Discussion on the postulates of quantum mechanics

Note Title 12-01-2011

1) Ψ & probability of the system being given by $\boxed{|\Psi|^2 dx}$

2) Classical observables $\longleftrightarrow$ Operators

Linear, Hermitean operators

superposition

"real" eigen value

Examples of Operators in QM

Classical observable	QM

✓ Position (x, y, z)

$\hat{x} = x \times f$

$\hat{y} = y \times f$

$\hat{z} = z \times f$

✓ Momentum $(\vec{p})$

$-i\hbar \vec{\nabla}$

$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$

$\hat{p}_y = -i\hbar \frac{\partial}{\partial y}$

(Angular momentum$)_x$

L_x

$\vec{L} = \vec{r} \times \vec{p} \longrightarrow$ momentum operator

$\hookrightarrow$ position operator

$$\hat{\vec{L}} = \begin{vmatrix} i & j & k \\ \hat{x} & \hat{y} & \hat{z} \\ \hat{p}_x & \hat{p}_y & \hat{p}_z \end{vmatrix}$$

$\hat{L}_x =$

$\hat{L}_y =$

$\hat{L}_z =$

$\hat{\mathcal{H}}$ = Hamiltonian

= Energy operator

$$\hat{\mathcal{H}} = E = KE + PE$$

$$= \frac{p^2}{2m} + V(\vec{r})$$

$V(r)$ example, $V = \frac{1}{2}kx^2$

$$\hat{V} = \frac{1}{2}kx^2$$

$$\vec{p}\cdot\vec{p} = -i\hbar\vec{\nabla}\cdot -i\hbar\vec{\nabla} = -\hbar^2\nabla^2$$

$$\hat{\mathcal{H}} = -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})$$

$$\hat{O}_p (c_1 f_1 + c_2 f_2) = c_1 \hat{O}_p f_1 + c_2 \hat{O}_p f_2$$

then $\hat{O}_p$ is a linear operator

$\frac{d}{dx}$ is a linear operator

$$\left(\hat{O}_p^{x}\right)^2 = \left(\frac{d}{dx} + x\right)^2$$

$$= \left(\frac{d}{dx} + x\right)\left(\frac{d}{dx} + x\right) f(x)$$

$$\overset{?}{=}$$

$$\frac{d}{dx}\,x\,f \neq x\,\frac{d}{dx}\,f$$

p_x

$$\hat{A}\,\hat{B}\,f \neq \hat{B}\,A\,f$$

$$\left[\frac{d}{dx}, y\right] = 0$$

$$\underbrace{(\hat{A}\,\hat{B} - \hat{B}\,A)}\cancel{f} = [\hat{A}, \hat{B}]\,\cancel{f}$$

$\rightarrow$ commutator $= [\hat{A}, B]$

If $[\hat{A}, \hat{B}] = 0$, $\underline{\hat{A}\ \&\ \hat{B}}$ commute

$$\Delta p_x\,\Delta x \geq \hbar/2$$

$$\Delta A \, \Delta B \geq \frac{1}{2} \left| \langle [A, B] \rangle \right|$$

$$[p_x, x] = \hbar \left[-i\hbar \frac{d}{dx}, x \right] f$$

(3) Measurement leads to eigen values $= \hbar f$

$\hat{O}_p f = c_k f \rightarrow$ eigen value equation

$$-i\hbar \frac{d}{dx} e^{ikx} = \hbar k \, e^{ikx} -$$

$p = \hbar k$

④

$$\langle O_p \rangle = \frac{\int \Psi^* \hat{O}_p \Psi \, d\tau}{\int \Psi^* \Psi \, d\tau}$$

$P(x)$

$$\langle x \rangle = \int P(x) \, x \, dx$$

$\Psi^* \quad \Psi$

Real expectation values –

Operator is Hermitian

$$\boxed{\int f^* \hat{O}_p\, g\, d\tau = \int (O_p f)^* g\, d\tau}$$

Then $\hat{O}_p$ is Hermitian

$$\int f^* \hat{O}_p f\, d\tau$$

$$\boxed{\begin{aligned} \hat{O}_p f &= a f \\ (\hat{O}_p f)^* &= a^* f^* \end{aligned}}$$

$$(a - a^*) \int f^* f\, d\tau = 0$$

Linear & Hermitian $\Rightarrow$ $a = a^*$