CYL110 2010-2011 Quantum Tutorial 1

Tutors: Kurur/Pandey

1 Blackbody Radiation (Sec. 8.1(a) and (b) of PWA)

1. The Planck blackbody spectrum giving the energy density ($\rho(\nu)$) in the frequency range ν and $\nu + d\nu$ is $\frac{8\pi\nu^2}{c^3}\frac{h\nu}{\exp(\frac{h\nu}{kT})-1}$. Express the Planck expression for the energy density in terms of λ.

2. Derive the Wien displacement law for the wavelength at which the blackbody energy density is a maximum: $\lambda_{\max}T$ = constant. (Hint: You will need to solve a transcendental equation $(5 - x) = 5\exp(-x)$, where $x = hc/\lambda_{\max}kT$. Use the "method of successive approximations" to do so.)

3. At what wavelength does the human body emit its maximum thermal radiation? List the assumptions you made in arriving at the answer.

4. Verify that $\nu_{\max} = T \times$constant by setting $d\rho(\nu)/d\nu = 0$. Confirm whether this result agrees with that obtained from $\lambda_{\max}T$ = constant by substituting $\lambda_{\max} = c/\nu_{\max}$. Explain your last observation.

5. Planck's distribution law gives the radiant energy density of thermal radiation emitted between ν and $\nu + d\nu$. Integrate the Planck distribution over all frequencies to obtain the total energy density in blackbody radiation:

$$\frac{U}{V} = \left(\frac{\pi^2 k^4}{15\hbar^3 c^3}T^4\right).$$

You will need to look up (unless, of course, you know how to evaluate it!) an integral of the type

$$\int_0^\infty \frac{x^s - 1}{\exp(x) - 1}dx.$$

2 Heat capacities (Section 8.1(c) of PWA)

6. According to the Einstein model, the total molar energy of a solid is

$$U_m = \frac{3N_A h\nu}{\exp(\frac{h\nu}{kT}) - 1}.$$

Obtain the high-temperature and low-temperature limit of this expression and compare your result with equations 8.8a and 8.8b of Atkins. What would the heat capacity be in each of these limits?

7. The vibrational frequency of NaCl(g) is 159.23 cm^{-1}. Calculate the molar heat capacity at 1000 K.