Math 2E03- Introduction to Modelling

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Multiple Compartments

Multiple compartments are simply chains of compartments (perhaps laked joined by rivers) whose outflow is the inflow of another comaprment. For instance, for three compartments we have where r_1 and c_1 are the input flow and particle concentartion into compartment A, and r_2 and c_A the output flow and concentration itself. We can see thatb r_2 and c_A are the input flow and concentration for compartment B, and r_3 and c_B are the input flow and concentration for compartment C, and r_4 and c_C are the input flow and concentration for compartment C. Each individual compartment can be modelled by our standard compartmental differential equation as though it were an independent compartment, but these are actually joined as a sysrem of ODEs since C is affected by the concentration of B, which in turn is affected by the concentration of A.

Thus we have

$$
\begin{aligned}
\frac{dA}{dt} &= r_1 c_1 - r_2 \frac{A}{V_A(t)} \\
\frac{dB}{dt} &= r_2 \frac{A}{V_A(t)} - r_3 \frac{B}{V_B(t)} \qquad (1) \\
\frac{dC}{dt} &= r_3 \frac{B}{V_B(t)} - r_4 \frac{C}{V_C(t)}
\end{aligned}
$$

Problem : Leads enters the body in food, air and water contaminated by automobiles and industrial emissions. It accumulates in the blood, in tissues and espically in the bones. Some lead is excerted through the uniary system and by hairs,nails and sweat , but enough remain in the body to impair mental and motor capacity . A model of this system can be obtained by tracking the concentration of lead in different parts of the body over time.Assuming that the body can be modelled by three compartments -blood, tissues and bones build a mathematical model to track the concentration of lead in the different part of the body. Assume that the lead can enter and leave only by the blood system.

Solution : Let E=the concentartion of lead in environment.

B=the mass of lead in the blood.

T=the mass of lead in Tissues.

O=the mass of lead in Bones.

g=input flow rate

a=from B to T, b=from T to B, c= from T to O, d=from O to T, e=from O to B, f=from B to O

h=output flow rate

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$$\frac{dB}{dt} = gE + \frac{bT}{V_T} + \frac{eO}{V_O} - \frac{(a+f+h)B}{V_B}$$

$$\frac{dT}{dt} = \frac{aB}{V_B} + \frac{dO}{V_O} - \frac{(c+b)T}{V_T}$$

$$\frac{dO}{dt} = \frac{cT}{V_T} + \frac{fB}{V_B} - \frac{(e+d)O}{V_O}$$

Multiple Species (Using chain decay Kinetics)

Here we are considering the chain decay, for instance -one radioisotopes decays to another, which in turn decays to another, and so on until a stable isotopes is reached. Each type of isotopes may be thought of as a compartment.

X represent the radioisotope, Y represent unstable isotope obtained by X and Z is stable isotope obtanied by Y.

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$$\frac{dX}{dt} = -\lambda_X X$$
$$\frac{dY}{dt} = \lambda_X X - \lambda_Y Y \qquad (3)$$
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The solution is

$$
\begin{aligned}
X &= X_0 e^{-\lambda_X t} \text{ Using } X(0) = X_0 \\
Y &= \frac{X_0 \lambda_X}{\lambda_Y - \lambda_X}(e^{-\lambda_X t} - e^{-\lambda_Y t}) \text{ Using } Y(0) = 0
\end{aligned}
\tag{5}
$$