CYL110 2009-2010 Quantum Tutorial 1

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1. The Planck blackbody spectrum giving the energy density ($\rho(\nu)$) in the frequency range ν and $\nu + d\nu$ may be written as $\frac{8\pi\nu^2}{c^3}\frac{h\nu}{\exp(\frac{h\nu}{kT})-1}$. Use this to determine the energy density in the wavelength range λ and $\lambda + d\lambda$.

2. Convince yourself that the high frequency limit of the energy density agrees with experimental observations.

3. Derive the Wien displacement law for the wavelength at which the blackbody energy density is a maximum: $\lambda_{\text{max}}T = \text{constant}$. (Hint: You will need to solve a transcendental equation $(5 - x) = 5\exp(-x)$ which you could using the "method of successive approximations.")

4. Derive Stefan-Boltzmann formula for the total energy density in blackbody radiation:
$$\frac{U}{V} = \left(\frac{\pi^2 k^4}{15\hbar^3 c^3}T^4\right).$$
You will need to look up (unless, of course, you know how to evaluate it!) an integral of the type
$$\int_0^\infty \frac{x^s - 1}{\exp(x) - 1}dx.$$

5. According to the Einstein model the total molar energy of a solid is
$$U_m = \frac{3N_A h\nu}{\exp(\frac{h\nu}{kT}) - 1}.$$
Obtain the high-temperature and low-temperature limit of this expression and compare your result with equations 8.8a and 8.8b of Atkins. What would the heat capacity be in each of these limits?

6. What are the results of operating on the following functions with the operator d/dx and d^2/dx^2: (a) $\exp(-ax^2)$, (b) $\cos(bx)$, (c) $\exp(ikx)$? Which functions are eigen functions of these operators? What are the corresponding eigen values?

7. Which of the following operators are linear? (a) d/dx; (b) $\sqrt{}$; (c) exponentiation; (d) integration.

8. Find the square of the operator $d/dx + \hat{x}$.

9. In algebra it can be easily shown that $(P + Q)(P - Q) = P^2 - Q^2$. What is the value of $(P+Q)(P-Q)$ if P and Q are operators? Under what conditions will this result be equal to $P^2 - Q^2$.

10. Find $[z^3, d/dz]$ and $[d^2/dx^2, ax^2 + bx + c]$.

11. Which of the following functions cannot be solutions of the Schrödinger equation for all values of x? Why not? (a) $A\sec(x)$; (b) $A\tan(x)$; (c) $A\exp(x^2)$; (d) $A\exp(-x^2)$.

12. The possible values obtained from a measurement of a discrete variable, x, are 1, 2, 3, and 4. (a) If the respective probabilities are 1/4, 1/4, 1/4, and 1/4, calculate the expectation values of x and x^2. (b) If the respective probabilities are 1/12, 5/12, 5/12, and 1/12, calculate the expectation values of x and x^2.

13. Determine the probability density of a particle as a function of its position if its wave function is $A\exp(ikx)$. What is the value of its momentum?

14. Normalize the following wave functions to unity: (a) $\sin(n\pi x/L)$ for the range $0 < x < L$, (b) c, a constant in the range $-L < x < L$, (c) $\exp(-r/a_0)$ in three dimensions, (d) $x\exp(-r/2a_0)$ in three dimensions.

15. Show that the wave functions $\psi_1(x) = \sin(n\pi x/L)$ and $\psi_2(x) = \cos(n\pi x/L)$, where n is a non-zero positive integer and L is a constant are orthogonal. The permitted values of x are $0 \le x \le L$.

16. Write down the Hamiltonian for the following systems: (a) a particle of mass m in a cubical box of side a; (b) a particle of mass m in a spherical box of radius a; (c) a particle of mass m moving on the x-axis subjected to a force directed towards the origin, of magnitude proportional to the distance from the origin; (d) an electron moving in the presence of a nuclear charge $+Ze$; (e) two electrons moving in the presence of a fixed nucleus of charge $+Ze$.