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COVID-19: Mathematical Modeling and Predictions

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Abstract

We investigate the problem of mathematical modeling of new corona virus (COVID19) spread in various countries; so that the estimation of new cases can be predicted and required preparation can be done. We propose a new mathematical model for COVID-19 spread. It is proved by the analytical and available data that USA and Italy are in third stage of COVID19; whereas India is in second stage of COVID19. By using the proposed model, an approximate estimation of new cases can be performed easily. The effect of lock-down is also considered in the paper. It is evident from the results that lock-down plays an important role in eradicating the COVID-19 spread. We also verify the proposed model for the cases reported in France and predict future number of cases.

Index Terms: COVID19, estimation, mathematical modeling

I. Introduction

Corona virus disease (COVID-19) is a new and a contagious disease caused by a new virus, known as novel corona virus. The cases were observed by various countries from November 2019 to till date. The disease affects lungs and causes respiratory illness with symptoms like the flu such as cold, throat infection, cough, fever, and in critical cases, difficulty in breathing. The active period of the novel corona virus is of fourteen days. It is suggested by medical authorities that one can protect himself/herself by washing hands frequently, avoiding touching the nose, ears and face, and by maintaining social distancing (1 meter or 3 feet) with other people. World Health Organization (WHO) declares that COVID-19 is a pandemic and releases guidelines to help countries maintain essential health services during the COVID-19 pandemic on 30th March 2020 [1]. It is stated by WHO that the COVID-19 pandemic is straining health systems worldwide. The rapidly increasing demand on health facilities and health care workers threatens to leave some health systems overstretched and unable to operate effectively. In this situation it is very essential to have an accurate prediction of new cases due to COVID19 so that the necessary preparation by hospitals and requisite actions by administration can be taken in advance. Further, a necessary course of action is also needed to plan so that the countries those are presently in Stage-III can move to Stage-II and finally in controlling stage. An important question for COVID19 is that how many days are required to infect the total population of a country if no measures has been taken and if some measures like lock-down has been taken then what will be the situation?

In this paper, we propose a mathematical model for the scenario of community spread of COVID19 (i.e., Stage-III). An approximate prediction for the spread of disease in coming days can be performed by using this model. We study the data of United States of America (USA), Italian Republic (Italy); and the Republic of India (India). This model may be used for other countries as well for prediction of number of COVID-19 cases in coming days. For this, we have considered the case of France for prediction of COVID-19 cases in coming days. These studies establish that fact that the proposed model can be used to predict the stage of COVID-19 also by

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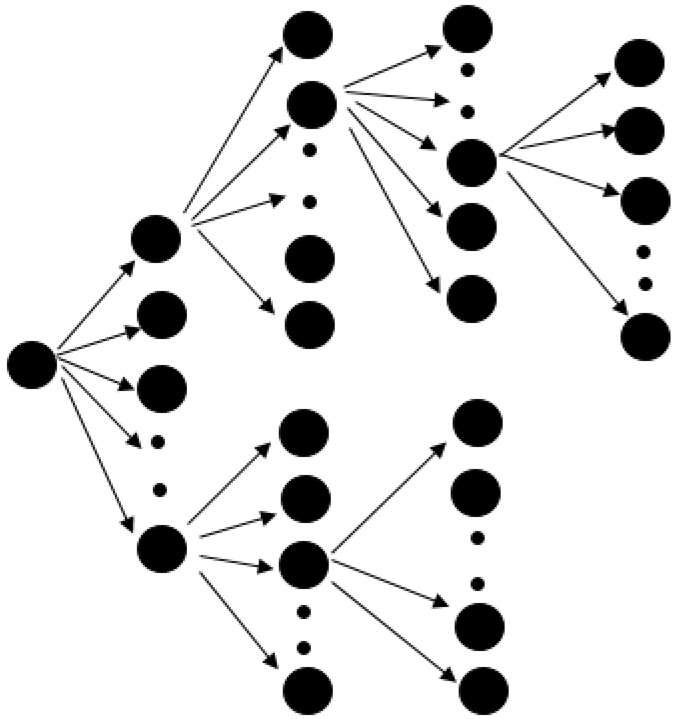

Fig. 1. Proposed model for spread of COVID-19.

matching the available data with analytical results. Further, the other statistics like number of deaths or number of recovered cases can also be predicted with a sufficient accuracy. The model is verified for the COVID-19 cases reported in France and for next few days the number of new cases is predicted for this country.

II. Proposed Model for COVID19 in Unconstrained Scenario

We assume that a positive corona patient comes into a country and comes into the contact of other people. Since it is an infectious disease, therefore, it spreads in others. We denote the positive corona patient as an active node. Since there is no identification of this active node, therefore, it is in contact will all other nodes. A tree based structure is shown to clarify the scenario in Fig. 1. It is clear from the figure that node A (an infected person) can infect a other nodes (person) in 1 unit time (unit may be day, hour, minute). Now all these new infected nodes are active and each of them can infect other a nodes and so on. Therefore, we can write total number of active nodes or infected persons as

$$\begin{aligned} T_{cases} = {} & 1 + a + (1+a)a + (1 + a + (1+a)a)a \\ & + (1 + a + (1+a)a + (1 + a + (1+a)a)a)a + \cdots \end{aligned} \tag{1}$$

After some algebra, from (1), we get

$$T_{cases} = 1 + a + \underbrace{(1+a)a + (1+a)^2 a + (1+a)^3 a + (1+a)^4 a + \cdots}_{\text{GP of } (N-2) \text{ terms}} \tag{2}$$

Here N is the number of terms in the series, indicating the N units of the time span over which the spread of disease is needed to be calculated. It can be observed from (2) that after first two terms, i.e., 1 and a, rest of the series follows the geometric progression (GP) with common ratio 1+a. By using the formula of sum of N terms of GP in (2) [2], we obtain

$$T_{cases} = 1 + a + \frac{(1+a)a((1+a)^{N-2}-1)}{a}\,. \tag{3}$$

After solving (3), we get

$$T_{cases} = (1+a)^{N-1}\,. \tag{4}$$

It can be observed from (4) that this series grows very fast. For example, if we assume that $a = 1$, i.e., one person can infect only a single person in one unit time, i.e., ($a = 1$); in that case total population of a country will be infected when total cases in (4) will be equal to or greater than the total population (P). Therefore, from (5), we get

$$P \leq 2^{N_p-1}\,, \tag{5}$$

where N_p is the smallest value of N which attains this inequality.

For example, the population of India is 130 crores, then approx. only 31 days are needed to spread the disease in unconstrained environment (without any restriction), when one person infects only a single person in one day. In case of Italy with 6 crores population, only 26 days are needed, when one person infects only a single person in one day. It shows that only five more days are required to infect 130 crores as compared to six crores. It can be concluded from this calculation that countries with large population or small population have almost the same threat from COVID-19 disease in unrestricted environment.

Now, we consider the effect of virus incubation period that is virus is active for k days. Therefore, if we assume that after k days the infected nodes and their effects are inactive, then total number of active cases can be written as

$$\begin{aligned} T_{cases} &= X_n - X_{n-k} - X_{n-k+1} \\ &= a(1+a)^{n-2} - a(1+a)^{n-k-2} - a(1+a)^{n-k-1},\ n > k, \end{aligned} \tag{6}$$

where X_n, X_{n-k}, and X_{n-k+1} are the nth, $n-k$ th, and $n-k+1$ th terms of series given by (2).

III. Results

In this section, we consider two countries, USA and Italy for the study of spread of COVID-19 disease. Both these countries are observing an enormous increase in the number of patients each day. It seems that the disease has been spread at community level and it is difficult to get control of it now. These two scenarios suit the assumption of unconstrained scenario as even after every measures taken placed, the number of infected persons is kept on increasing day-by-day. For checking the validity of our proposed model, we take the data from [3].

In Fig. 2, we have plotted the curves between the total number of COVID-19 cases versus days (till March 29, 2020) in Italy based on the actual data [3] and the proposed model. The rate of spread/infection is taken as 1.46 per day, i.e., one infected node takes 1.46 days to infect another node. Also, we consider $a = 1$, i.e., one node can infect only one node in 1.46 days. Further, the

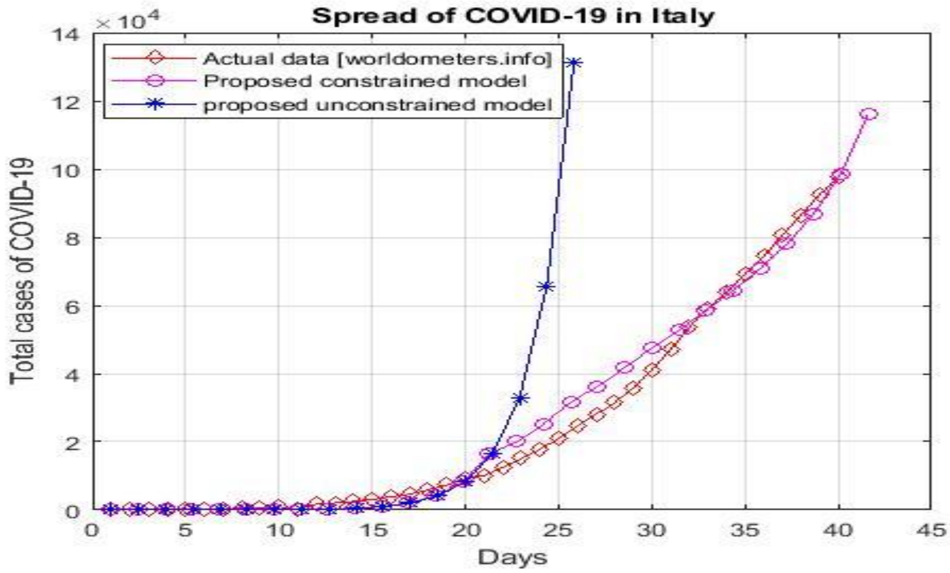

Fig. 2. COVID-19 spread in Italy.

value of k=14, i.e., it takes 14 days for an infected node to recover. It is seen from the actual data that the mortality rate of Italy is very high, therefore, after 14 days, more than the recovered nodes needed to be removed from the system. Hence, for each day after 14th day, we actually remove all nodes active from the 12th day onwards. Further, Italy has taken a lock-down action throughout the country, therefore due to the lock-down, the spreading of disease has been slowed down. This slow down is taken into account by using a correction factor of C from 15th day onwards. It can be seen from the figure that the proposed model with C = 10692 gives a very close indications of the COVID-19 spread over the considered range of days. Further, we have plotted the plot for unconstrained scenario in the same figure. it can be seen form the figure that in an unconstrained scenario, where no lock-down step has been taken, the situation would be much worst than the present day.

In Fig. 3, we have considered the COVID-19 spread in USA. The actual data is taken from [3]. The proposed model is used to plot the number of cases each day in the figure. In USA, the mortality rate is lower than Italy, therefore, we use the following relation obtained form (6):

$$T_{cases} = \begin{cases} X_n, & \text{for } n \leq k, \\ X_n - X_{n-k+1} - C, & \text{for } n > k \end{cases} \quad (7)$$

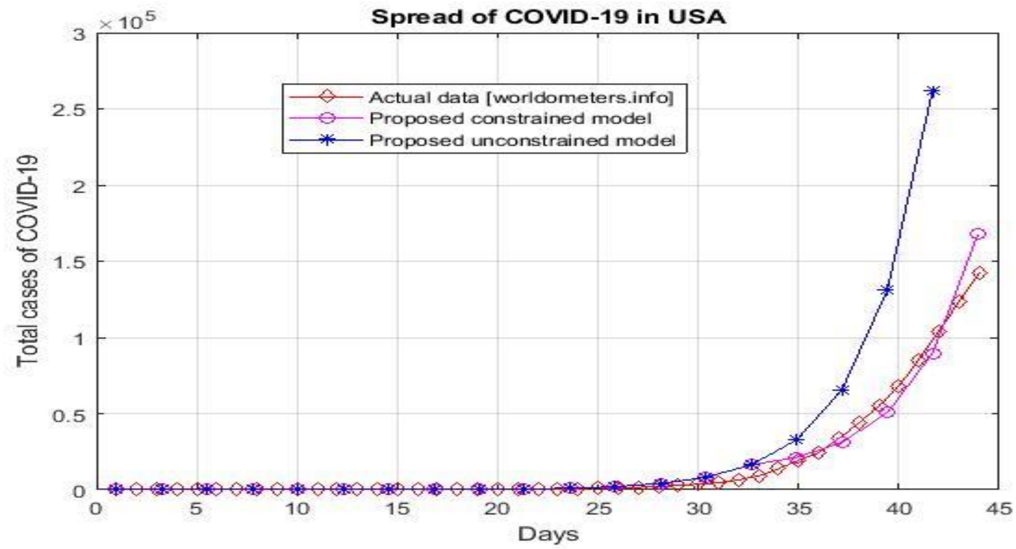

Fig. 3. COVID-19 spread in USA.

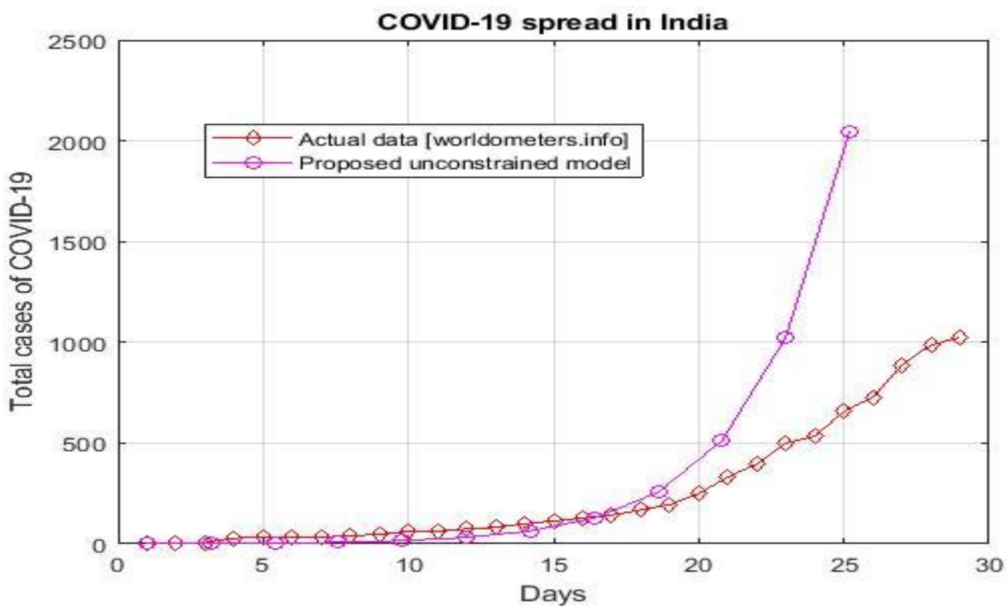

Fig. 4. COVID-19 spread in India.

The value of a is taken as unity and C as 11500; whereas, the quarantine period of k = 14 days is assumed. The rate of spread of disease is taken as one person in 2.26 days. A close match of the proposed model and actual data is seen from the figure. Further, the proposed model can be used to predict the cases in coming days. An unconstrained plot for the total cases is also shown in the figure, which indicates that the spread of COVID-19 can be very drastic but die to the measure taken for testing and quarantine, the spread is still well below the bound.

Let us now use the proposed model to check the current situation of COVID-19 spreading in India. We have plotted the actual cases from [3] (till March 29, 2020) in Fig. 4. Further, we have also plotted the values of infected cases for each day by using the proposed unconstrained model. It can be seen from the figure that India is doing very well in keeping the pandemic away from taking a big leap in terms of infected cases. This is a result of strict measures taken from the beginning that the spread of the disease is very limited in India.

We have also verified the proposed model for the COVID cases reported in [3] till April 01, 2020 in Fig. 5. The value of a is taken as unity and C as 10300; whereas, the quarantine period of k = 14 days is assumed. The rate of spread of disease is taken as one person in 1.92 days. It can be seen from the figure that the number of cases found by using the proposed model is in close agreement with the actual data. Further, by using the proposed model, we can predict the number of cases in coming days, as seen in Fig. 5. For example, the number of COVID cases predicted on April 02, 03, 04 are approx.. 6700, 7800, 9200.

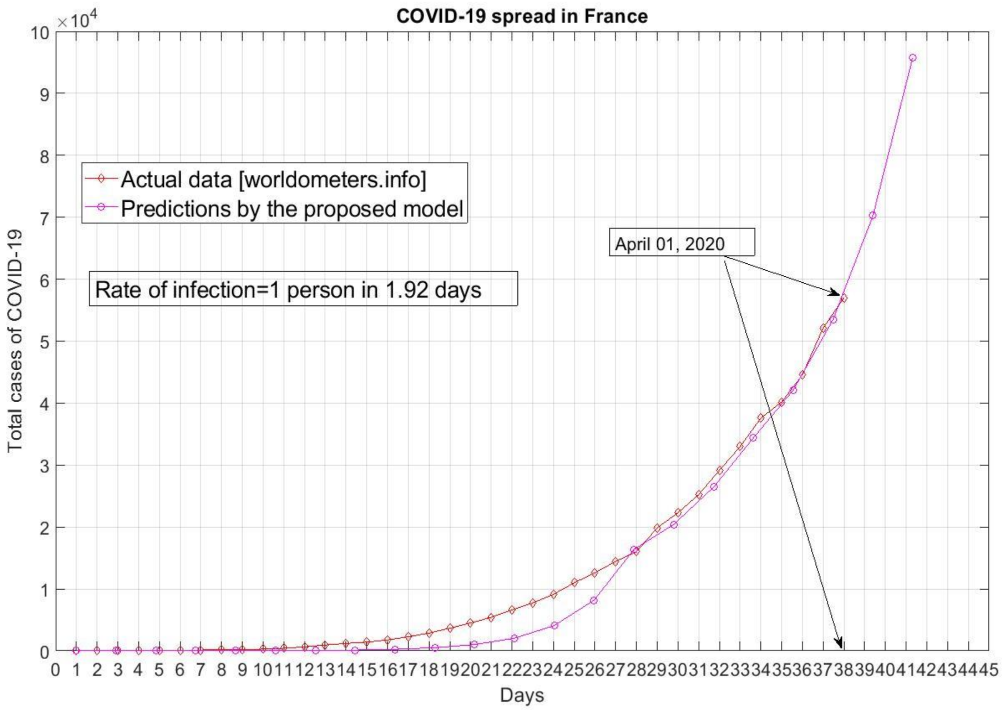

Fig. 5. COVID-19 spread in France.

IV. CONCLUSIONS

A simple but accurate model has been proposed to predict the COVID-19 cases reported for the countries facing the Stage-III of the disease. The proposed model has shown a close match with the available data for different countries. By using the model, it is also possible to check whether a country has entered Stage-III or not. It has been observed from the numerical results that the disease transmission rate is the highest in Italy which has resulted in devastated scenario of COVID-19 disease spread in the country. France, though having a similar population size as that of Italy, has a lower disease transmission than Italy and still has time to control the spread of the disease. On the other hand, India is well below the Stage-III and can control the spread by employ strict disciplinary mechanisms like lock-down and compulsory COVID-19 testing.

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IV. References

[1] https://www.who.int/emergencies/diseases/novel-coronavirus-2019 [Accessed on March 30, 2020].
[2] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. San Francisco, CA, USA: Academic, 2000.
[3] https://www.worldometers.info/coronavirus/countries [Accessed on March 30, 2020].
[4] Arti MK and Kushagra Bhatnagar, “Modeling and Predictions for COVID 19 Spread in India,” ResearchGate, DOI: DOI: 10.13140/RG.2.2.11427.81444, published on April 01, 2020.