CYL110 2009-2010 Quantum Tutorial 3

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1. Calculate the moment of inertia of $H^{35}Cl$, $H^{37}Cl$, and $D^{35}Cl$ all of which have an equilibrium bond length of 1.275 Å. Determine the energies of the first three rotational states for $H^{35}Cl$. Use this information to predict these values for $D^{35}Cl$.

2. Show by direct operation that the functions $\sin\theta\exp(i\phi)$, $\sin\theta\exp(-i\phi)$, and $\cos\theta$ are eigen functions of $\hat{L_z}$. What are the eigen values?

3. Use the operator for $\hat{L^2}$ in polar coordinates to show that the function $(3\cos^2\theta - 1)$ is an eigen function of this operator. What is the eigen value? What is the quantum number l for this function?

4. The normalization of $Y_l^m(\theta,\phi) = NP_l^m(\cos\theta)\exp(im\phi)$ is performed as follows:

$$N^2\int_{\theta=0}^{\pi}(P_l^m)^*P_l^m\sin\theta d\theta\int_{\phi=0}^{2\pi}\exp(-im\phi)\exp(im\phi)d\phi = 1,$$

   while its orthogonality to $Y_{l'}^{m'}$ implies that

$$\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}Y_l^m(Y_{l'}^{m'})^*\sin\theta d\theta d\phi = 0.$$

   (a) Show that $Y_1^{-1}(\theta,\phi)$ is normalized and it is orthogonal to $Y_2^1(\theta,\phi)$.

   (b) Determine the normalization constant for the function given in problem 2.

5. For angular momentum with quantum number $l = 3$, how many m-values are there? What is the semi-angle of the cone subtended by the angular momentum vector if its z-projection is $2\hbar$?

6. From the definition of angular momentum, $\vec{L} = \vec{r}\times\vec{p}$, and following the procedure outlined in the class obtain the Cartesian form of the y-component of the angular momentum operator.

7. Show that $[\hat{L_y}, \hat{L_z}] = i\hbar\hat{L_x}$. (Hint: Use the information from the previous problem.)

8. Show that $[\hat{L_x}, y] = i\hbar z$.

9. Determine the positions of the first three rotational transitions for $H^{35}Cl$ and $D^{35}Cl$.

10. In the far infrared spectrum of $H^{79}Br$, there is a series of lines separated by 16.72 cm^{-1}. Calculate the values of the moment of inertia and the internuclear separation in $H^{79}Br$.

11. The $J = 0 \rightarrow J = 1$ line in the microwave absorption spectrum of $^{12}C^{16}O$ and of $^{13}C^{16}O$ was found to be at 3.84235 cm^{-1} and 3.67337 cm^{-1}. Calculate (a) the bond length of $^{12}C^{16}O$, (b) the relative atomic mass of ^{13}C.