Recap

Postulate 1: Wavefunction (Ψ) - has a probabilistic interpretation. Ψ and its first derivative have to be finite, single valued and continuous. Normalization of the wavefunction

Postulate 2: Classical observables represented by Operators. *Op f(x) = g(x)*
Position (*x*) operator by multiplication with *x* and momentum by -i (hbar) d/dx
Operators in QM are Linear (*Op (a f(x) + b g(x)) = a Op f(x) + b Op g(x)*), to allow for superposition, and Hermitian, so that they have real eigenvalues.

Self check: Write the operator for the total energy of a particle of mass *m* suspended from a spring with a spring constant *k* and experiencing a restoring force proportional to the displacement from equilibrium.

$$E = T + V$$

$$\frac{1}{2} m v_x^2 = \frac{p_x^2}{2m} \qquad \hat{T} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

$F = -kx$ (restoring force proportional to displacement)

$$F = -\frac{\partial V}{\partial x} \Rightarrow V = -\int F\,dx$$

$$V = \frac{1}{2}kx^2 \qquad \hat{V} = \frac{1}{2}kx^2$$

$\hat{\mathcal{H}}$ (Energy operator or Hamiltonian)

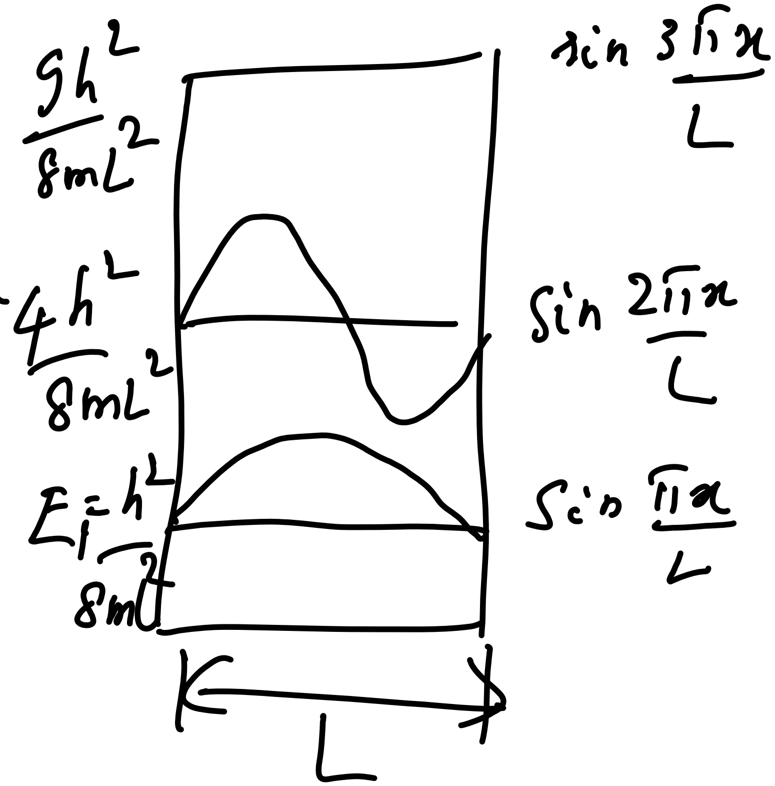
$$= \frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2$$

Measurement

Discussion on postulate 3: Measurement

Consider a particle in a box of length L

- Energies & wavefunction recollect from PHL100

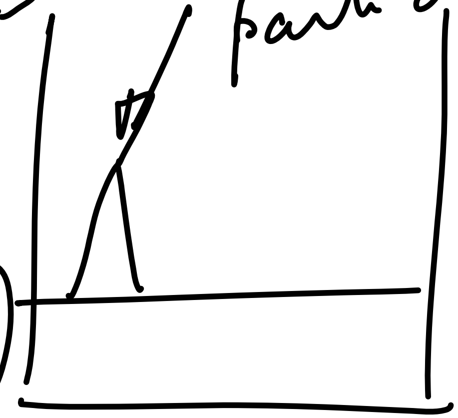


wave function $\Psi = \sum c_i \psi_i$ of PIB

of particle

$\psi_i = \sqrt{\frac{2}{L}} \sin\left(\frac{i \pi x}{L}\right)$

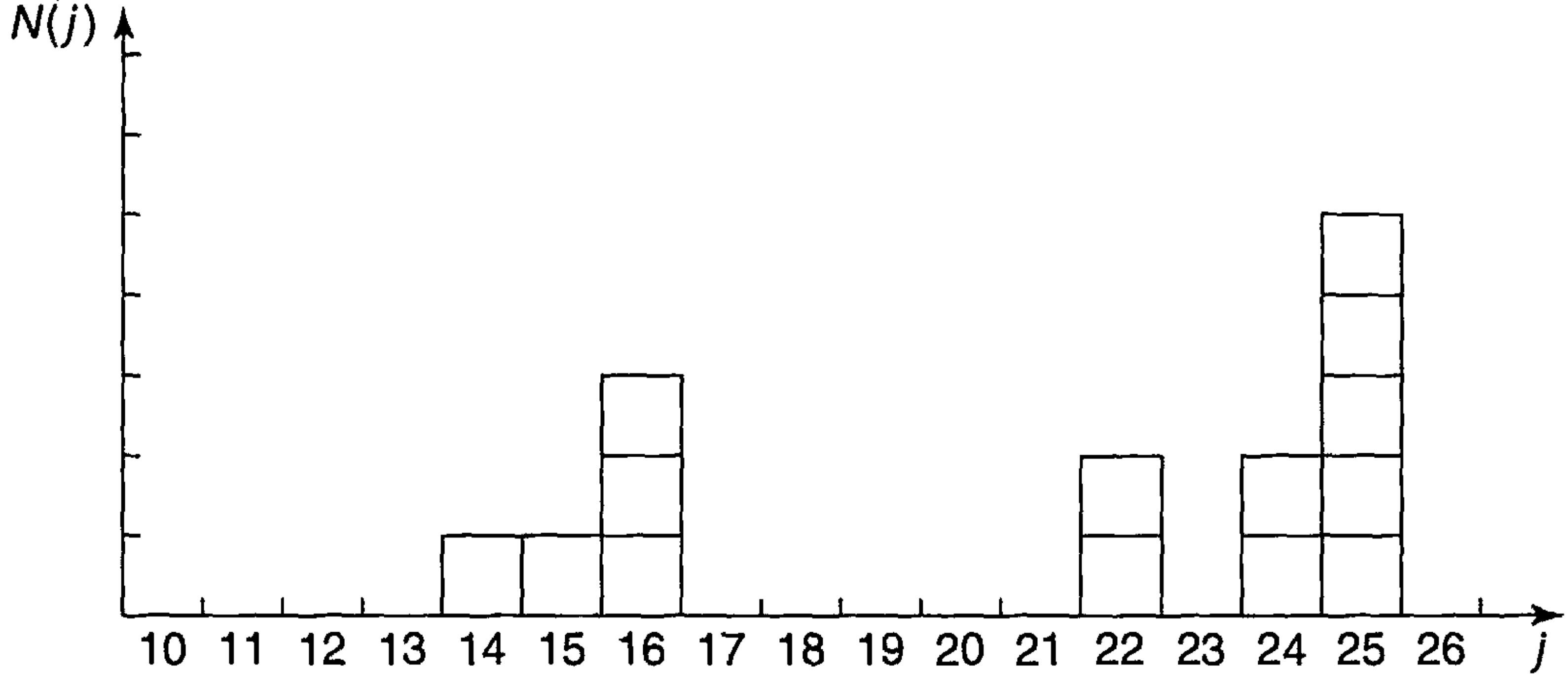
particle



Make measurement of energy of the particle

- postulate says you can only get $\rightarrow$ $\frac{h^2}{8mL^2}$, $\frac{4h^2}{8mL^2}$, $\frac{9h^2}{8mL^2}$

because $\mathcal{H}\psi_i = E_i \psi_i$



Consider the prob distribution above (marks obtained by students in a class, for example)

Average marks given by

$$\boxed{\langle x \rangle = \sum P_i \times x_i}$$

For a continuous distribution this is

$$\langle x \rangle = \int x \, P(x) \, dx$$

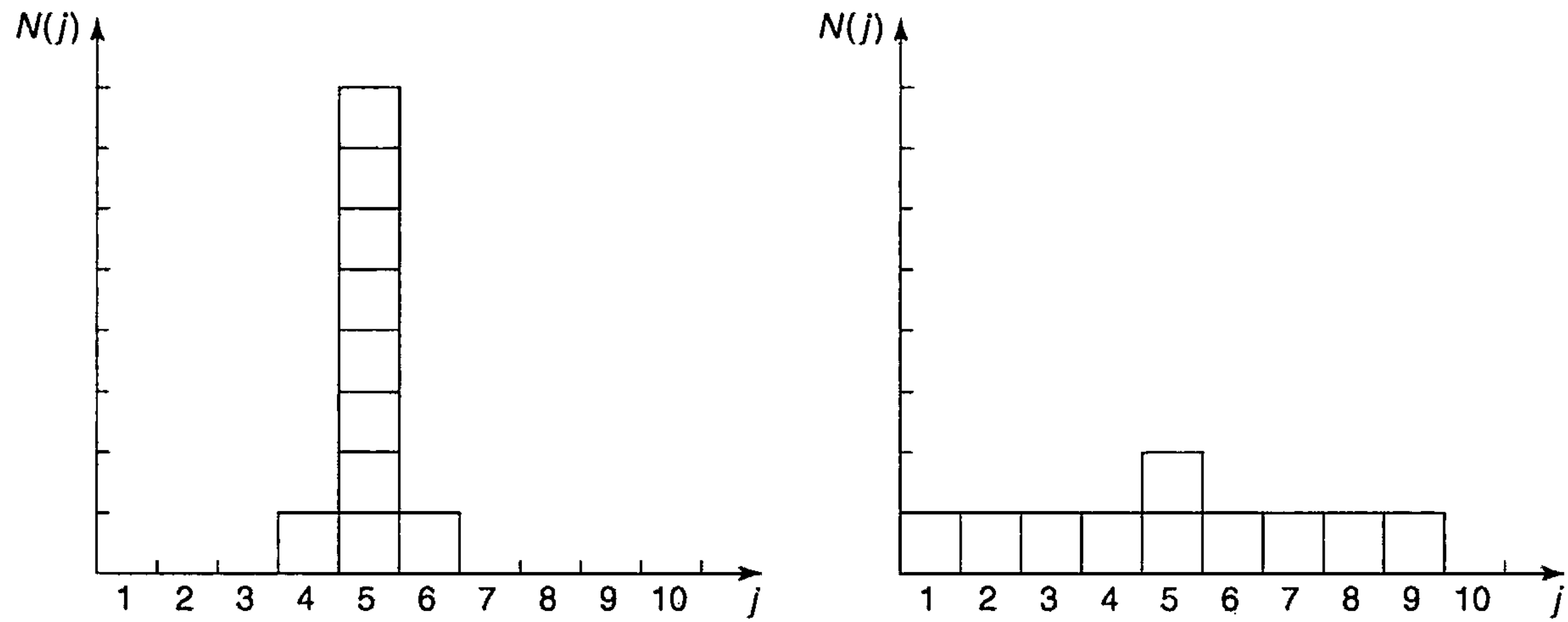
In QM, $P(x) = \psi^* \psi$ and

$$\langle x \rangle = \int \psi^* \, x \, \psi \, dx$$

expectation value / average value

Note that the average value (21) is not a mark obtained by any student - 14, 15, 16, 22, 24, and 25 are possible "eigen" values

Remember that expectation, which suggests most probable, is the average.



Consider the two distributions given above: have same average, median & most prob. value. How can we characterize the difference in the width of these two distributions.

$$\langle x^2 \rangle = \sum P(x)\, x^2 \text{ or } \int x^2 P(x)\, dx$$

In QM,

$$\langle x^2 \rangle = \int \psi^* x^2 \psi \, dx$$

or $\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$

$$\langle x - \langle x \rangle \rangle = 0$$

(a measure of the width)

↳ standard deviation

$$\langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

$$\geq 0 \quad \boxed{\Delta O_p = \sqrt{\langle O_p^2 \rangle - \langle O_p \rangle^2}}$$

uncertainty in the measurement of O_p

Example: Consider a free particle with

$$\psi = A e^{ikx}$$

$$p_x \psi = -i\hbar \frac{\partial}{\partial x} A e^{ikx}$$

eigen value

$$\cancel{p_x} = (\hbar k) A e^{ikx}$$

$$\langle p_x \rangle = \hbar k$$

$$p_x^2 \psi = -\hbar^2 \frac{d^2}{dx^2} A e^{ikx}$$

$$\langle p_x^2 \rangle = \hbar^2 k^2 \qquad = \hbar^2 k^2 \left(A e^{ikx}\right)$$

$$\langle p_x^2 \rangle - \langle p_x \rangle^2 = 0$$

$$\Delta p_x = 0$$

$$\psi = A e^{ikx}$$

$$\Delta x = \infty$$

$$\psi^* \psi = |A|^2$$

Time dependent Schrödinger equation

$$\boxed{i\hbar \frac{\partial \Psi}{\partial t} = \hat{\mathcal{H}}\, \Psi}$$

Look for solutions of the type

1) "Separation of variables" $\Psi = \boxed{\psi(r)\,\phi(t)}$

2) $\mathcal{H} = \hat{T} + \hat{V}$

independent of time

$\mathcal{H}\left(-i\hbar \frac{d}{dx},\ x\right)$

$$i\hbar \frac{\partial}{\partial t} \psi \varphi = \mathcal{H} \psi \varphi$$

$$\frac{1}{\psi \varphi} \left[i\hbar \, \psi \frac{\partial \varphi}{\partial t} = \varphi \hat{\mathcal{H}} \psi \right]$$

$$\frac{i\hbar}{\varphi} \frac{\partial \varphi}{\partial t} = \frac{1}{\psi} \hat{\mathcal{H}} \psi$$

$$\frac{i\hbar}{\varphi} \frac{\partial \varphi}{\partial t} = E$$

$$\frac{1}{\psi} \hat{\mathcal{H}} \psi = E$$

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi$$

$$\boxed{\psi(t) = \psi_0 \, e^{-\frac{iEt}{\hbar}}}$$

$$\boxed{\hat{\mathcal{H}}\psi = E\psi}$$ — Time Independent Schrödinger equation

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