Math 2E03- Introduction to Modelling

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Conti. from previous lecture

We solve this equation under the assumption that the pop. is in a stable age distribution. Thus, we assume that growth is expontial at the rate r, where we again use the symbol r to represent the intrinsic rate of increses of the pop. This can be expressed as

$$B(t) = e^{rt}B(0) \tag{1}$$

Now using (5) from the previous lecture and (1) we get the Euler's equation

$$1 = \int_0^\infty e^{-rx} l(x) m(x) dx \tag{2}$$

Note that this equation can be used to find r if we know the life table for a particular organism.

However, the birth rates and survival probabilities cannot be measured over continuous time , only at discrete intervals. When this pop. is used to estimate r, ecologist usually use a discrete approximation obtained by replacing the integral with a sum

$$1 = \sum_{x=0}^{\infty} e^{-rx} l_x m_x \tag{3}$$

An estimate for r can be found from this equation numerically.

Net productive rate :: There is another quantity that is often calculated from the life table, the net production rate R_0 which is average total number of offspring produced by a single individual in her lifetime.

$$R_0 = \int_0^{\infty} l(x)m(x)dx \tag{4}$$

Stable age distribution :: Denote by $c(x)$ a densuty function for the fraction of individuals of age x in the stable age distribution. This means that the fraction of individuals between ages x and $x + dx$ is given by $\int_x^{x+dx} c(z)dz \approx c(x)dx$.

Thus

$$c(x) = \frac{\text{number of individuals of age } x}{\text{total number of individuals}} \tag{5}$$

Where the total number of individuals is given by

$$\int_0^{\infty} (\text{number of individuals of age } x)dx \tag{6}$$

which will be

$$c(x) = \frac{B(t-x)l(x)}{\int_0^\infty B(t-z)l(z)dz} \approx \frac{e^{-rx}l_x}{\sum_0^\infty e^{-rx}l_x} \tag{7}$$

Here we have used the age specific birth and death rates to calculate the growth rate of a pop. For many organism this is an appropriate approach, but for another organism it is not. For example, many insects go through a series of developmental stages (egg, larva, pupa and adult) that vary in duration acording to food availablity, temprature and moisture.