CYL110 2012–13 **Quiz 1** Version A

Name: ______________________ Entry No.: __________ Group: ______

Section 1. True/False: Write T(rue)/F(alse) in the space provided (5×3 pts.)

<u>False</u> The function $f(x) = \exp(-|x|)$ in the interval $-\infty$ to ∞ is an acceptable quantum mechanical wavefunction.

<u>True</u> The de Broglie wavelength of an H_2 molecule traveling at the root-mean square speed of molecules in H_2 gas at temperature T is greater than that of an O_2 molecule traveling at the root-mean-square speed of molecules in O_2 gas at the same T.

<u>False</u> Observables corresponding to $\hat{A}$ and $\hat{E}$ are complementary, where $\hat{A} = x\left(\frac{\partial}{\partial x}\right)$ and $\hat{E} = x^2\left(\frac{\partial^2}{\partial x^2}\right)$.

<u>False</u> To explain the departure of the heat capacities of monoatomic metals obtained from the Dulong-Petit expression at low temperatures, Einstein's assumption that each atom oscillates about its equilibrium position with a single frequency provided a perfect match to the experimentally found values.

<u>True</u> If violet light with $\lambda = 400$ nm does not cause the photoelectric effect in a certain metal, then it is certain that red light with $\lambda = 700$ nm will not cause the photoelectric effect in that metal.

Section 2. Fill in the blanks (5 pts. per blank)

1. If $\hat{a} = \frac{(\hat{x}+i\hat{p}_x)}{\sqrt{2}}$ and $\hat{a}^* = \frac{(\hat{x}-i\hat{p}_x)}{\sqrt{2}}$, then $[\hat{a}, \hat{a}^*] = \underline{\hbar \text{ or } \hbar\hat{I}}$.

2. The ground-state wavefunction of a hydrogen atom is $\psi = \left(\frac{1}{\pi a_0^3}\right)^{1/2}\exp(-r/a_0)$, where $a_0 = 53$ pm (Bohr radius). The probability that the electron will be found somewhere within a small sphere of radius 1.0 pm centered on the nucleus is $\underline{9.0 \times 10^{-6}}$. If the same sphere is located at $r = a_0$, the probability that the electron is inside it is $\underline{1.2 \times 10^{-6}}$.

3. The normalization constant for the wavefunction $x\exp(-r/2a_0)$ in 3-D is $\underline{\frac{1}{\sqrt{32\pi a_0^5}}}$.
$\int_0^\infty x^n e^{-ax} dx = n!/a^{n+1}$

4. Suppose we have a particle-in-a-1D-box and we know that the particle is in its $n = 1$ or $n = 2$ eigenstate. Suppose further that the particle is twice as likely to be in the $n = 2$ eigenstate than the $n = 1$ state. An appropriate normalized wavefunction for this system is $\underline{\sqrt{\frac{2}{L}}\frac{1}{\sqrt{3}}\left[\sin\left(\frac{\pi x}{L}\right) + \sqrt{2}\sin\left(\frac{2\pi x}{L}\right)\right]}$. If the energy of 10^6 such systems were measured and the average energy computed from the data, the average energy in terms of $h^2/8mL^2$, where m and L are the mass of the particle and the length of the 1D box, respectively, is $\underline{\frac{3h^2}{8mL^2}}$.

5. If the translation operator $\hat{T}_h$ is defined as $\hat{T}_h f(x) = f(x+h)$, then $(\hat{T}_1^2 - 3\hat{T}_1 + 2)x^2 = \underline{-2x+1}$.