CYL110 2010-2011 Quantum Tutorial 2

Tutors: Kurur/Pandey

1. What are the results of operating on the following functions with the operator d/dx and d^2/dx^2: (a) $\exp(-ax^2)$, (b) $\cos(bx)$, (c) $\exp(ikx)$? Which functions are eigen functions of these operators? What are the corresponding eigen values?

2. Which of the following operators are linear? (a) d/dx; (b) $\sqrt{}$; (c) exponentiation; (d) integration; (e) the translation operator $\hat{T}f(x) = f(x+1)$; (f) the operator $\hat{L}f(x) = f(x) + 1$.

3. Consider the operator $\hat{I}$, defined by $\hat{I}f(x) = f(x)$. (a) Show that $\hat{I}$ is a linear operator. (b) Find the eigenfunctions and the corresponding eigenvalues of $\hat{I}$.

4. In algebra it can be easily shown that $(P+Q)(P-Q) = P^2 - Q^2$. What is the value of $(P+Q)(P-Q)$ if P and Q are operators? Under what conditions will this result be equal to $P^2 - Q^2$.

5. Which of the following operators are Hermitian? (a) $3 - 4i$ (b) $\partial/\partial x$ (c) xpx.

6. Find $[z^3, d/dz]$ and $[d^2/dx^2, ax^2 + bx + c]$.

7. Write down the Hamiltonian for the following systems: (a) a particle of mass m in a cubical box of side a; (b) a particle of mass m in a spherical box of radius a; (c) a particle of mass m moving on the x-axis subjected to a force directed towards the origin, of magnitude proportional to the distance from the origin; (d) an electron moving in the presence of a nuclear charge $+Ze$; (e) two electrons moving in the presence of a fixed nucleus of charge $+Ze$.

8. Which of the following functions cannot be solutions of the Schrödinger equation for all values of x? Why not? (a) $A\sec(x)$; (b) $A\tan(x)$; (c) $A\exp(x^2)$; (d) $A\exp(-x^2)$.

9. The possible values obtained from a measurement of a discrete variable, x, are 1, 2, 3, and 4. (a) If the respective probabilities are 1/4, 1/4, 1/4, and 1/4, calculate the expectation values of x and x^2. (b) If the respective probabilities are 1/12, 5/12, 5/12, and 1/12, calculate the expectation values of x and x^2.

10. Determine the probability density of a particle as a function of its position if its wave function is $A\exp(ikx)$. What is the value of its momentum?

11. Normalize the following wave functions to unity: (a) $\sin(n\pi x/L)$ for the range $0 < x < L$, (b) c, a constant in the range $-L < x < L$, (c) $\exp(-r/a_0)$ in three dimensions, (d) $x\exp(-r/2a_0)$ in three dimensions.