$$Y_0^0 = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

$$Y_1^{-1}(\theta, \varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}} \cdot e^{-i\varphi} \cdot \sin\theta$$

$$Y_1^0(\theta, \varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}} \cdot \cos\theta$$

$$Y_1^1(\theta, \varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}} \cdot e^{i\varphi} \cdot \sin\theta$$

$$Y_2^{-2}(\theta, \varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}} \cdot e^{-2i\varphi} \cdot \sin^2\theta$$

$$Y_2^{-1}(\theta, \varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}} \cdot e^{-i\varphi} \cdot \sin\theta \cdot \cos\theta$$

$$Y_2^0(\theta, \varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}} \cdot (3\cos^2\theta - 1)$$

$$Y_2^1(\theta, \varphi) = \frac{-1}{2}\sqrt{\frac{15}{2\pi}} \cdot e^{i\varphi} \cdot \sin\theta \cdot \cos\theta$$

$$Y_2^2(\theta, \varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}} \cdot e^{2i\varphi} \cdot \sin^2\theta$$

Identify l and m in

$$\frac{3}{8}\sqrt{\frac{5}{2\pi}} \cdot e^{2i\varphi} \cdot \sin^2\theta \cdot (7\cos^2\theta - 1)$$

$$\frac{-3}{32}\sqrt{\frac{1001}{\pi}} \cdot e^{5i\varphi} \cdot \sin^5\theta \cdot \cos\theta$$

Given the θ part you can identify

l & $|m|$

$$L^2 Y_{\ell m} = \ell(\ell+1)\hbar^2 Y_{\ell m}$$

$$L_z Y_{\ell m} = m\hbar Y_{\ell m}$$

s, p, d, etc orbitals

H- atom

$$\hat{\mathcal{H}} = \hat{T}_{nuc} + \hat{T}_{el} + V$$

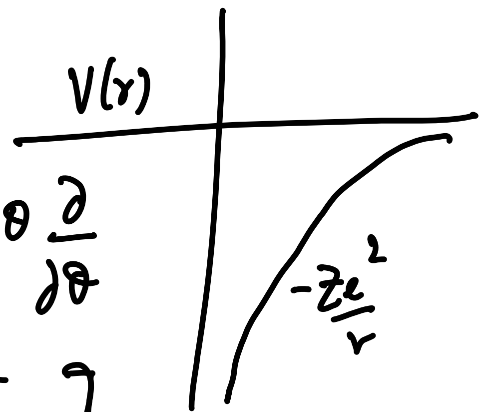
electrostatic e attraction

$$= \hat{T}_M + \hat{T}_\mu + V(|r_e - r_N|) \qquad \frac{1}{\mu} = \frac{1}{m_N} + \frac{1}{m_e}$$

$\hookrightarrow$ behaves like a free particle

$$\hat{\mathcal{H}} = -\frac{\hbar^2}{2\mu}\nabla_\mu^2 - \frac{Ze^2}{r} \qquad \frac{1}{4\pi\epsilon_0} = 1$$

$$\nabla_\mu^2 = \frac{1}{r^2}\left\{\left[\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}\right] + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right\}$$



Schrödinger equation

$$\hat{\mathcal{H}}\psi = \left(-\frac{\hbar^2}{2M}\left\{\nabla_\mu^2\right\} - \frac{Ze^2}{r}\right)\psi = E\psi$$

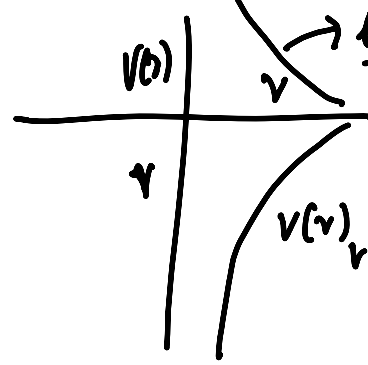
$$L^2 Y_{lm} = l(l+1)\hbar^2 Y_{lm}$$

$$\psi = \boxed{R(r)\, Y_{lm}(\theta,\varphi)}$$

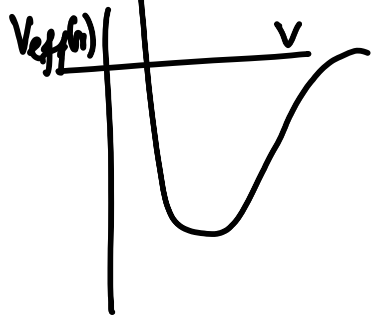
$$\hat{\mathcal{H}}\psi = \left(-\frac{\hbar^2}{2\mu r}\left\{\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}\right\} + \frac{\hat{L}^2}{2\mu r^2} - \frac{Ze^2}{r}\right)\psi = E\psi$$

$$\to \frac{l(l+1)\hbar^2}{r^2}$$

$$V_{eff}(r) = \frac{l(l+1)\hbar^2}{2\mu r^2} - \frac{Ze^2}{r}$$



$$V(r)\big|_{r=0} = -\infty$$



As $r \to 0$, $V_{eff}(r) \to +\infty$

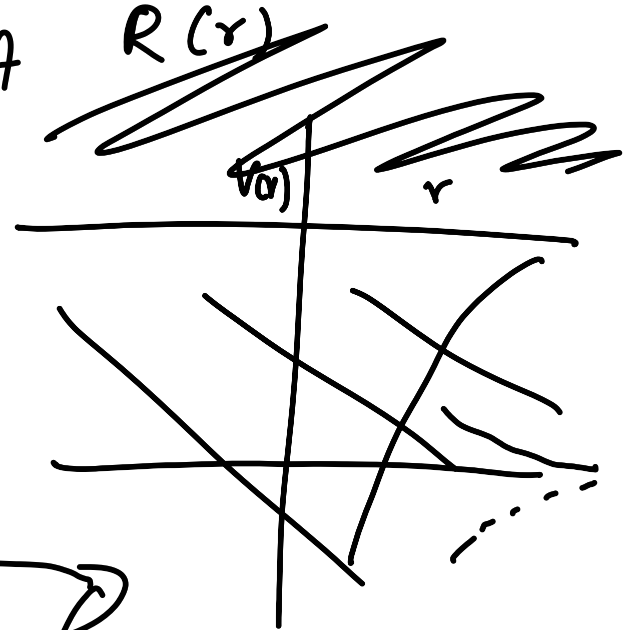
$R(r) \to 0$ as $r \to 0$

For $l = 0$ (spherically symmetric state)

$V = -\infty$ as $r \to 0$ so $R(r)$ is finite at $r = 0$

V_{eff}^{min} is a function of l

Qualitative form of $R(r)$



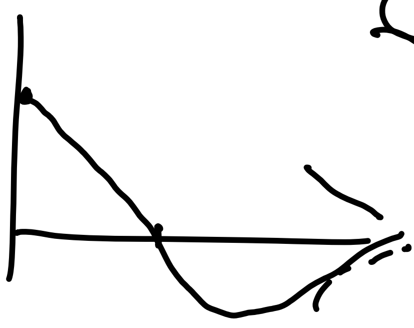
$\ell = 0$

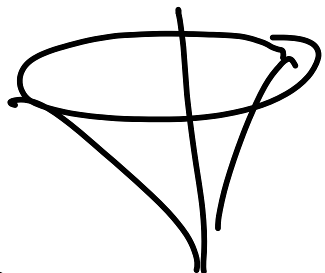
$E < 0$

g.s

$R(r)$

1^{st} excited state





$$R(r) = \frac{g(r)}{r}$$

$$\left\{ -\frac{\hbar^2}{2\mu r^2} \left[\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} \right] - \frac{Ze^2}{r} \right\} R(r) = E R(r)$$

Look for the solution at large r

$$R(r) = \boxed{\frac{g(r)}{r}}$$

$$-\frac{\hbar^2}{2\mu} \frac{d^2 g}{dr^2} - \frac{Ze^2}{r} g(r) = E\, g(r)$$

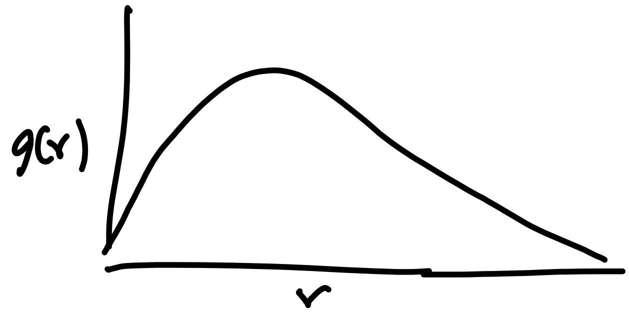
$$b^2 = -\frac{2\mu E}{\hbar^2} \; ; \; b = \frac{\sqrt{-2\mu E}}{\hbar}$$

$$g(r) \sim A e^{-br} + \cancel{B e^{+br}}$$

$$g(r) = A r e^{-br}$$

$$E_1 = \frac{\mu Z^2 e^4}{2\hbar^2}$$

$g_1(r)$



$(ar - br^2)\, e^{-br}$

$g_2(r)$