M2E03 - Solution of Assignment - 1

1.(a) $N(t) = N(0)\, e^{rt}$

$e^{rt} = \dfrac{N(t)}{N(0)}$

$t = \dfrac{1}{r} \ln\left(\dfrac{N(t)}{N(0)}\right)$

1.(b) $N(0) = 10$

$r = .1$ (per day)

(i) $N(t) = 10,000$ $\quad t = \frac{1}{.1} \ln\left(\frac{10,000}{10}\right)$

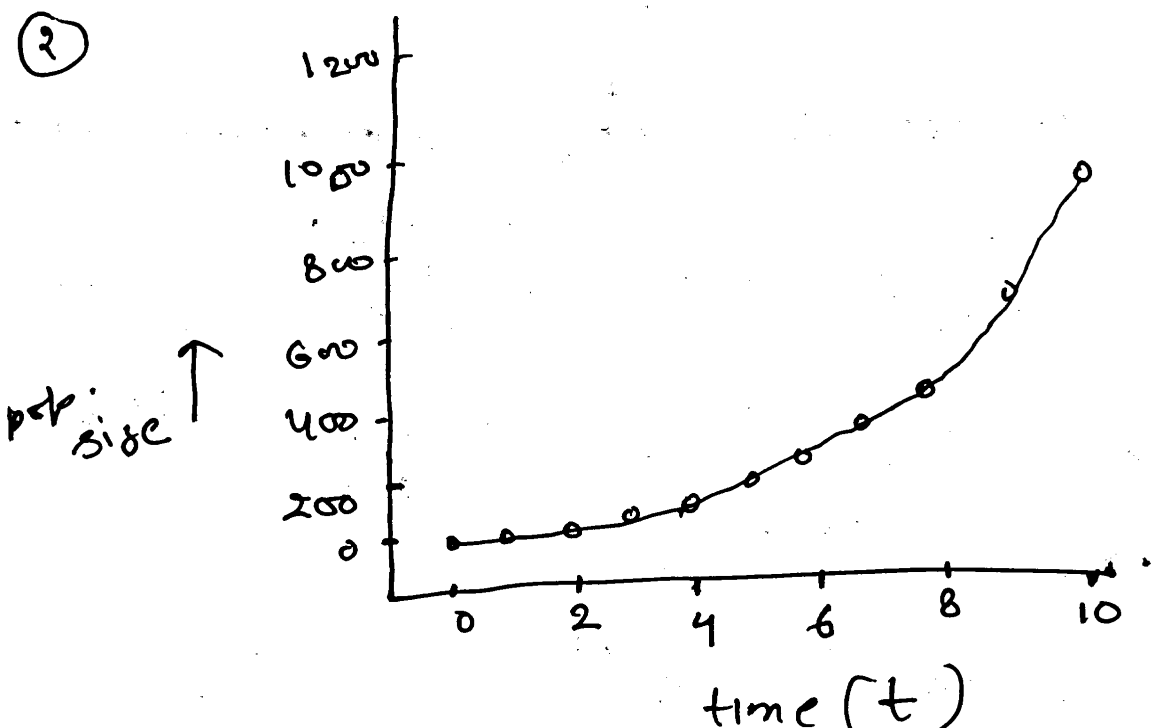
$\simeq 69.07$ days

(ii) $N(t) = 10,000,000 \Rightarrow t \simeq 138.15$ days

(iii) $N(t) = 10,000,000,000 \Rightarrow t \simeq 207.23$ days

1 (c) $\Rightarrow t_{\text{double}} = 6.93$ days

(2)



(Roughly

(3) $No = 10 \quad R = 2 \ (d)$

$t = 3 \times 12 = 36$ (three 20 minute periods per hour)

$Nt = 10 \times 2^{36}$

$= 687194767360.$

(4) $d = R = \frac{15}{6} = 2.5$ for a period of two weeks

$t = \frac{10}{2} = 5$ (the number of two week. periods in 10 weeks)

$No = 6$

$N_t = 6 \times 2.5^5$

$= 585.9$ individuals

(5) $d = 2$ (per three day)

$-r_m$ per three days $\quad d = e^r$

$r = \ln d$

$= \ln 2$

r_m per day $= \frac{\ln 2}{3} = \frac{.6931}{3} = .2310.$

r_m per week is $\quad 7 \times .2310 = 1.617$

d per day $= e^{.2310} = 1.26$

d per week $= e^{1.6173} = 5.04$

6 — (a)

$$L = \begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 0 \end{bmatrix} \qquad \lambda \simeq \underline{1.28}.$$

b let $\begin{bmatrix} n_0(t) \\ n_1(t) \end{bmatrix}$ be the population at time t

$n_0(t) = \#$ of 0 year

$n_1(t) = \#$ of 1 year

$$\begin{bmatrix} n_0(t) \\ n_1(t) \end{bmatrix} = L \begin{bmatrix} n_0(t-1) \\ n_1(t-1) \end{bmatrix}$$

$$\begin{bmatrix} n_0(0) \\ n_1(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$ then

$$\begin{bmatrix} n_0(1) \\ n_1(1) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} n_0(2) \\ n_1(2) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} n_0(3) \\ n_1(3) \end{bmatrix} = \begin{bmatrix} 5/2 \\ 1/2 \end{bmatrix}$$

If $\begin{bmatrix} n_0(0) \\ n_1(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

then $$\begin{bmatrix} n_0(3) \\ n_1(3) \end{bmatrix} = \begin{bmatrix} 9/8 \\ 5/8 \end{bmatrix}$$

C.

$$\lambda \simeq 1.28$$

let $\begin{bmatrix} u \\ v \end{bmatrix}$ be eigenvector associated with λ

$$L \begin{bmatrix} u \\ v \end{bmatrix} = \lambda \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \lambda \begin{bmatrix} u \\ v \end{bmatrix}$$

the bottom row gives $\frac{u}{2} = \lambda v$

$$v = \frac{1}{2\lambda} u$$

Choose $u = 1$ $\quad v = \frac{1}{2\lambda} = \simeq .39$

so stable age distribution is $\begin{bmatrix} 1 \\ .39 \end{bmatrix}$

7. $r \simeq .25$

$$c_x = \frac{e^{-rx} l_x}{\sum_{x=0}^{\infty} e^{-rx} l_x}$$

x	c_x
0	.48
1	.23
2	.15
3	.09
4	.05
5	0

$R_0 = 2.4$.