Quiz + Discussion (2-3 PM)

Recall defⁿ of Pseudo center (Z):

$$\forall x \in V(G),\ ecc_G(z) \leq ecc_G(x).$$

In last class,

we saw an $O(m \log n)$ time algo to compute pseudo-center Z of $O(\log^2 n)$ size.

<u>This class</u>:

An $O(m \log^2 n)$ time algorithm to compute a 2-eccentricity spanner having $O(n \log^2 n)$ edges.

Algorithm to compute eccentricity spanner

1. $H = (V, E_H = \phi)$
2. Compute pseudo-center Z of $O(\log^2 n)$ vertices.
3. $\forall z \in Z$: add to H edges of In-BFS(z) and Out-BFS(z)

Correctness of H:

Fix a pair (x, y).

Let $z \in Z$ satisfy $d_G(Z, y) = d_G(z, y)$

Then,

$$d_H(x,y) \leq d_H(x, z) + d_H(z, y)$$
$$= d_G(x, z) + d_G(z, y)$$
$$= d_G(x, z) + d_G(Z, y)$$
$$\leq ecc_G(x) + ecc_G(Z)$$
$$\leq 2\, ecc_G(x) \quad (\text{as } Z \text{ is pseudo-center})$$

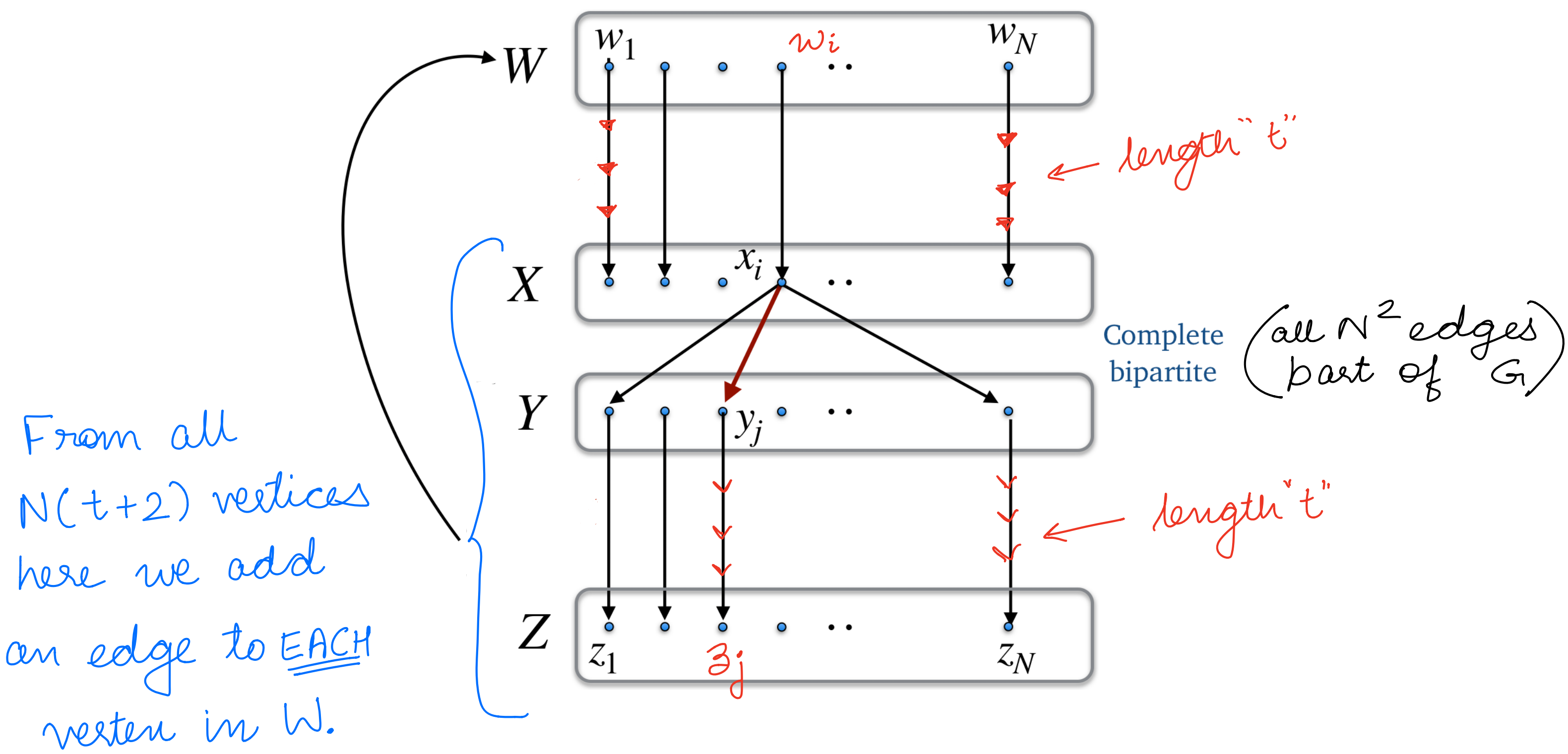
So, $ecc_H(x) = \max_y d_H(x, y) \leq 2\, ecc_G(x)$

Thus, H is a 2-eccentricity spanner.

To show tightness of our construction, we will prove following theorem.

Theorem: For any n, there exists $\Theta(n)$ vertex graph G whose eccentricity spanner of stretch less than 2 requires $\Omega(n^2)$ edges.

Proof: Consider following construction of G



$$|V(G)| = 2 \cdot N \cdot (t+1) = n \text{ (say)}$$

$$ecc_G(x_i) = t+1$$

If H is sparser than G, then it cannot contain all N^2 edges from $(X)\times(Y)$.

Let $H = G \setminus (x_i, y_j)$ for some $i, j \in [1, N]$.

Then,

$$ecc_H(x_i) = 2(t+1) \quad \text{bcoz } d_H(x_i, z_j) = 2(t+1).$$

For t to be some constant, $N = \Theta(n)$.

Thus, if want H to have stretch strictly less than 2, then H must contain $\Omega(n^2)$ edges.

$\square$