CYL100 2013–14 II semester Homework 1

Handed out: January 10, 2014 Due in: January 17, 2014

Note: Any data you require will be found in the back pages of Atkins's book

1. What are the results of operating on the following functions with the operator d/dx and d^2/dx^2: (a) $\exp(-ax^2)$, (b) $\cos(bx)$, (c) $\exp(ikx)$? Which functions are eigen functions of these operators? What are the corresponding eigen values?

2. Which of the following operators are linear? (a) d/dx; (b) $\sqrt{}$; (c) exponentiation; (d) integration.

3. In algebra it can be easily shown that $(P+Q)(P-Q) = P^2 - Q^2$. What is the value of $(P+Q)(P-Q)$ if P and Q are operators? Under what conditions will this result be equal to $P^2 - Q^2$.

4. Find $[z^3, d/dz]$ and $[d^2/dx^2, ax^2 + bx + c]$.

5. Prove by induction that $[\hat{x}^n, \hat{p}_x] = i\hbar n\hat{x}^{n-1}$ and that $[\hat{x}, \hat{p}_x^n] = i\hbar n\hat{p}_x^{n-1}$ where n is a positive integer.

6. Which of the following functions cannot be solutions of the Schrödinger equation for all values of x? Why not? (a) $A\sec(x)$; (b) $A\tan(x)$; (c) $A\exp(x^2)$; (d) $A\exp(-x^2)$.

7. Normalize the following wave functions to unity: (a) $\sin(n\pi x/L)$ for the range $0 < x < L$, (b) c, a constant in the range $-L < x < L$, (c) $\exp(-r/a_0)$ in three dimensions, (d) $x\exp(-r/2a_0)$ in three dimensions.

8. Construct the squares of the following operators: (a) $\hat{D}\phi(x) = \partial\phi(x)/\partial x$, (b) $\hat{\triangle}\phi(x) = -\partial^2\phi(x)/\partial x^2$, (c) $\hat{Q}\phi(x) = \int_0^1 dx'\phi(x')$, (d) $\hat{F}\phi(x) = F(x)\phi(x)$, (e) $\hat{B}\phi(x) = \frac{1}{3}\phi(x)$, (f) $\hat{P}\phi = \phi^3 - 3\phi^2 - 4$, (g) $\hat{G}\phi(x) = (d/dx + x)\phi(x)$. Which of these operators are linear?

9. Write down the Schrödinger for the following systems: (a) a particle of mass m in a cubical box of side a; (b) a particle of mass m in a spherical box of radius a; (c) a particle of mass m moving on the x-axis subjected to a force directed towards the origin, of magnitude proportional to the distance from the origin; (d) an electron moving in the presence of a nuclear charge $+Ze$; (e) two electrons moving in the presence of a fixed nucleus of charge $+Ze$.

10. Let s be the number of spots shown by a die thrown at random. (a) Calculate $\langle s\rangle$. (b) Calculate Δs.

11. A particle is known to be in the state
$$\psi(x,t) = A\exp\left[-\frac{(x-x_0)^2}{4a^2}\right]\exp\left(\frac{ip_0x}{\hbar}\right)\exp(i\omega_0 t)$$
    (a) Determine A (You might find the following integral useful: $\int_{-\infty}^{\infty} e^{-\beta^2/2}d\beta = \sqrt{2\pi}$).
    (b) Determine the expectation value of position of the particle.
    (c) What is the uncertainty in the position?
    (d) What is the expectation value of the momentum? And its uncertainty?

12. A particle is in a state described by the wave function $\psi = (\cos\chi)\exp(ikx) + (\sin\chi)\exp(-ikx)$ where χ is a parameter. What is the probability that the particle will be found with a linear momentum (a) $+k\hbar$, (b) $-k\hbar$? What form would the waveunction have if it were 90% certain that the particle had linear momentum $+k\hbar$?