CYL501 Homework 2

November 15, 2012

1. Consider a hypothetical atom that has just two states: a ground state with energy zero and an excited state with energy 2 eV. Draw a graph of the partition function for this system as a function of temperature, and evaluate the partition function numerically at T = 300 K, 3000 K, 30,000 K, and 300,000 K.

2. Imagine a particle that can be in only three states, with energies -0.05 eV, 0, and 0.05 eV. This particle is in equilibrium with a reservoir at 300 K.
   (a) Calculate the partition function for this particle.
   (b) Calculate the probability for this particle to be in each of the three states.
   (c) Because the zero point for measuring energies is arbitrary, we could just as well say that the energies of the three states are 0, +0.05 eV, and +0.10 eV, respectively. Repeat parts 3a and 3b using these numbers. Explain what changes and what doesn't.

3. Estimate the probability that a hydrogen atom at room temperature is in one of its first excited states (relative to the probability of being in the ground state). Don't forget to take degeneracy into account. Then repeat the calculation for a hydrogen atom in the atmosphere of the star γ UMa, whose surface temperature is approximately 9500 K.

4. A water molecule can vibrate in various ways, but the easiest type of vibration to excite is the "flexing" mode in which the hydrogen atoms move toward and away from each other but the HO bonds do not stretch. The oscillations of this mode are approximately harmonic, with a frequency of 4.8×10^{13} Hz. (a) Calculate the probability of a water molecule being in its ground state and in each of the first two excited states, assuming that it is in equilibrium with a reservoir (say the atmosphere) at 300 K. (Hint: Calculate Z by adding up the first few Boltzmann factors, until the rest are negligible.) (b) Repeat the calculation for a water molecule in equilibrium with a reservoir at 700 K.

5. Suppose you have 10 atoms of kururium: 4 with energy 0 eV, 3 with energy 1 eV, 2 with energy 4 eV, and 1 with energy 6 eV. (a) Compute the average energy of all your atoms. (b) Compute the probability that one of your atoms chosen at random would have energy E, for each of the four values of E that occur.