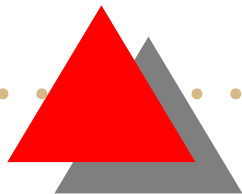




*Math 2E03- Introduction to
Modelling*

Instructor– Dr. Mani Mehra

Department of Mathematics and Statistics
McMaster Univ.





A survey of models containing DCEs

Teena is saving up for a new Barbie doll by stuffing her weekly allowances of \$2 in her mattress. Each week, her current savings are equal to the previous week's saving plus two dollars. If a represent the amount of money she has, then mathematically the amount of money sha has on the n^{th} week is

$$(1) \quad a_n = a_{n-1} + 2$$

This is known as difference equation. More geneareally,

$$(2) \quad a_n = f(a_{n-1})$$



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What is difference between difference equations (DCs) and differential equations (DEs)?





DCs deal with finitely-spaced steps, while DEs are based on continuous changes in the variables. Example of DCs is as in equation (2) .



The world of high finance

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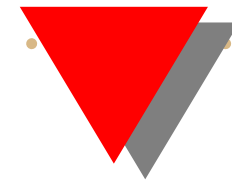


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The Solution Compound interest differ from the simple interest...how? If a_n is current amount, where n is our index indicating how many compounding periods have gone by since the money was deposited at $n = 0$. 12% was the advertise price so $i = .12$. Finally since the bank divides the advertised rate by the number of yearly compounding periods, introduce $N = 12$.

$$(7) \quad a_n = a_{n-1} + \frac{i}{N} a_{n-1}$$



In more generalization sense

$$(8) \quad a_n = \left(1 + \frac{i}{N}\right)^n a_0$$

If amount is a function of some other time variables- for instance a_t where t is measured in years. Since interest is compounded N times in a year, $n = Nt$ and

$$(9) \quad a_t = \left(1 + \frac{i}{N}\right)^{Nt} a_0$$





Interest with Money Deposits/Withdrawal

Up until now, we have assumed that the amount of money in a particular account is not touched by its owner-the money is simple left in the account to accumulate interest.

If a particular amount b is added or subtracted from the account each month.?



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The solution is

$$(15) \quad a_n = \left(1 + \frac{i}{N}\right)^n \left(a_0 + \frac{Nb}{i}\right) - \frac{Nb}{i}$$






Problem : Craig deposits \$500 annually into an account which pays 8% interest, compounded annually. He initially opened the account by placing \$1000 into it. How much money has he put into it after 10 years.?

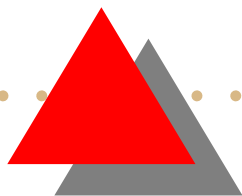



Problem : Craig deposits \$500 annually into an account which pays 8% interest, compounded annually. He initially opened the account by placing \$1000 into it. How much money has he put into it after 10 years.?

Solution : He placed \$6000 into the account. Finally he had \$9402.21.



Problem of Fibonacci's rabbits: Assume that each pair of rabbits breeds twice, once at age 1 month and once at age 2 months. Also assume that each mating produces 1 pair each time. Let a_n be the number of newborn pairs at the n^{th} month, beginning with one newborn pair ($a_0 = 1$). This in the first month one pair will be born ($a_1 = 1$), and the following month two newborn pairs will be born—one from the first (a_0) pair, and one from the second a_1 pair:

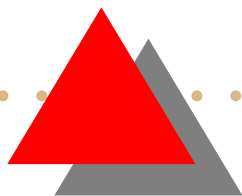





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$$\begin{aligned} a_2 &= a_0 + a_1 = 2 \\ a_3 &= a_1 + a_2 = 3 \\ a_4 &= a_2 + a_3 = 5 \end{aligned}$$

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The DCE for all $n \geq 2$

$$a_n = a_{n-1} + a_{n-2}$$

(21)





Linear and Nonlinear operations

Consider some unknown operator Φ applied to a variable or function x .
The Φ is only a linear operations if the following holds true.

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