M2E03 - INTRODUCTION TO MODELLING ASSIGNMENT 3

DUE:NOON,FRIDAY 14 NOVEMBER

"Please drop the hand written assignment in locker C16 (near to HH105 in Hamilton hall)"

LATE ASSIGNMENTS WILL RECEIVE A GRADE OF ZERO.

1. Spphie and Craig have just been married, and are looking for a place to settle down and raise kids. They figure out they can afford a whopping $\$2000$ per month for housing payments (not including the cost of living, maintenance, etc.), and they have $\$30000$ kicking about in a saving account. Sophie has always dreamt of owning her own house, and wants to buy one right away.

   - Determine the priciest house that they could afford, assuming that they use the full $\$30000$ as a down payment and wish to enter into a 10 year mortgage. The lending rate at their friendly neighborhood bank is 6%

2. At time $t = 0$ a tank contains $Q_0 lb$ of salt dissolved in $100gal$ of water. Assume that water containing $\frac{1}{4}lb$ of salt/gal is entering the tank at a rate of $rgal/min$ and that the well stirred mixture is draining from the tank at the same rate. Set up the initial value problem that describes this process. Find the amount of salt $Q(t)$ in the tank at any time and also find the limiting amount Q_L that is present after a long time.

3. Consider a pond that initially contains 10 million gal of water. Water containing an undesirable chemical flow in to pond at the rate of $5gal/yr$, and the mixture in the pond flows out at the same rate. The concentration $\gamma(t)$ of chemical in the incoming water varies periodically with time according to the expression $\gamma(t) = 2+2sin(2t)g/gal$. Construct a mathematical model of this flow process and determine the amount of chemical in the pond at any time.

4. The chemical reaction is

$$2NO + Cl_2 \rightarrow 2NOCl$$

   Suppose initial concentration of NO, Cl and $2NOCl$ are a, b and c respectively. Set up the mathematical model for reaction rate of all reactants and products in terms of solvable ODEs.

5. A simple model for a predator-prey interaction is as follows.

$$\frac{dx}{dt} = x(1 - .5y)$$
$$\frac{dy}{dt} = y(-.75 + .25x)$$

   Determine whether each equilibrium is stable or unstable.

6. Construct a simple model for a predator-prey interaction as follows. Let H be the population size for the predator species and L be the population size for prey species.

   - Set up equation for the prey species in the absence of the predator so that intrinsic rate of growth of the prey is 4 and the carrying capacity of the prey is 10.
   - Set up equation for the predator species in the absence of prey so that predator undergoes exponential decay $L(t) = L(0)e^{-3t}$
   - Modify the equation of predator species to include prey's effect on the predator. Do this so that L grows if $H > 5$ and L shrinks if $H < 5$.