Math 2E03- Introduction to Modelling

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Graphical approach to represent Logistic model

This approach has the advantage of displaying the dynamics for all the values of the pop. density and of displaying easily the effect of small changes in the model. We graph (Given in class) the rate of change of the pop. $\frac{dN}{dt}$ against the pop. size N. From the figure we can easily find equilibria. We can also determine stability and conclude small changes in the model do not lead to qualitative changes in the behavior.

Cyclic behavior means whose number increases and decreases in a relatively regular fashion. This type of behavior can not be explained by logistic model.

Logistic growth with lags

The temprateure in the room in which you are sitting is probably controlled by a thermostat. If the temprature gets too low, the heat immediately goes on, if it gets too high the heat goes off. The temprature remaines relatively constant. However if someone checked the temprature once per hour?????.

The simplest model incorporating delayed regulation is the equation

$$\frac{dN}{dt} = rN(t)[1 - N(t-T)/K] \tag{1}$$

Discrete time density dependence models

The time delayed logistic model led to oscillations, but even more complex behavior is possible for the equivalent models in discrete time. A class of models appropriate for describing animals that live 1 year, reproduce, and then die, takes the form.

$$N_{t+1} = F(N_t) \tag{2}$$

Where the function F gives the pop.numbers next year in terms of this year pop. This equation has an implicit time delay of 1 year We are bringing the effect of density dependence in this form of $F(N)$

$$F(N) = N \exp[r(1 - N/K)] \tag{3}$$

Now the model will look like

$$N_{t+1} = N_t \exp[r(1 - N/K)] \tag{4}$$

Least square method for curve fitting

Measuring the population at several different times can yield a set of data which we need to represent with an equation rather than as separate points. To do this we use a process called line or data fitting, and in this reading we will explain one of these methods, a process called linear least squares fitting. If the square of the deviations is minimized, the "best line" can be calculated.
(Mathematical explanation is given in the class)