CYL501 Final Exam

Time: 2 hrs Date: 22 Nov. 2011

1. (a) Freeze-drying is a dehydration process typically used to preserve a perishable material. Freeze-dried things that you may be familiar with are "Bru" coffee and "Maggi." Freeze-drying works by freezing the material and then reducing the surrounding pressure to allow the frozen water in the material to sublime directly from the solid phase to the gas phase. Explain freeze-drying with the help of a phase diagram.

   (b) The definition of a thermodynamic quantity from the CRC Handbook of Physics and Chemistry, 58^{th} edition, is given below.

   The quantity of heat required to change one gram of liquid to vapor without change in temperature, measured in calories per gram.

   i. What is the quantity that is defined above?
   ii. Is the quantity intensive or extensive? Why?
   iii. Is there any way, in your opinion, that the definition could be improved?

2. (a) A gas obeys the van der Waals equation of state. If its molar internal energy is given by $U = cT - a/V$ (in which V is the molar volume, a is one of the constants in the equation of state, and c is a constant), calculate C_p and C_v. You are given that $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$.

   (b) Decide, with appropriate justification, whether the following statements are true/false.

   i. The internal energy of a system and its surroundings is not conserved during an irreversible process, but is conserved for a reversible process.
   ii. If the total Gibbs energy of two phases are equal at constant pressure and temperature, then the two phases are in equilibrium with each other.

3. Consider four molecules labelled a, b, c, d that can exist in a set of quantum states with energies $\epsilon_0 = 0$, $\epsilon_1 = 1$, $\epsilon_2 = 2$, $\cdots$ with a total energy, E, of 6.

   (a) Explain microstate and macrostate with the help of this example.
   (b) Which configuration is the most probable?
   (c) What is W in the most probable configuration?
   (d) Make a plot of W as a function of configuration.

4. Consider a two-level system with energies 0 and ϵ.

   (a) What is the lowest value the partition function could have? Justify.
   (b) Is the situation in question 4a ever realizable? Explain.
   (c) Under what conditions is the population in the two levels equal.
   (d) Lasers are nowadays found in many devices. The CD and DVD drive are two examples. More population in the excited state than in the ground state is required for laser action. If this situations occurs in the two-level system, what inference can you draw?

5. Consider a mixture of two monoatomic ideal gases with N_A molecules of gas A and N_B molecules of gas B.

   (a) Write the expression for the partition function for the mixture $Q(N_A, N_B, V, T)$ in terms of the single molecule partition functions q_A and q_B.

   (b) If

   $$q_j(V,T) = \left(\frac{2\pi m_j kT}{h^2}\right)^{3/2} V, \qquad j = A, B$$

   obtain an expression for the energy E.

   (c) Also derive Dalton's law of partial pressures. You are given that $P = -\left(\frac{\partial A}{\partial V}\right)_T$.

6. Imagine a particle that can be in only three states, with energies -0.05 eV, 0, and 0.05 eV. This particle is in equilibrium with a reservoir at 300 K. ($k = 8.62 \times 10^{-5}$ eV/K)

   (a) Calculate the partition function for this particle.
   (b) Calculate the probability for this particle to be in each of the three states.
   (c) How would your result for parts 6a and 6b be affected if the energies of the three states were 0, $+0.5$ eV, and $+0.10$ eV?