① The difference equation for a mortgage payment type situation will be

$$a_n = \left(1 + \frac{i}{N}\right) a_{n-1} + b \quad — ①$$

in this case, b will be negative since each monthly payment subtracts from the total amount owed each. In ten years there will be $k = 10 \times 12 = 120$ payments, after which amount owed should be zero.

The solution of difference eq ① is

$$a_n = \left(1 + \frac{i}{N}\right)^n \left(a_0 + \frac{Nb}{i}\right) - \frac{Nb}{i}$$

(a_k should be 0 — a_0 the original amount ~~owed~~ loaned. (a_0 is also the cost of the house minus the \$ 3000 down payment).

Solving for a_0 $a_k = 0$

$$\left(1 + \frac{i}{N}\right) a_0 = \left[1 - \left(1 + \frac{i}{N}\right)^k\right] \frac{Nb}{i}$$

$$a_0 = \frac{\left[1 - \left(1 + \frac{.06}{12}\right)^{120}\right] \frac{12(-\$2000)}{.06}}{\left(1 + \frac{.06}{12}\right)^{120}}$$

$$\cong 18000$$

(2)

$$\frac{dQ}{dt} = \text{rate in} - \text{rate out}$$

$$= \frac{r}{4} - \frac{Q}{100} r$$

$$Q(0) = Q_0$$

$$\frac{dQ}{dt} + \frac{Qr}{100} = \frac{r}{4} \qquad 3$$

integrating factor is $e^{\frac{rt}{100}}$

Solution is

$$Q \cdot e^{\frac{rt}{100}} = \int \frac{r}{4} e^{\frac{rt}{100}}\, dt + C$$

$$= \frac{\cancel{r}}{4} e^{\frac{rt}{100}} \cdot \frac{100}{\cancel{r}} + C$$

$$Q(t) = 25 + C e^{-\frac{rt}{100}}$$

at $t = 0$ $\quad C = Q_0 - 25$

$$\Rightarrow \quad Q(t) = 25 + (Q_0 - 25) e^{-\frac{rt}{100}} \qquad 8$$

as $t \to \infty$ $\quad Q(t) \to 25$

So limiting value Q_L is 25 $\qquad$ 10

$\cancel{\text{If } r = 2,\ Q = 2Q_L = 50}$

$\cancel{Q(t) = 25 + 25 e^{-\frac{2t}{100}}}$

$\cancel{Q(t) = 2\%\ \text{of}\ Q_L \ldots \frac{r}{100}\frac{25}{4} = 0.5}$

③ $\frac{dQ}{dt}$ = rate in − salt out

$$= 5(2 + 2\sin 2t) - 5 \cdot \frac{Q(t)}{10^7}$$

$$\frac{dQ}{dt} + \frac{Q(t)}{2\times 10^6} = 10(1 + \sin 2t).$$

Let $k = \frac{1}{2\times 10^6}$

$$IF = e^{kt}.$$

$$Q \cdot e^{kt} = 10\int (e^{kt} + e^{kt}\sin 2t)\,dt + K$$

$$Q \cdot e^{kt} = \frac{10e^{kt}}{k} + 10\int e^{kt}\sin 2t\,dt + c$$

Let $I = \int e^{kt}\sin 2t\,dt$

$$e^{kt}\frac{-\cos 2t}{2} + \int k e^{kt}\frac{\cos 2t}{2}\,dt$$

$$= -e^{kt}\frac{\cos 2t}{2} + \frac{k}{2}\left(e^{kt}\frac{\sin 2t}{2} - \int k e^{kt}\frac{\sin 2t}{2}\right.$$

$$I\left[1 + \frac{k^2}{4}\right] = -e^{kt}\frac{\cos 2t}{2} + \frac{k}{4}e^{kt}\sin 2t$$

$\Rightarrow$

$$Q = \frac{10}{k} + 10\left[\frac{-\frac{\cos 2t}{2} + \frac{k}{4}\sin 2t}{1 + \frac{k^2}{4}}\right] + c\,e^{-kt}$$

where

$$k = \frac{1}{2\times 10^6}$$

(4) Let a, b, c is concentration of [NO] [Cl_2] — [$NOCl_2$]

$$\frac{da}{dt} = -k a^2 b \quad — (1)$$

$$\frac{db}{dt} = -k a^2 b \quad — (2)$$

$$\frac{dc}{dt} = k a^2 b \quad — (3) .$$

(1) - (2) $\quad \frac{d(a-b)}{dt} = 0$

$\Rightarrow \quad a - b = c \quad$ where c is constant of integration.

$c = a_0 - b_0$

$\Rightarrow \quad a - b = a_0 - b_0$

Now $\quad \frac{da}{dt} = -k a^2 [a - a_0 + b_0]$

$\frac{db}{dt} = -k a^2 [a - a_0 + b_0]$

$\frac{dc}{dt} = k a^2 [\cancel{= a - a_0 + b_0}] b$

$\frac{dx}{dt} \quad \frac{dy}{dt}$

(2) + (3) $\quad b + c = b_0 + c_0$

(1) + (3) $\quad a + c = a_0 + c_0$

$\Rightarrow \quad \frac{dc}{dt} = k [c_0 + a_0 - c]^2 [b_0 + c_0 - c]$

$x(t=0) = a_0$

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$$\frac{dx}{dt} = x(1 - .5y)$$

$$\frac{dy}{dt} = y(-.75 + .25x)$$

$\Rightarrow$ $x = 0$ $y = 2$

$y = 0$ $x = 3$

points of equilibrium $(0\,0)$ $(3\,2)$ —— 3

around $(0\,0)$

$x = 0 + \epsilon$

$y = 0 + \delta$

$$\dot{\epsilon} = \epsilon(1 - .5\delta)$$

$$\dot{\epsilon} \simeq \epsilon$$

$$\dot{\delta} = \delta(-.75 + .25\epsilon)$$

$$\simeq -.75\delta$$

$\epsilon = Ae^{t}$ [A integration constant]

$\delta = Be^{-.75t}$ [B " "]

$$\begin{bmatrix}\epsilon\\ \delta\end{bmatrix} = A\begin{bmatrix}1\\0\end{bmatrix}e^{t} + B\begin{bmatrix}0\\1\end{bmatrix}e^{-.75t} \Rightarrow$$ $(0\,0)$ is unstable —— 3

around $(3\,2)$

$x = 3 + \epsilon$

$y = 2 + \delta$

$$\dot{\epsilon} = (3 + \epsilon)\left(1 - .5(2 + \delta)\right)$$

$$= (3 + \epsilon)(\cancel{1} - \cancel{1} - .5\delta)$$

$$= -1.5\delta - .5\epsilon\delta$$

$$\simeq -1.5\delta$$

$$\dot{\delta} = (2 + \delta)(-.75 + .25(3 + \epsilon)$$

$$(2 + \delta)(.25\epsilon)$$

$$\simeq .5\epsilon$$

$\ddot{\epsilon} = -1.5\,\delta = -1.5(.5\epsilon)$

$\ddot{\epsilon} + .75\,\epsilon = 0.$

$\ddot{\delta} = .5\,\dot{\epsilon} = -.75\,\delta = 0$

$\Rightarrow \quad \ddot{\delta} + .75\,\delta = 0.$

$\epsilon = A\cos(\sqrt{.75}\,t) + B\sin(\sqrt{.75}\,t)$

δ will also be in this form

$\Rightarrow$ (3,2) is stable point

10

6

$$\frac{dH}{dt} = 4H(1-aH)$$

for carrying capacity

$f(a) = 0. \Rightarrow 1-aH=0$

~~$H = \frac{1}{a}$~~ $\quad H = \frac{1}{a} = 10$

$a = \frac{1}{10}.$

$$\Rightarrow \frac{dH}{dt} = 4H\left(1-\frac{H}{10}\right)$$

second part. $\quad \frac{dL}{dt} = -3L$

Third part. $\quad \frac{dL}{dt} < 0 \quad H<5$

$\qquad > 0 \quad H>5$

$\frac{dL}{dt} = 0$ for $H=5$

$\Rightarrow \frac{dL}{dt} = L(-3+cH)$

for $H=5 \quad c = \frac{3}{H}$

$= 3/5$

$\Rightarrow \frac{dL}{dt} = L\left(-3+\frac{3H}{5}\right)$