

CYL 110: 2009-2010 Thermodynamics Tutorial 2
18th – 22nd January 2010

1. Derive a general relation between C_p and C_v to get the general result

$$C_p - C_v = \left[P + \left(\frac{\partial U}{\partial V} \right)_T \right] \left(\frac{\partial V}{\partial T} \right)_p$$

2. Show for a van der Waals gas $C_p - C_v = \alpha^2 TV / \kappa_T$. You will need to use an identity (to be derived later in this course)

$$\pi_T = T \left(\frac{\partial P}{\partial T} \right)_V - P$$

3. A mole of ideal gas is compressed adiabatically to one-half of its original volume when the temperature of the gas is increased from 273 K to 433 K. Assuming \bar{C}_v is independent of the temperature, calculate the value of \bar{C}_v for this gas.

4. Show that the work involved in a reversible, adiabatic pressure change of one mole of an ideal gas is given by

$$w = c_v T_1 \left[\left(\frac{P_2}{P_1} \right)^{R/C_p} - 1 \right]$$

where T_1 is the initial temperature and P_1 and P_2 are the initial and final pressures, respectively.

5. A sample of 1.00 mol perfect gas molecules with $C_{p,m} = 7/2R$ is put through the following cycle: (a) Constant volume heating to twice its initial volume, (b) Reversible, adiabatic expansion back to its initial temperature, (c) reversible isothermal compression back to 1.00 atm. Calculate q , w , ΔU , and ΔH for each step and overall.

6. Take nitrogen to be a van der Waals gas with $a = 1.390 \text{ dm}^6 \text{ atm mol}^{-2}$ and $b = 0.03913 \text{ dm}^3 \text{ mol}^{-1}$, and calculate ΔH_m when the pressure on the gas is decreased from 500 atm to 1.00 atm at 300 K. For a van der Waals gas, $\mu_{JT} = \{(2a/RT) - b\}/C_{p,m}$. Assume $C_{p,m} = 7/2R$.

7. (a) What is the total differential of $z = x^2 + 2y^2 - 2xy + 2x - 4y - 8$? (b) Show that $\partial^2 z / \partial y \partial x = \partial^2 z / \partial x \partial y$ for this function. (c) Let $z = xy - y + \ln x + 2$. Find dz and show that it is exact.

8. Calculate the work done during the isothermal reversible expansion of a gas that satisfies the virial equation of state, $P\bar{V} = RT + B\left(\frac{1}{\bar{V}}\right) + C\left(\frac{1}{\bar{V}}\right)^2 + \dots$. Evaluate (a) the work for 1.0 mol Ar at 273 K ($B = -21.7 \text{ cm}^3 \text{ mol}^{-1}$) and (b) the same amount of a perfect gas. Let the expansion be from 500 cm^3 to 1000 cm^3 in each case.

9. A gas obeys the equation of state $V_m = RT/p + aT^2$ and its constant-pressure heat capacity is given by $C_{p,m} = A + BT + Cp$, where a , A , B , and C are constants independent of T and p . Obtain expressions for (a) the Joule–Thomson coefficient and its constant-volume heat capacity.

10. Calculate the heat needed to raise the temperature of air in a house from 20 to 25 °C. Assume that the house contains 600 m^3 of air, which should be taken to be a perfect diatomic gas. The density of air is 1.21 kg m^{-3} at 20 °C Calculate ΔU and ΔH for the heating of the air.

11. Find the maximum inversion temperature of the gas whose equation of state is $\left(P + \frac{x}{TV^2}\right)(V - y) = RT$ where x and y are constants.

12. When a fluorocarbon gas was allowed to expand reversibly and adiabatically to twice its volume the temperature fell from 298.15 to 248.44 K and its pressure fell from 1522.2 Torr to 613.85 Torr. Evaluate C_v and C_p . You may assume that the gas behaves perfectly.

13. Verify that the enthalpy of a perfect gas is independent of its pressure while it is dependent upon pressure for a van der Waals gas. [Use the approximate form of van der Waals equation $PV = nRT + nPb - (a/RT)$].

14. An important application of adiabatic cooling/heating is in atmospheric physics. Pressure varies with altitude as $dP/dh = -g\rho$, where ρ is the density. Calculate the change in temperature at the top of a mountain 2 km above the valley floor assuming the process is adiabatic and air behaves ideally.

15. Which of these cyclic integrals must vanish for a closed system with P-V work only? (a) $\oint PdV$, (b) $\oint (PdV + VdP)$, (c) $\oint VdV$, (d) $\oint HdT$, (e) $\oint dq_{rev}$, (f) $\oint dq_{rev}/P$.