# Class of 11 Jan.: tying up loose ends

Narayanan Kurur

Department of Chemistry
IIT Delhi

Jan. 12, 2011

For photons,

$$E = pc \qquad E = h\nu = \frac{hc}{\lambda}$$

implies

$$p = \frac{h}{\lambda}$$

- de Broglie showed that this relation also applies to material particles
- Confirmed later experimentally by Davisson and Germer and G. P. Thomson

# A classical plane wave satisfies the DE $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

Plane wave solution $y = A\sin(kx - \omega t)$

$$\frac{\partial y}{\partial x} = kA\cos(kx - \omega t)$$

$$\frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t)$$

One differential equation satisfied by the plane wave is

$$\frac{\partial y}{\partial x} = -\frac{k}{\omega}\frac{\partial y}{\partial t}.$$

Symmetrical opposite going wave $A\sin(kx + \omega t)$ not a solution of this DE Taking second derivatives of $y$ we find the differential equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 y}{\partial t^2}$$

# Schrödinger eq. is analogous to the classical wave eq.

Consider a free particle ($V = 0$) of momentum $p$.

$$E = \frac{\hbar^2 k^2}{2m} = \hbar\omega$$

By analogy with the plane wave its solution is $\Psi = A\sin(kx - \omega t)$

$$\frac{\partial^2\Psi}{\partial x^2} = -k^2 A\sin(kx - \omega t)$$

Because $\omega = \frac{\hbar k^2}{2m}$ take only the first derivative with respect to $t$

$$\frac{\partial\Psi}{\partial t} = -\omega A\cos(kx - \omega t)$$

Unfortunately because of the use of sin and cos

$$\frac{\partial^2\Psi}{\partial x^2} \neq \frac{2m}{\hbar}\frac{\partial\Psi}{\partial t}$$

# Using a complex $\Psi(x,t)$ gives Schrödinger equation

Use $\Psi(x,t) = A\exp(i(kx - \omega t))$.

$$\frac{\partial^2 \Psi}{\partial x^2} = -k^2 A\exp(i(kx-\omega t)) = -\frac{p^2}{\hbar^2}\Psi$$

$$\frac{\partial \Psi}{\partial t} = -i\omega A\exp(i(kx-\omega t)) = -\frac{iE}{\hbar}\Psi$$

or

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{2mE}{\hbar^2}\Psi = -\frac{2m}{\hbar}\frac{\partial \Psi}{\partial t}$$

# DE of a particle experiencing a potential

$$p^2 = 2m(E - V) = \hbar^2 k^2$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E - V)\Psi = -i\frac{2m}{\hbar}\frac{\partial \Psi}{\partial t} + \frac{2m}{\hbar^2}V\Psi$$

which can be written as

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = E\Psi$$

or

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial \Psi}{\partial t}.$$