Math 2E03- Introduction to Modelling

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A survey of models containing DCEs

Teena is saving up for a new Barbie doll by stuffing her weekly allowences of $\$2$ in her mattress. Each week, her current savings are equal to the previous week's saving plus two dollars. If a represent the amount of money she has, then mathematically the amount of money sha has on the n^{th} week is

$$a_n = a_{n-1} + 2 \tag{1}$$

This is known as difference equation. More genearally,

$$a_n = f(a_{n-1}) \tag{2}$$

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What is difference between difference equations (DCs) and differential equations (DEs)?

DCs deal with finitely-spaced steps, while DEs are based on continuous changes in the variables. Example of DCs is as in equation (2) .

The world of high finance

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The Solution Compound interest differ from the simple interest...how? If a_n is current amount, where n is our index indicating how many compounding periods have gone by since the money was deposited at $n = 0$. 12% was the advertise price so $i = .12$. Finally since the bank divides the advertised rate by the number of yearly compounding periods, introduce $N = 12$.

$$a_n = a_{n-1} + \frac{i}{N} a_{n-1} \tag{7}$$

In more generalization sense

$$a_n = \left(1 + \frac{i}{N}\right)^n a_0 \tag{8}$$

If amount is a function of some other time variables- for instance a_t where t is measured in years. Since interest is compounded N times in a year, $n = Nt$ and

$$a_t = (1 + \frac{i}{N})^{Nt} a_0 \tag{9}$$

Interest with Money Deposits/Withdrawl

Up until now, we have assumed that the amount of money in a particular account is not touched by its owner-the money is simple left in the account to accumulate interest.

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$$a_n = \left(1 + \frac{i}{N}\right) a_{n-1} + b \tag{14}$$

The solution is

$$a_n = \left(1 + \frac{i}{N}\right)^n \left(a_0 + \frac{Nb}{i}\right) - \frac{Nb}{i} \tag{15}$$

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Solution : He placed $\$6000$ into the account. Finally he had $\$9402.21$.

Problem of Fibonacci's rabits: Assume that each pair of rabbits breeds twice, once at age 1 month and once at age 2 months. Also assume that each mating produces 1 pair each time. Let a_n be the number of newborn pairs at the n^{th} month, beginning with one newborn pair $(a_0 = 1)$. This in the first month one pair will be born $(a_1 = 1)$, and the following month two newborn pairs will be born-one from the first (a_0) pair, and one from the second a_1 pair:

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$$\begin{aligned} a_2 &= a_0 + a_1 = 2 \\ a_3 &= a_1 + a_2 = 3 \\ a_2 &= a_2 + a_3 = 5 \end{aligned} \tag{18}$$

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\end{aligned}
$$

The DCE for all $n \geq 2$

$$a_n = a_{n-1} + a_{n-2} \qquad (21)$$

Linear and Nonlinear operations

Consider some unknown operator Φ applied to a variable or function x. The Φ is only a linear operations if the following holds true.

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