CYL100 2013–14 I semester Homework 4

Handed out: September 06, 2013 Due in: September 11, 2013

1. Calculate the minimum uncertainty in the speed of a ball of mass 500 g that is known to be within 1.0 μm of a certain point on a bat.
2. What is the minimum uncertainty in the position of a bullet of mass 5.0 g that is known to have a speed somewhere between 350.00001 m s^{-1} and 350.00000 m s^{-1}.
3. Calculate the de Broglie wavelength of a Helium atom travelling at 1000 m s^{-1} and compare it with that of a mass of 1.0 g travelling at the same speed. Comment on the significance of your results.
4. Write down the Schrödinger for the following systems: (a) a particle of mass m in a cubical box of side a; (b) a particle of mass m in a spherical box of radius a; (c) a particle of mass m moving on the x-axis subjected to a force directed towards the origin, of magnitude proportional to the distance from the origin; (d) an electron moving in the presence of a nuclear charge $+Ze$; (e) two electrons moving in the presence of a fixed nucleus of charge $+Ze$.
5. Which of the following functions cannot be solutions of the Schrödinger equation for all values of x? Why not? (a) $A\sec(x)$; (b) $A\tan(x)$; (c) $A\exp(x^2)$; (d) $A\exp(-x^2)$.
6. Determine $\psi^*\psi$ for the following wave functions: (a) $\cos\theta + i\sin\theta$ and (b) $\exp(-x^2)$.
7. The possible values obtained from a measurement of a discrete variable, x, are 1, 2, 3, and 4. (a) If the respective probabilities are 1/4, 1/4, 1/4, and 1/4, calculate the expectation values of x and x^2. (b) If the respective probabilities are 1/12, 5/12, 5/12, and 1/12, calculate the expectation values of x and x^2.
8. Determine the probability density of a particle as a function of its position if its wave function is $A\exp(ikx)$. What is the value of its momentum?
9. A particle is in a state described by the wave function $\psi = (\cos\chi)\exp(ikx) + (\sin\chi)\exp(-ikx)$ where χ is a parameter. What is the probability that the particle will be found with a linear momentum (a) $+k\hbar$, (b) $-k\hbar$? What form would the waveunction have if it were 90% certain that the particle had linear momentum $+k\hbar$?
10. Normalize the following wave functions to unity: (a) $\sin(n\pi x/L)$ for the range $0 < x < L$, (b) c, a constant in the range $-L < x < L$, (c) $\exp(-r/a_0)$ in three dimensions, (d) $x\exp(-r/2a_0)$ in three dimensions.
11. (a) Calculate the energy levels for $n = 1$, 2, and 3 for an electron in an infinite potential well of width 0.25 nm. (b) If an electron makes a transition from $n = 2$ to $n = 1$ what will be the wavelength of the emitted radiation?
12. (a) Evaluate the probability of locating a particle in the middle third of 1-D box. (b) Find the probability that a particle in a box L wide can be found between $x = 0$ and $x = L/n$ when it is in the nth state.
13. Consider two wave functions which describe any two different states of a particle in a box. Show that these satisfy the relation $\int_0^L \psi_i^*\psi_j dx = \delta_{ij}$, where $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$.
14. Verify the uncertainty principle for the particle in a box.
15. Describe the color of carrots using the particle in a box model. (Hint: Consider the π electrons to be confined to a box whose length is the length of the molecule. Use 1.54 Å as a C$-$C and 1.35 Å as a C=C bond length.)
16. A particle is confined to a two dimensional square box of length L. What are the allowed energy levels?
17. Below are some general statements about wave functions for stationary states of unique energy for a particle bound in a one-dimensional potential well $V(x)$. Decide whether each statement is true or false. Name one or more counterexamples for false statements. Be careful: except where noted, these are meant to be general statements, true, for example, even if there is a classically forbidden region *inside* the well. The phrase "outside the well" for any given energy E means a continuous classically forbidden region ($E < V(x)$) extending to infinity.
    (a) There are no nodes in the wave function outside the well.
    (b) There are no nodes in classically forbidden regions.
    (c) If the potential has only one relative minimum, the ground state probability function $|\psi|^2$ has only one maximum.
    (d) The ground state probability function has no nodes.
    (e) The ground state probability function has only one maximum.
    (f) The probability function for any state is greater at positions of higher potential than at positions of lower potential.
    (g) The probability function in a classically forbidden region is greater at positions of higher potential than at positions of lower potential.
    (h) For a given region outside the well, the probability function is smaller as one goes farther from the well.