CYL110 II Semester 2010 - 2011

Quiz I Version A

Date: Jan. 25, 2011 Time: 0.5 hour

Name: ______________________________ Entry #: _______________ Group #: ___

Section 1. True/False (2 each)

_______ One of the consequences of waveparticle duality is that it sets limits on the amount of information that can be obtained about a quantum system at any one time.

_______ If $y_1(x,t)$ and $y_2(x,t)$ are solutions of the wave equation $\frac{\partial^2 y_i}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y_i}{\partial t^2}$, where $i = 1$ or 2, and v is a constant, then $y_{\text{sum}} = ay_1(x,t) + by_2(x,t)$, where a and b are constants, is also a solution to the wave equation.

_______ A postulate of quantum mechanics says that $|\psi(x)|^2 dx$ is the probability that the particle is between x and $x + dx$ from which one can conclude that $\int_{-\infty}^{\infty} |\psi(x)|^2 dx$ must be exactly 1.

_______ The quantum mechanical expression for the kinetic energy is $\int_{-\infty}^{\infty} -\frac{\hbar^2 k^2}{2m}\frac{\partial^2}{\partial x^2}\left(\Psi(x,t)^*\Psi(x,t)\right)dx$.

_______ $< x^2 >$ is always greater than $< x >^2$.

Section 2. Fill in the blanks (5 each)

1. Two operators are defined $\hat{A} = \hat{x} + i\hat{p_x}$ and $\hat{A}^\dagger = \hat{x} - i\hat{p_x}$. In terms of $\hat{A}$ and $\hat{A}^\dagger$, the operator $\hat{x^2}$ is _________________________. The uncertainty product for the simultaneous measurement of $\hat{A}$ and $\hat{A}^\dagger$ is ___________________________.

2. A system is in the state $\psi = \frac{1}{\sqrt{3}}\psi_1 + \sqrt{\frac{2}{3}}\psi_2$, where ψ_1 and ψ_2 are eigenfunctions of the Schrödinger equation with energies E_1 and E_2 respectively. The probability to measure E_1 is _. The expectation value of $< E^2 >$ is __________.

3. If the function $\psi(x) = a\exp(-bx^2)$ is an eigenfunction of the Hamiltonian $\frac{d^2}{dx^2} - kx^2$, with the eigenvalue $-\sqrt{k}$ (**this eigenvalue is redundant**), the constant b is ____. Given that $\int_{-\infty}^{\infty}\exp(-\beta x^2)dx = \sqrt{\frac{\pi}{\beta}}$, the constant a is _________________. The position of a system in this state is _.

4. The probability current or flux is defined as $j(x,t) = \frac{\hbar}{2mi}\left(\psi^*(x,t)\frac{\partial\psi(x,t)}{\partial x} - \frac{\partial\psi^*(x,t)}{\partial x}\psi(x,t)\right)$. If $\psi(x,t)$ is normalized, the dimensions of $j(x,t)$ are ____.

Answer Key Version A

Section 1. True/False (2 each)

True One of the consequences of waveparticle duality is that it sets limits on the amount of information that can be obtained about a quantum system at any one time.

True If $y_1(x,t)$ and $y_2(x,t)$ are solutions of the wave equation $\frac{\partial^2 y_i}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y_i}{\partial t^2}$, where $i = 1$ or 2, and v is a constant, then $y_{\rm sum} = ay_1(x,t) + by_2(x,t)$, where a and b are constants, is also a solution to the wave equation.

True A postulate of quantum mechanics says that $|\psi(x)|^2 dx$ is the probability that the particle is between x and $x + dx$ from which one can conclude that $\int_{-\infty}^{\infty} |\psi(x)|^2 dx$ must be exactly 1.

False The quantum mechanical expression for the kinetic energy is $\int_{-\infty}^{\infty} -\frac{\hbar^2 k^2}{2m}\frac{\partial^2}{\partial x^2}\left(\Psi(x,t)^*\Psi(x,t)\right)dx$.

False $< x^2 >$ is always greater than $< x >^2$.

Section 2. Fill in the blanks (5 each)

1. Two operators are defined $\hat{A} = \hat{x} + i\hat{p_x}$ and $\hat{A}^\dagger = \hat{x} - i\hat{p_x}$. In terms of $\hat{A}$ and $\hat{A}^\dagger$, the operator $\hat{x^2}$ is $\frac{1}{4}\left(\hat{A}^2 + \hat{A}\hat{A}^\dagger + \hat{A}^\dagger\hat{A} + \hat{A}^{\dagger 2}\right)$. The uncertainty product for the simultaneous measurement of $\hat{A}$ and $\hat{A}^\dagger$ is $\Delta A \Delta A^\dagger \geq \frac{1}{2} < |[\hat{A}, \hat{A}^\dagger]| >\geq \hbar$.

2. A system is in the state $\psi = \frac{1}{\sqrt{3}}\psi_1 + \sqrt{\frac{2}{3}}\psi_2$, where ψ_1 and ψ_2 are eigenfunctions of the Schrödinger equation with energies E_1 and E_2 respectively. The probability to measure E_1 is $\frac{1}{3}$. The expectation value of $< E^2 >$ is $\frac{1}{3}E_1^2 + \frac{2}{3}E_2^2$.

3. If the function $\psi(x) = a\exp(-bx^2)$ is an eigenfunction of the Hamiltonian $\frac{d^2}{dx^2} - kx^2$, with the eigenvalue $-\sqrt{k}$ (**this eigenvalue is redundant**), the constant b is $\sqrt{\frac{k}{2}}$. Given that $\int_{-\infty}^{\infty}\exp(-\beta x^2)dx = \sqrt{\frac{\pi}{\beta}}$, the constant a is $\left(\frac{\pi}{2b}\right)^{1/4} = \left(\frac{\pi}{\sqrt{k}}\right)^{1/4}$. The position of a system in this state is 0.

4. The probability current or flux is defined as $j(x,t) = \frac{\hbar}{2mi}\left(\psi^*(x,t)\frac{\partial\psi(x,t)}{\partial x} - \frac{\partial\psi^*(x,t)}{\partial x}\psi(x,t)\right)$. If $\psi(x,t)$ is normalized, the dimensions of $j(x,t)$ are T^{-1}.