > S \end{cases}}$$

$\min(\underline{t+E}, \underline{S})$

$$Cr(\text{Algo1}) = \max_t \left(\frac{\text{Time taken by Algo1}}{\min(t+E, S)} \right)$$

— How far are we from best offline algo.

Scenarios

$\overbrace{t \le S-E}^{1}$	$\overbrace{t \in [S-E, S]}^{2}$	$\overbrace{t \ge S}^{3}$
ratio $= \frac{t+E}{t+E} = 1$	ratio $= \frac{t+E}{S}$	ratio $= \frac{2S}{S}$

$Cr = 2$

Algo 2 $\alpha = S-E$

Best

Wait for "S-E" seconds.

If elevator comes : Take elevator

else : Take stairs

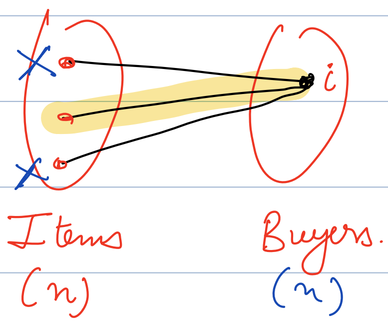
H.W (1)

$Cr = \left(2 - \frac{E}{S}\right)$

$t \leq S - E$ | $t \geq S - E$

H.W. (2) For any det algo for Elevator problem $Cr \geq 2 - \frac{E}{S}$

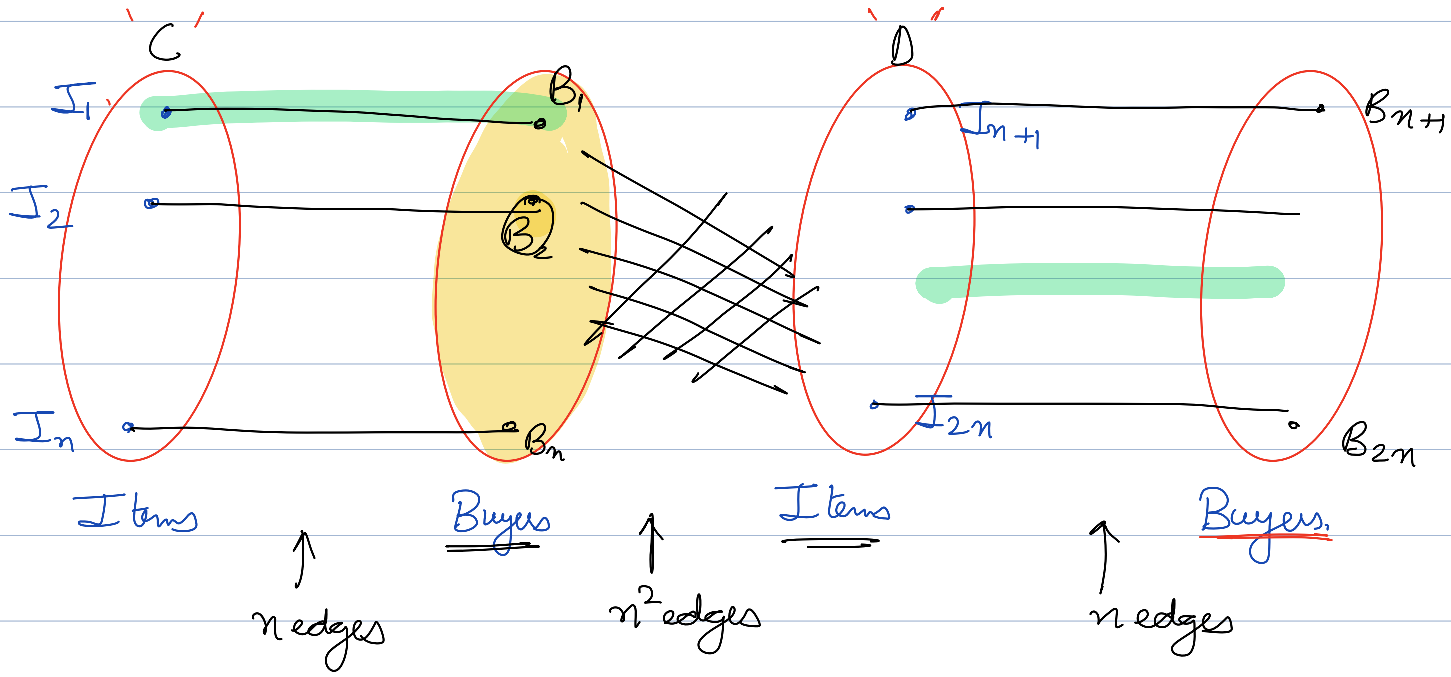
Online Matching



Algo

For $i = 1$ to n:

match buyer B_i "randomly" to any of available goods he/she wants to buy.



Offline scenario – what is $|\text{maximum-matching}| = 2n$

Online scenario :

ques – what is expected size of matching in online setting.

Random variable $X_i = \begin{cases} 1 & \text{if buyer } B_i \text{ is matched in set } C \\ 0 & \text{o/w} \end{cases}$

For $i \leq n$

CLAIM : $\left(\sum_{i=1}^{n} X_i + n \right)$ – size of online matching.

Exp size of online matching $= \sum_{i=1}^{n} E(X_i) + n$

$E(X_1) = \text{Prob}(B_1 \text{ is matched in set } C) = \frac{1}{n+1}$

$E(X_2) = \text{Prob}(B_2 \text{ is matched in set } C) = \begin{cases} \frac{1}{n+1} & \text{if first buyer matched in set } C \\ 1/n & \text{o/w} \end{cases}$

$\leq \frac{1}{n}$

$E(X_3) \leq 1/n-1$

$$E(X_n) \le 1/2$$

Exp size of online matching $\le \left(\frac{1}{n+1} + \frac{1}{n} + \dots + \frac{1}{2}\right) + n$

$$= O(\log n) + n$$

$$\underline{\underline{Cr}}(\text{rand Algo}) \le \sup_n \left(\frac{n + O(\log n)}{2n} \right) = \frac{1}{2} \quad \text{— (i)}$$

$Cr(\text{rand algo}) \ge \frac{1}{2}$ since we compute maximal matching. — (ii)

$$Cr(\text{rand algo}) = \frac{1}{2}$$