CYL110 2009-2010 Quantum Tutorial 4

Instructors: Chowdhury/Kurur/Sapra

1. A hydrogen-like atom can be formed from a proton and a negative muon whose mass is approximately 206 times that of the electron. What are the energies and most probable radius for the $1s$ and $2p$ levels of this atom?

2. Using the uncertainty principle argue that free electrons cannot exist in the nucleus. The diameter of a typical nucleus is 10^{-14} m.

3. For a hydrogen atom in the ground state find the classically forbidden region and calculate the probability of finding the electron in this region.

4. Compute the average value of r, the most probable value of r, and the root-mean-square value of r for the $1s$ and $2p$ levels of the hydrogen atom. Compare the three kinds of values and explain the origin of their differences.

5. Show that the hydrogenlike atomic wave function ψ_{210} is normalized and that it is orthogonal to ψ_{200}.

6. Calculate the probability that an electron described by a hydrogen $1s$ wave function will be found within one Bohr radius of the nucleus.

7. Prove that $< V >= 2 < E >$ and, consequently, that $< K > = - < E >$, for a $2s$ electron.

8. Compute $< r >$ in the $2s$, $2p$ states of the hydrogen atom. Compare your result with the general formula

$$< r_{nl} >= \frac{a_0}{2}\left[3n^2 - l(l+1)\right].$$

9. Where do the maxima in $r^2\psi_{2s}^2(r)$ occur?

10. What combinations of the d ($l = 2$) atomic orbitals will produce the Cartesian function $d_{xz} = xzR_{nl}(r)$ and $d_{xy} = xyR_{nl}(r)$.

11. If we were to ignore the inter-electronic repulsion in helium, what would be its ground state energy and wave function? The experimental ground state energy of He is -79.0 eV.