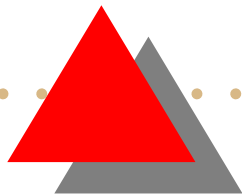




*Math 2E03- Introduction to
Modelling*

Instructor– Dr. Mani Mehra

Department of Mathematics and Statistics
McMaster Univ.





Conti. from previous lecture


What will be the pop. after t years. We start with the change in pop. over 1 year, $N_1 = LN_0$ and deduce the long term description

(1)
$$N_t = L^t N_0$$

To understand the long-term dynamics, we need a simple expression for $L^t N_0$.

we can express the pop. after an arbitrary number of years, $L^t N_0$, in a simple form using eigenvectors and eigenvalues. Let λ_0 and λ_1 be the two eigenvalues and v_0 and v_1 be the two corresponding eigenvectors. First to express the initial pop. size as the sum of the two eigenvectors

(2)
$$N_0 = a_0 v_0 + a_1 v_1$$



Using this expression for the initial pop., we can find the pop. after t years as

$$\begin{aligned} N_t &= L^t N_0 = L^t (a_0 v_0 + a_1 v_1) \\ (3) \quad &= a_0 L^t v_0 + a_1 L^t v_1 \\ &= a_0 \lambda^t v_0 + a_1 \lambda^t v_1 \end{aligned}$$

Assume that we have labeled the eigenvalues so that $|\lambda_0| > |\lambda_1|$, then λ_0 is what we have been calling the growth rate of the pop. After a long time, as t become large, the term $a_0 \lambda^t v_0$ in the expression eq. (3) for the pop. size after an arbitrary number of years, will be much larger than the term $a_1 \lambda^t v_1$. In fact ignoring the term, we will have

$$N_t \approx a_0 \lambda^t v_0$$



Iteroparity versus semelparity

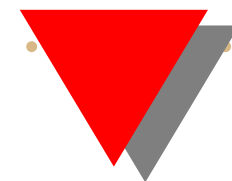
An organism that reproduces just once during its lifetime is called semelparous, while an organism that reproduces many times is called iteroparous. Examples of semelparous organism are salmon and periodical cicadas. Oak trees, humans, and turtles are examples of iteroparous species.



More general age class

We will consider the more general case where there is an arbitrary number of age classes, noting only that the length of each age class must be the same. We call the description of age-specific birth and death rates the life table. The quantities that we will use in the life table are as follow:

- l_x is the fraction of newborn individuals that survive to age x .
- S_x is the probability of surviving from age x to $x + 1$.
- m_x is the average birth rate for an individuals of age x .
- $B(t)$ is the total birth rate in the population. To find the total birth rate we sum the births to individuals of all possible ages.



This leads to

$$(4) \quad B(t) = \int_0^{\infty} (\text{rate of births to parents of age } x) dx$$

We now determine the birth rate in equation (4), using the following reasoning.

- Rate of births to parents of age $x = m(x)$ times number of individuals of age x .
- Number of individuals of age $x =$ number of births x years ago times fraction of newborn who survive to age x .
- Number of individuals of age $x = B(t - x)l(x)$

We substitute into equation (4) to obtain a single equation relating the birth rate now to the birth rates in the past

$$(5) \quad B(t) = \int_0^{\infty} B(t - x)l(x)m(x)dx$$

