



*Math 2E03- Introduction to  
Modelling*

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*Conti. from previous lecture*


We solve this equation under the assumption that the pop. is in a stable age distribution. Thus, we assume that growth is exponential at the rate  $r$ , where we again use the symbol  $r$  to represent the intrinsic rate of increases of the pop. This can be expressed as

$$(1) \quad B(t) = e^{rt} B(0)$$

Now using (5) from the previous lecture and (1) we get the Euler's equation

$$(2) \quad 1 = \int_0^{\infty} e^{-rx} l(x) m(x) dx$$

Note that this equation can be used to find  $r$  if we know the life table for a particular organism.



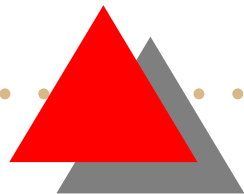
However, the birth rates and survival probabilities cannot be measured over continuous time, only at discrete intervals. When this pop. is used to estimate  $r$ , ecologists usually use a discrete approximation obtained by replacing the integral with a sum


$$(3) \quad 1 = \sum_{x=0}^{\infty} e^{-rx} l_x m_x$$

An estimate for  $r$  can be found from this equation numerically.

**Net productive rate** :: There is another quantity that is often calculated from the life table, the net production rate  $R_0$  which is the average total number of offspring produced by a single individual in her lifetime.

$$(4) \quad R_0 = \int_0^{\infty} l(x) m(x) dx$$





**Stable age distribution ::** Denote by  $c(x)$  a density function for the fraction of individuals of age  $x$  in the stable age distribution. This means that the fraction of individuals between ages  $x$  and  $x + dx$  is given by  $\int_x^{x+dx} c(z)dz \approx c(x)dx$ .

Thus

$$(5) \quad c(x) = \frac{\text{number of individuals of age } x}{\text{total number of individuals}}$$

Where the total number of individuals is given by

$$(6) \quad \int_0^{\infty} (\text{number of individuals of age } x)dx$$



which will be

$$(7) \quad c(x) = \frac{B(t-x)l(x)}{\int_0^\infty B(t-z)l(z)dz} \\ \approx \frac{e^{-rx}l_x}{\sum_0^\infty e^{-rx}l_x}$$

Here we have used the age specific birth and death rates to calculate the growth rate of a pop. For many organism this is an appropriate approach, but for another organism it is not. For example, many insects go through a series of developmental stages (egg, larva, pupa and adult) that vary in duration according to food availability, temperature and moisture.