

Correction in the Hermite polynomial recursion relation

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$

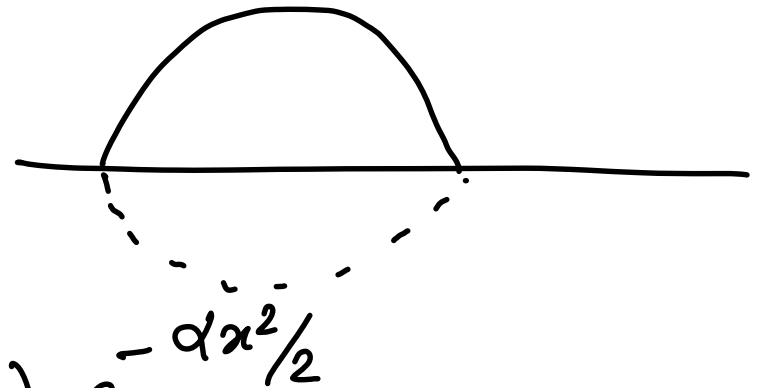
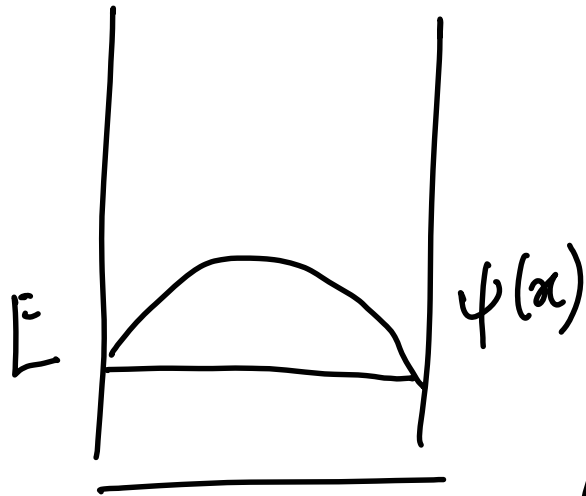
$$H_0(x) = 1$$

$$H_1(x) = 2x$$

o

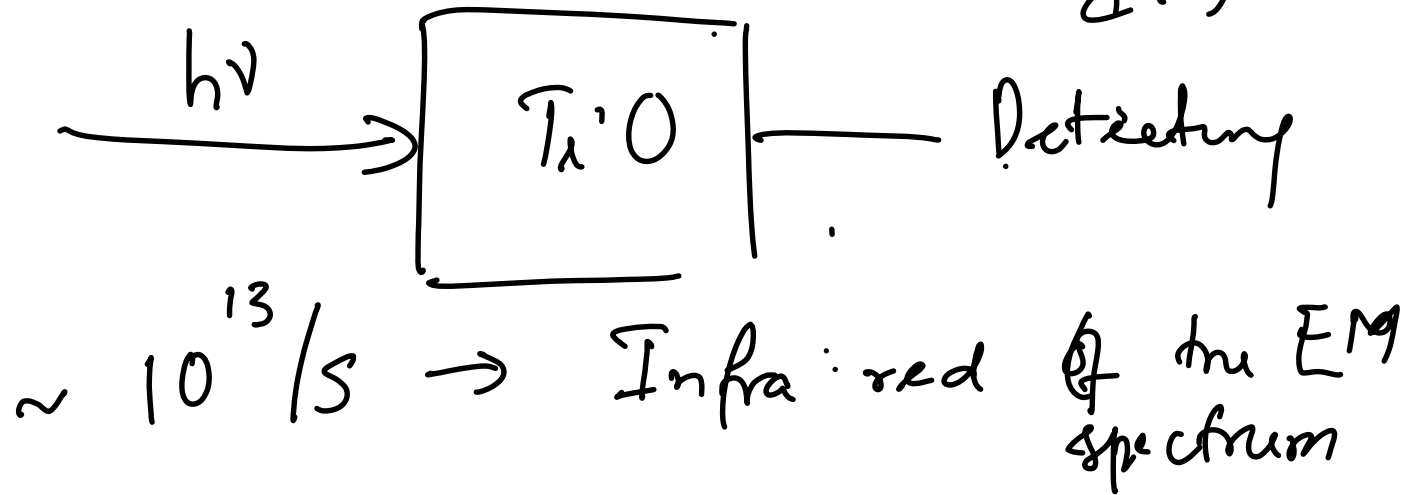
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$$(4x^2 - 2) e^{-\alpha x^2/2}$$

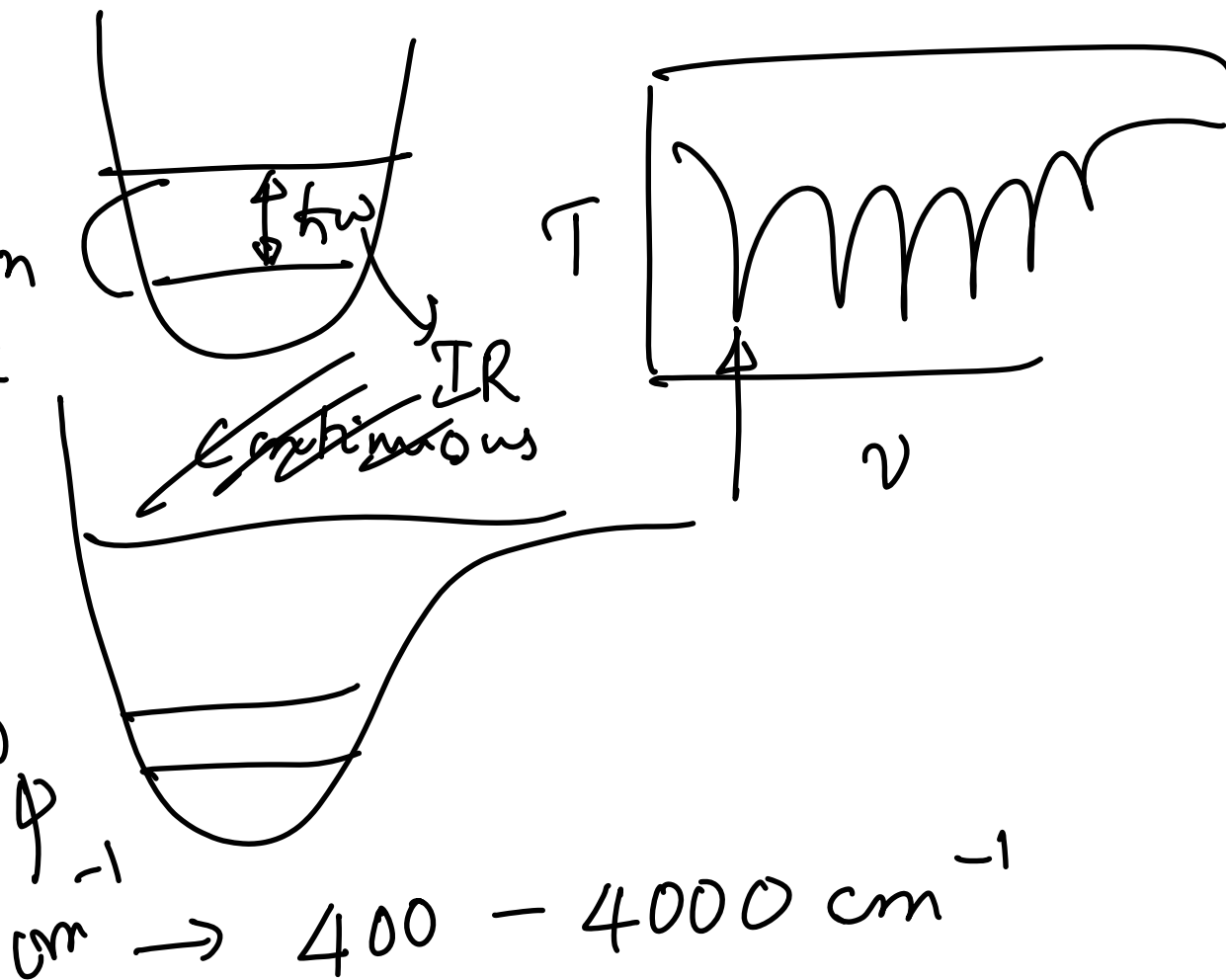
$k \sim 1000 \text{ N/m}$   
 $w = \sqrt{\frac{k}{\mu}}$



$\Delta l = \pm 1$   
 selection rule

$\omega = 2\pi\nu$

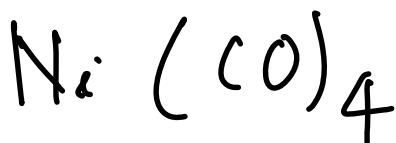
$\Delta E = \hbar\omega$   
 $= h\nu$   
 $= hc\tilde{\nu}$



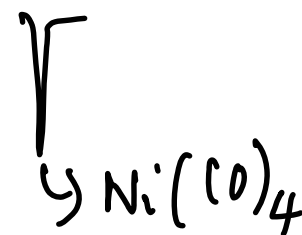


$$\omega = \sqrt{\frac{k}{\mu}}$$

4000 2100  $\text{cm}^{-1}$  ← 400



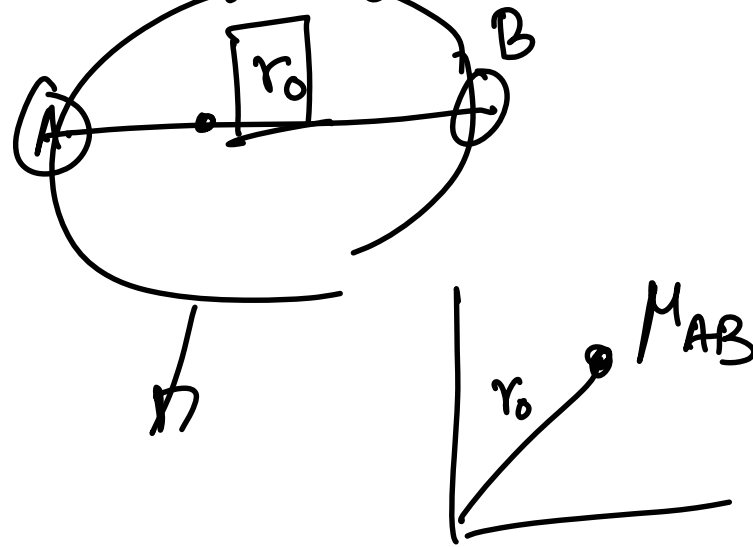
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# Rotational motion

Reduced mass at a fixed distance  
(equilibrium bond length) undergoing rotational motion



$$\mathcal{H} = \hat{T} + V = 0 + M$$

$$\hat{T} = \frac{\hat{p}^2}{2m} = \frac{\hat{L}^2}{2I} \quad I = \mu r_0^2$$

$$L^2 = \vec{L} \cdot \vec{L}$$

$$\vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$L_x, L_y, L_z$$

Verify at home!

$$\hat{L}_x = (y p_z - z p_y) -$$

$$\hat{L}_y = (z p_x - x p_z) -$$

$$\left[ i\hbar \frac{\partial}{\partial x}, x \right]$$

$$[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z$$

$$[p_x, x] = ?$$

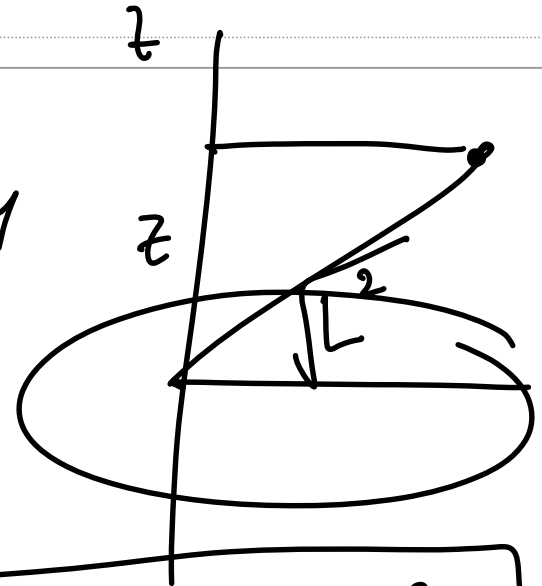
$$-i\hbar$$

cyclic permutations

$$[L_y, L_z] = i\hbar \hat{L}_x$$

$$[L^2, L_x] = 0 = [L^2, L_y] = [L^2, L_z]$$

$L^2$  &  $L_z$  → completely arbitrary



$$\hat{L} = \frac{\hat{L}_z}{z}$$

$$\hat{L}_z = -\frac{\hbar}{2} \left( \frac{\partial \sin \theta}{\partial \theta} \frac{\partial}{\partial \theta} \right) + \frac{1}{2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

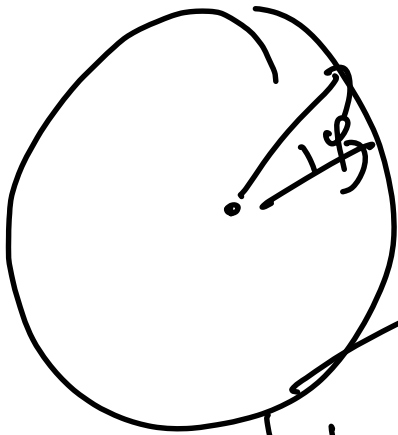
Motion confined to the xy-plane  $\sin \theta = 1$

$$L = L_z = -\hbar^2 \frac{\partial^2}{\partial \phi^2}$$



Rotor confined to a plane

$$\hat{H} = -\frac{\hbar^2}{2\mu r^2} \frac{\partial^2}{\partial \varphi^2}$$



$$\hat{H} \psi = E \psi$$

$\Delta L_z \Delta \varphi \sim \hbar/2$