1.

$$\frac{dy}{dt} = r_{in} c_{in} - \frac{y}{V} r_{out}$$

$$= 5 \times 1 - \frac{y}{V} 100$$

$$\frac{dy}{dt} + \frac{y}{V} 100 = 5$$

$$IF = e^{\int \frac{100}{V} dt}$$

$$= e^{\frac{100}{V} t}$$

$$y \cdot e^{\frac{100t}{V}} = \int 5 \cdot e^{\frac{100t}{V}} dt + K$$

$$y\, e^{\frac{100t}{V}} = 5 \cdot \frac{e^{\frac{100t}{V}}}{\frac{100}{V}} + K$$

$$y = \frac{V}{20} + K e^{-\frac{100t}{V}}$$

using condition. $t = 0$. $\frac{y}{V} = 0$

$\Rightarrow y = 0$.

$$0 = \frac{V}{20} + K \qquad K = -\frac{V}{20}$$

$$\Rightarrow y = \frac{V}{20}\left[1 - e^{-\frac{100t}{V}}\right]$$

$$\frac{y}{V} = \frac{1}{20}\left[1 - e^{-\frac{100t}{V}}\right]$$

$$.0141 = \frac{1}{20}\left[1 - e^{-\frac{100t}{V}}\right]$$

$V \approx 448.14h$

(2) suppose we pump in nitrogen 3 times. (2)

The initial amount of methane is m_0

we can assume that when the first amount of nitrogen (ρ) is pumped in

Each time an amount of gas ρ, will escape out of other opening of the tank

$$\Rightarrow m_1 = m_0 - \rho.$$

II^{nd} time $m_2 = m_1 - \rho \cdot \frac{m_1}{V}$

(Because the amount of methane escaping will be the concentration of methane times ρ)

$$m_3 = m_2 - \rho \cdot \frac{m_2}{V}$$

(3) $V(t+\Delta t, x) = r V(t, x-\Delta x) + (1-r-\ell) V(t, x) + \ell V(t, x+\Delta x)$

$V + \Delta t V_t = r\left[V - \Delta x V_x + \frac{\Delta x^2}{2} V_{xx}\right] + (1-r-\ell)\left[V\right] + \ell\left[V + \Delta x V_x + \frac{\Delta x^2}{2} V_{xx}\right]$

a) $r = \frac{1}{2} \quad \ell = \frac{1}{2}$

$\Rightarrow \quad \Delta t V_t = (r+\ell) \cdot \frac{\Delta x^2}{2} \cdot V_{xx}.$

$V_t = \frac{\Delta x^2}{2 \cdot \Delta t} \cdot V_{xx}$

as $\Delta x \to 0 \quad \Delta t \to 0$.

$V_t = D \cdot V_{xx}$

solution is

$V(x,t) = \sum A_n \sin \frac{n\pi x}{L} e^{-\left(\frac{n\pi}{2}\right)^2 D t}$

integration over 0 to L

$\int_0^L V(x,0) \sin \frac{m\pi x}{L} dx = \sum A_n \int_0^L \sin \frac{n\pi x}{L} \sin \frac{n\pi x}{L} dx$

$\int_0^L x \sin \frac{m\pi x}{L} dx = A_m \cdot \frac{L}{2}$

(4)

$$A_m \cdot \frac{L}{2} = \int_0^L x \sin \frac{m\pi x}{L}\, dx$$

$$= \left[-x \cdot \frac{\cos \frac{m\pi x}{L}}{\frac{m\pi}{L}} \right]_0^L + \int_0^L \frac{\cos \frac{m\pi x}{L}}{\frac{m\pi}{L}} \cdot dx$$

$$= -L \frac{\cos m\pi}{\frac{m\pi}{L}} + \frac{L}{m\pi} \cancel{\left. \frac{\sin \frac{m\pi x}{L}}{\frac{m\pi}{L}} \right|}$$

$$0$$

$$= -L^2 \cdot \frac{\cos m\pi}{m\pi}$$

$$= -\frac{L^2}{m\pi} (-1)^m$$

$$A_m = \frac{-2L}{m\pi} (-1)^m$$

$$\Rightarrow v(x, t) = -\cancel{\quad}_{100} \sum \frac{(-1)^m}{m\pi} \sin \frac{m\pi x}{20} e^{-\left(\frac{m\pi}{2}\right)^2 \cdots}$$

(b) ~~$v_0 = 8$~~ In this case

$$A_m \cdot \frac{L}{2} = \int_0^L \sin \frac{\pi x}{L} \cdot \sin \frac{m\pi x}{L}\, dx$$

So if $m = 1$ $\quad A_1 \cdot \frac{L}{2} = \frac{L}{2} \Rightarrow A_1 = 1$

$\Rightarrow$ ... πx ... $-\left(\frac{\pi}{2}\right)^2 D t$