Math 2E03- Introduction to Modelling

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- Extend the last model to include competition between two populations of algae that vie for the same food source. And what qualitative information can you determine about the model?.

- Sol.: We introduce a new species, H, to our model and assume that H exists under the same conditions that V does then mathematical equation for this model will look like:

$$\dot{V} = rV - gV^2 - cV - fH$$
$$\dot{H} = r^{'}H - g^{'}H^2 - c^{'}H - f^{'}V \tag{1}$$

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$$\begin{aligned} \dot{V} &= rV - gV^2 - cV - fH \\ \dot{H} &= r^{'}H - g^{'}H^2 - c^{'}H - f^{'}V \end{aligned} \tag{2}$$

We can make some assumption regarding the parameters in this system. First off, it is logical to assume that the c and $c^{'}$ parameters are roughly the same -the water carries them off equally (this would not be the case if one of the species was better able to grip onto the surface of the rock or plant). If we further assume that the intra-competitive terms g and $g^{'}$ are roughly the same, and the inter competitive terms f and $f^{'}$ are also roughly the same. Then we see that the key determining factor is their growth rates r and $r^{'}$.

If these are the same and the the initial population are the same, then they should be able to coexist with one another. A significant difference in growth rates and/or large difference in initial populations would result in the more advantaged species taking over while the other dies out.

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Warfare and other amusing pasttimes: (Combat situation)

There is nothing really special about modelling warfare-this is simply example of the interactions of two or more different species, with positive and negative effects taken into account.

Problem: In the Vietname war a guerrilla force (a member of an irregular force that fights a stanger force by harassment) opposed a conventional force (Those foeces capable of conducting operations using nonnuclear weapons). Build a system of differential equations to model the interaction between the two forces. What can you say about the equilibria of this system. Conventional wisdom indicates that a conventional force can defeat a guerrilla force only if the initial ratio of former to the later is significantly greater than 1. Investigate this notion using your model.

Sol.: We think of the conventional forces as being essentially out in the open, wanting to shoot anyone from the opposing team, while the guerrillas skulk about in the bushes, picking off conventional. Thus the more guerrillas there are, the more conventional will be dying. If c represent the conventional forces and g the guerrilla forces then mathematical model will be

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$$\dot{g} = -Bcg$$

The first ODE equals zero only if g is 0, while the second equals 0 if either c or g is 0. Thus our equilibria, where both equal zero , is again a line: the line $g = 0$. From a practical sense, this means that there can only be stability if there are no guerrillas.

Finding the relation between g and c

$$\frac{dc}{dg} = \frac{A}{Bc}$$

Solution of this differential equation is $g = \frac{1}{2}\frac{B}{A}c^2 + K$.

Let the initial numbers of conventional and guerrilla soldiers be c_0 and g_0, respectively. Also let $h = \frac{B}{2A}$.

$$\implies g = h(c^2 - c_0^2) + g_0$$

Now for conventional force to win, we must have some left standing after the guerrilla forces are wiped out. In other words, $c > 0$ when $g = 0$. We will get the condition $\frac{A}{Bg_0} >> \frac{1}{2}$ to win the conventional force.

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