# CYL501 2010-11 Homework 1
# Due date: August 13, 2010

August 7, 2010

1. For each of the following processes deduce whether each of the quantities $q$, $w$, $\Delta U$, $\Delta H$ is positive, zero, or negative. (a) Reversible melting of solid benzene at 1 atm and the normal melting point. (b) Reversible melting of ice at 1 atm and 0 °C. (c) Reversible adiabatic expansion of a perfect gas. (d) Reversible isothermal expansion of a perfect gas. (e) Adiabatic expansion of a perfect gas into a vacuum. (f) Reversible heating of a perfect gas at constant $P$. (g) Reversible cooling of a perfect gas at constant $V$.

2. How high can a person climb on an ounce of chocolate if its combustion releases 628 kJ?

3. In this problem we will derive a general relation between $C_P$ and $C_V$. Start with $U = U(P,T)$ and write

$$dU = \left(\frac{\partial U}{\partial P}\right)_T dP + \left(\frac{\partial U}{\partial T}\right)_P dT. \tag{1}$$

We could also consider $V$ and $T$ to be independent variables of $U$ and write

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT. \tag{2}$$

Now take $V = V(T,P)$ and substitute its expression for $dV$ into the second equation to obtain

$$dU = \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial P}\right)_T dP + \left[\left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P + \left(\frac{\partial U}{\partial T}\right)_V\right] dT. \tag{3}$$

Compare equation 1 to equation 3 to obtain the expressions

$$\left(\frac{\partial U}{\partial P}\right)_T = \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial P}\right)_T$$

and

$$\left(\frac{\partial U}{\partial T}\right)_P = \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P + \left(\frac{\partial U}{\partial T}\right)_V$$

Last, substitute $U = H - PV$ and use the definitions of $C_P$ and $C_V$ to obtain an expression for $C_P - C_V$.

4. When a rubber band is stretched, it exerts a restoring force, $\mathscr{F}$, which is a function of its length, $L$, and its temperature, $T$. The work involved is given by

$$w = \int \mathscr{F}(L,T)dL.$$

Consider a rubber band with the equation of state

$$\mathscr{F} = aT\left[\frac{L}{L_0} - \left(\frac{L_0}{L}\right)^2\right]$$

where $L_0$ is the original length, and $a$ is a constant equal to $1.3 \times 10^{-2}$ N K$^{-1}$. How much work is performed when the band is stretched isothermally and reversibly from its length of 10 cm to 20 cm, the temperature being 20 °C? Write the expressions for heat capacities in this case. Which would be bigger?