

# M2E03 - Introduction to Modelling –Practice Problems

Solution will be given in the class as well as posted on the web

4. If a uniform stretched string vibrates in a fixed plane containing its equilibrium position, and the transverse displacement  $u$  of the string in this plane remains small, then  $u$  is the solution of wave equation under the following set up

$$u_{tt} = c^2 u_{xx} \text{ where } c = \sqrt{T/\sigma}, \text{ with } T \text{ tension and } \sigma \text{ is linear density}$$
$$u(x, 0) = x, u_t(x, 0) = 0$$

and string of finite length  $L$  be clamped rigidly at each end. Then find out the solution of this model.

5. Consider a large tank holding 1000L of pure water into which a brine solution of salt begins to flow at a rate of 6L/min. The solution inside the tank is kept well stirred and is flowing out of the tank at a rate of 5L/min. If the concentration of salt in the brine entering the tank is 1kg/L, determine the concentration of salt in the tank as a function of time.
6. In 1790 the population of the United states was 3.93 million, and in 1890 it was 62.95 million. Now assume that US population predict the exponential growth. Therefore find out the US pop. as a function of time.
7. Suppose a fish hatchery removes a fixed number  $p$  of fishies every generation (the fish have a natural growth rate of  $r$ ). Is it possible for the population to remain the same from generation to generation?. If so, what is this population in terms of the stated parameters? Given an initial population  $a_0$ , drive an expression for the population of the  $n^{th}$  generation.
8. Carbon dating is often used to determine the age of a fossil. For example, a humanoid skull was found in a cave in South Africa along with the remains of campfire. It is determined that only 2% of the original amount of carbon-14 remains in the burnt wood of the campfire. Estimate the age of skull if the half life of carbon-14 is about 5600 years.
9. A white wine at room temperature  $70^\circ F$  is chilled in ice ( $32^\circ F$ ). If it takes 15 min for the wine to chill to  $60^\circ F$ , how long will it take for the wine to reach  $56^\circ F$ .