Math 2E03- Introduction to Modelling

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We now seek a solution involving the linear combination of solutions

$$U(x,t) = \sum_{n=1}^{\infty} \left[A_n \sin(\frac{n\pi}{L}ct) + B_n \cos(\frac{n\pi}{L}ct)\right] \sin\frac{n\pi}{L}x$$

Let's impose our initial condition

$$U(x,0) = \sum_{n=1}^{\infty} \left[0 + B_n \cos(\frac{n\pi}{L}t)\right] \sin\frac{n\pi}{L}x$$

$$f(x) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi}{L}x$$

Now imposing another initial condition

$$\frac{\partial U(x,t)}{\partial t} = \sum_{n=1}^{\infty} \left[A_n \left(\frac{nc\pi}{L}\right) \cos\left(\frac{nc\pi}{L}t\right) - B_n \left(\frac{nc\pi}{L}\right) \sin(\frac{n\pi}{L}t) \right] \sin\frac{n\pi}{L}x$$

$$\frac{\partial U(x,0)}{\partial t} = \sum_{n=1}^{\infty} \left[A_n \left(\frac{nc\pi}{L}\right) - 0 \right] \sin\frac{n\pi}{L}x = 0$$

$$\implies A_n = 0$$

We now have our actual solution

$$U(x,t) = \sum_{n=1}^{\infty} \left[B_n \cos(\frac{n\pi}{L}t) \right] \sin\frac{n\pi}{L}x$$

$$f(x) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi}{L}x$$

The former is our big solution and the latter allows us to find B_n if we know $f(x)$.