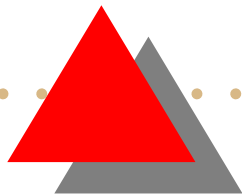




*Math 2E03- Introduction to  
Modelling*

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We now seek a solution involving the linear combination of solutions

$$U(x, t) = \sum_{n=1}^{\infty} \left[ A_n \sin\left(\frac{n\pi}{L}ct\right) + B_n \cos\left(\frac{n\pi}{L}ct\right) \right] \sin \frac{n\pi}{L}x$$

Let's impose our initial condition

$$U(x, 0) = \sum_{n=1}^{\infty} \left[ 0 + B_n \cos\left(\frac{n\pi}{L}t\right) \right] \sin \frac{n\pi}{L}x$$

$$f(x) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi}{L}x$$





Now imposing another initial condition

$$\frac{\partial U(x, t)}{\partial t} = \sum_{n=1}^{\infty} \left[ A_n \left( \frac{n\pi}{L} \right) \cos \left( \frac{n\pi}{L} t \right) - B_n \left( \frac{n\pi}{L} \right) \sin \left( \frac{n\pi}{L} t \right) \right] \sin \frac{n\pi}{L} x$$

$$\frac{\partial U(x, 0)}{\partial t} = \sum_{n=1}^{\infty} \left[ A_n \left( \frac{n\pi}{L} \right) - 0 \right] \sin \frac{n\pi}{L} x = 0$$

$$\implies A_n = 0$$

We now have our actual solution

$$U(x, t) = \sum_{n=1}^{\infty} \left[ B_n \cos \left( \frac{n\pi}{L} t \right) \right] \sin \frac{n\pi}{L} x$$

$$f(x) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi}{L} x$$

The former is our big solution and the latter allows us to find  $B_n$  if we know  $f(x)$ .

