CYL110 2010-2011 Quantum Tutorial 3

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1. (a) Calculate the energy levels for $n = 1$, 2, and 3 for an electron in an infinite potential well of width 0.25 nm. (b) If an electron makes a transition from $n = 2$ to $n = 1$ what will be the wavelength of the emitted radiation?

2. (a) Evaluate the probability of locating a particle in the middle third of 1-D box. (b) Find the probability that a particle in a box L wide can be found between $x = 0$ and $x = L/n$ when it is in the nth state.

3. Consider two wave functions which describe any two different states of a particle in a box. Show that these are orthonormal.

4. Verify the uncertainty principle for the particle in a box.

5. Discuss the source and nature of degeneracy for a particle in a $2-$D and $3-$D box.

6. A particle is confined to a two dimensional box of length L and $2L$. What are the allowed energy levels?

7. Many proteins contain metal porphyrin molecules. These molecules are planar and contain 26 π electrons. If the length of the molecule is $\sim$1000 pm, then what is the predicted lowest energy absorption of the porphyrin molecule?

8. Consider a particle confined to move in the region $-a/2 \leq x \leq a/2$ and whose wavefunction is $\Psi(x,t) = \cos(\pi x/a)\exp(-i\omega t)$. [$8 \times 5$]

   (a) Find the potential $V(x)$ and hence write the Hamiltonian.

   (b) What is the probability of finding the particle in the region $-a/6 \leq x \leq a/6$?

   (c) What is the expectation for the energy of the particle in this state?

   (d) What is the uncertainty in the energy measurement?

   (e) An operator $\hat{\mathcal{P}}$ is defined as $\hat{\mathcal{P}}\psi(x) = \psi(-x)$. Show that $\hat{\mathcal{P}}$ is an acceptable quantum mechanical operator.

   (f) Is $\Psi(x,t)$ an eigenfunction of $\hat{\mathcal{P}}$? If so, what is its eigenvalue?

   (g) Decide whether $\hat{\mathcal{P}}$ commutes with the Hamiltonian.

   (h) If another wavefunction of the particle is $\Psi'(x,t) = \sin(2\pi x/a)\exp(-i\omega' t)$, determine whether $\Psi(x,t)$ and $\Psi'(x,t)$ are orthogonal.