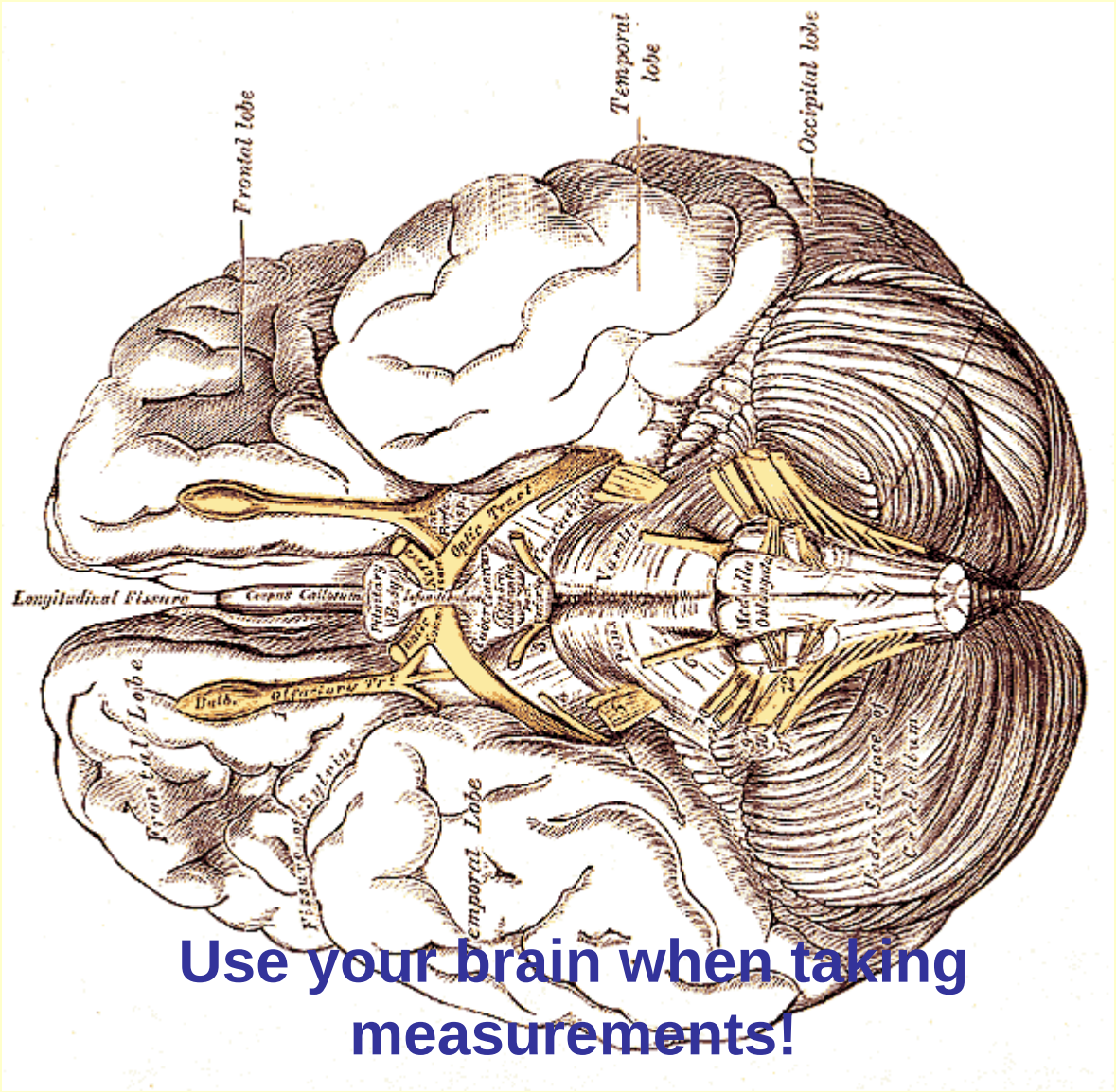
Measurements with Uncertainty

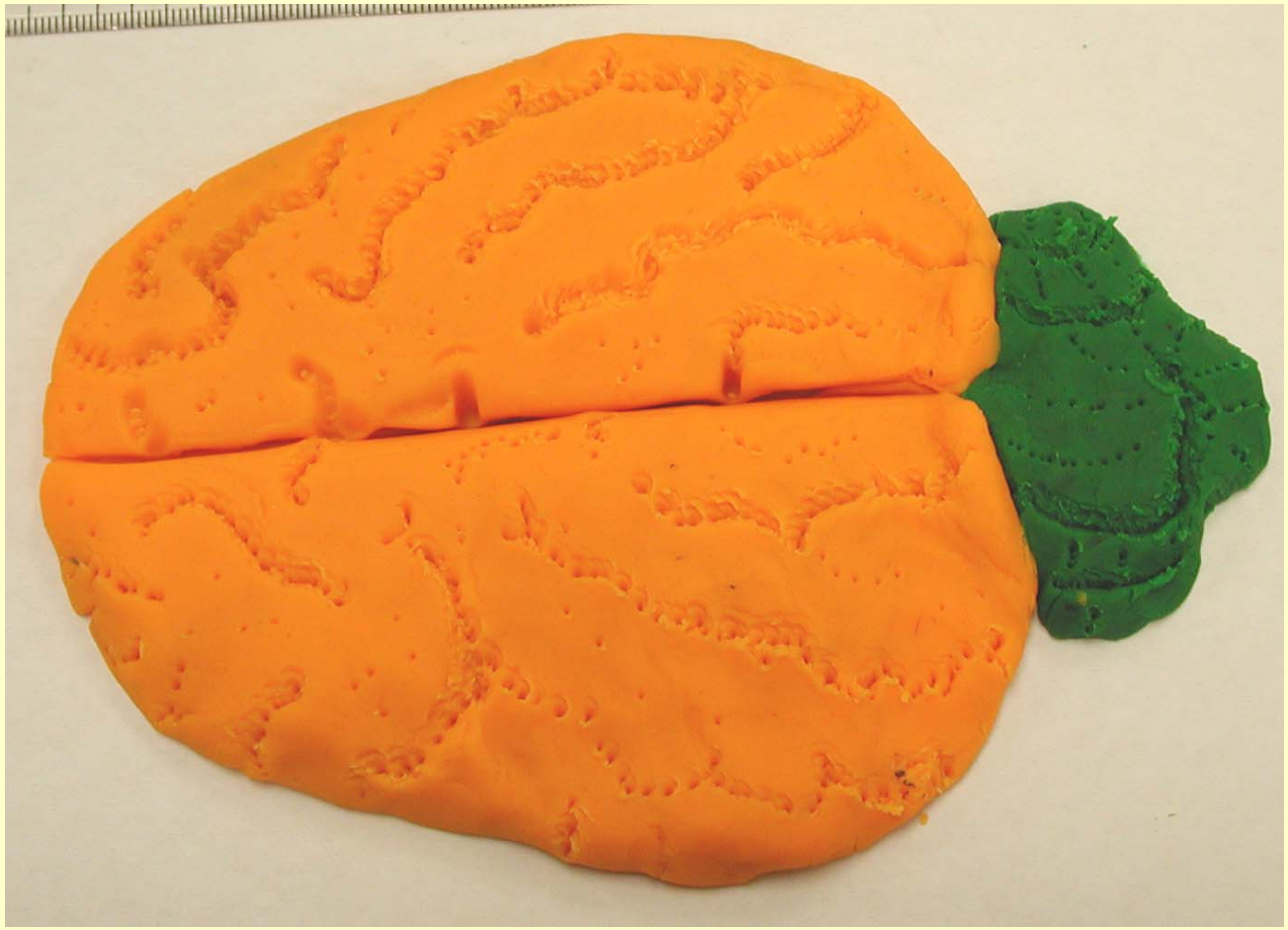
How can we estimate the size of the brain?



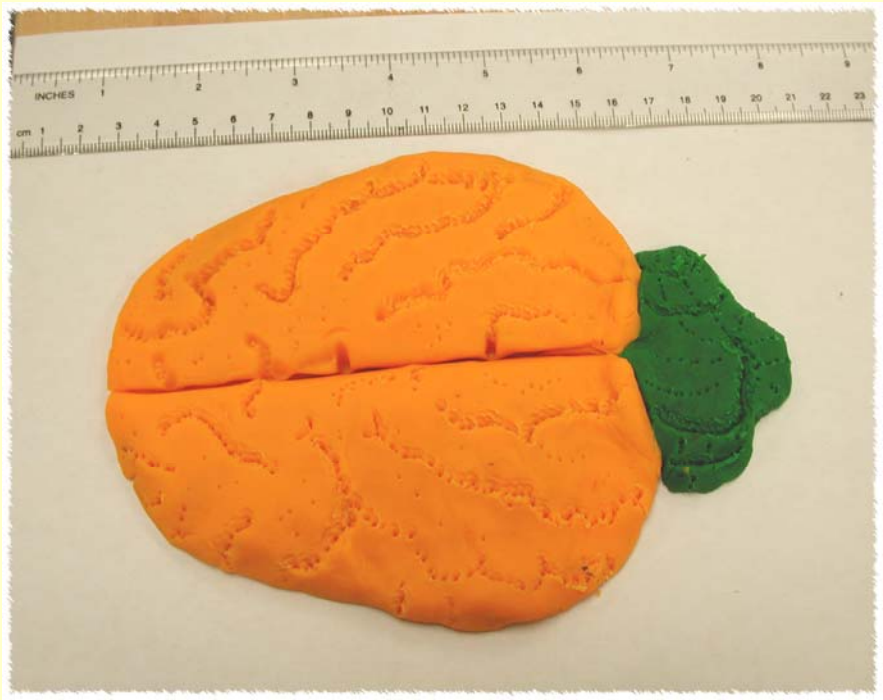
Use your brain when taking measurements!

Now how do you measure brain size? make a model...

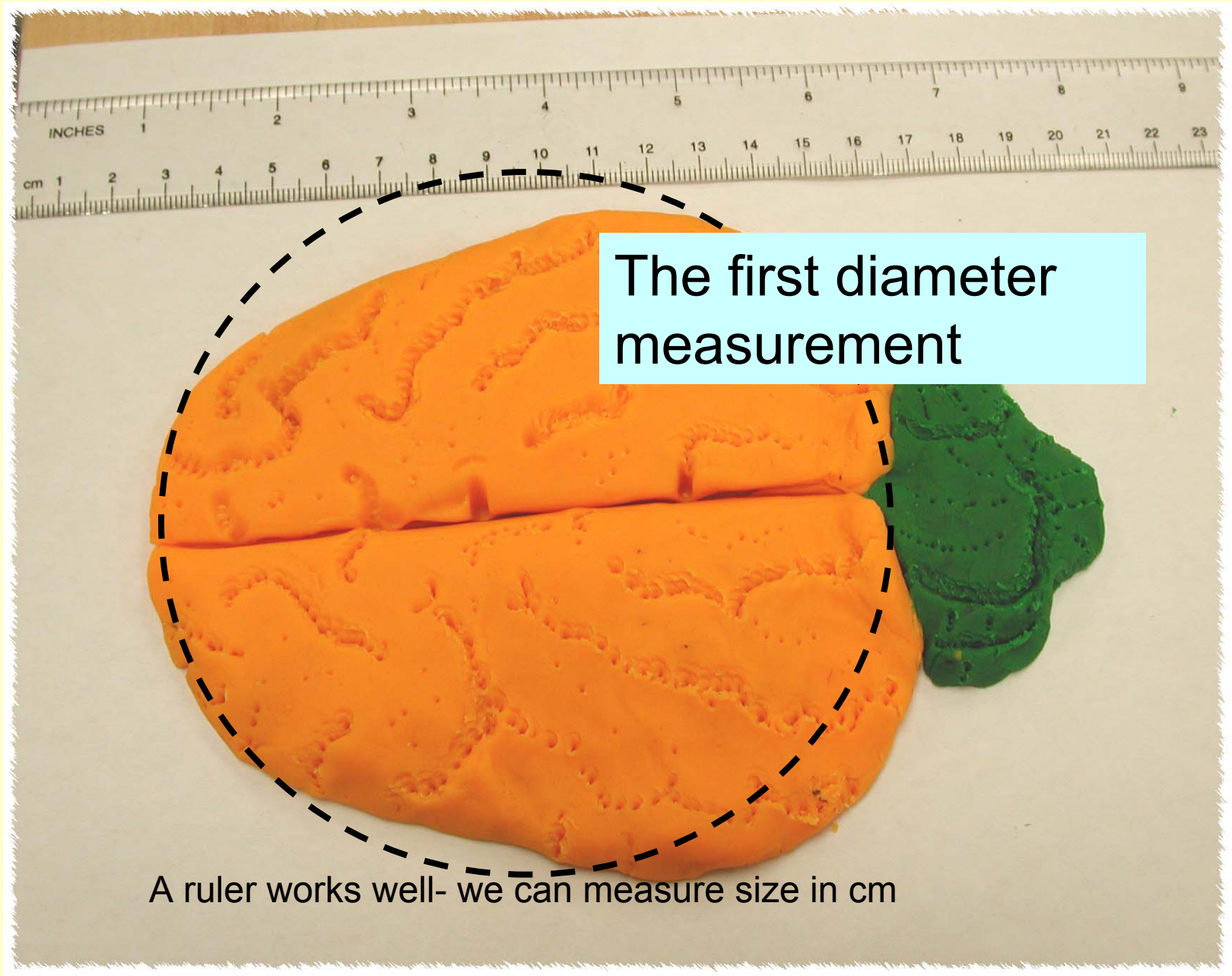
Our “carrot brain” will give us some measurements



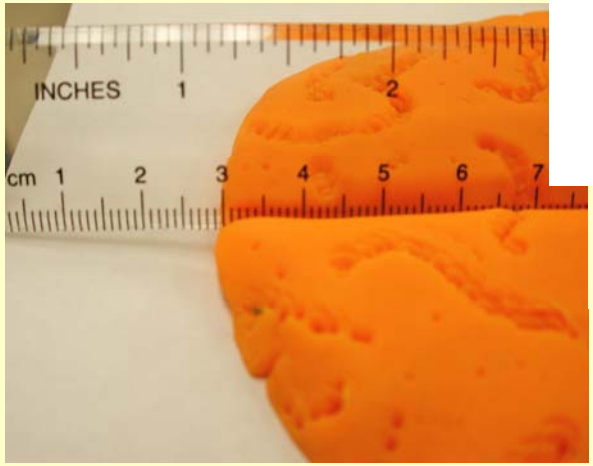
Our brain model

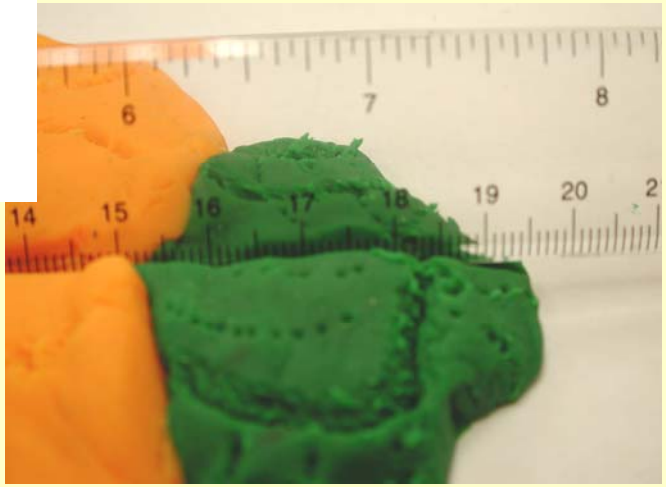


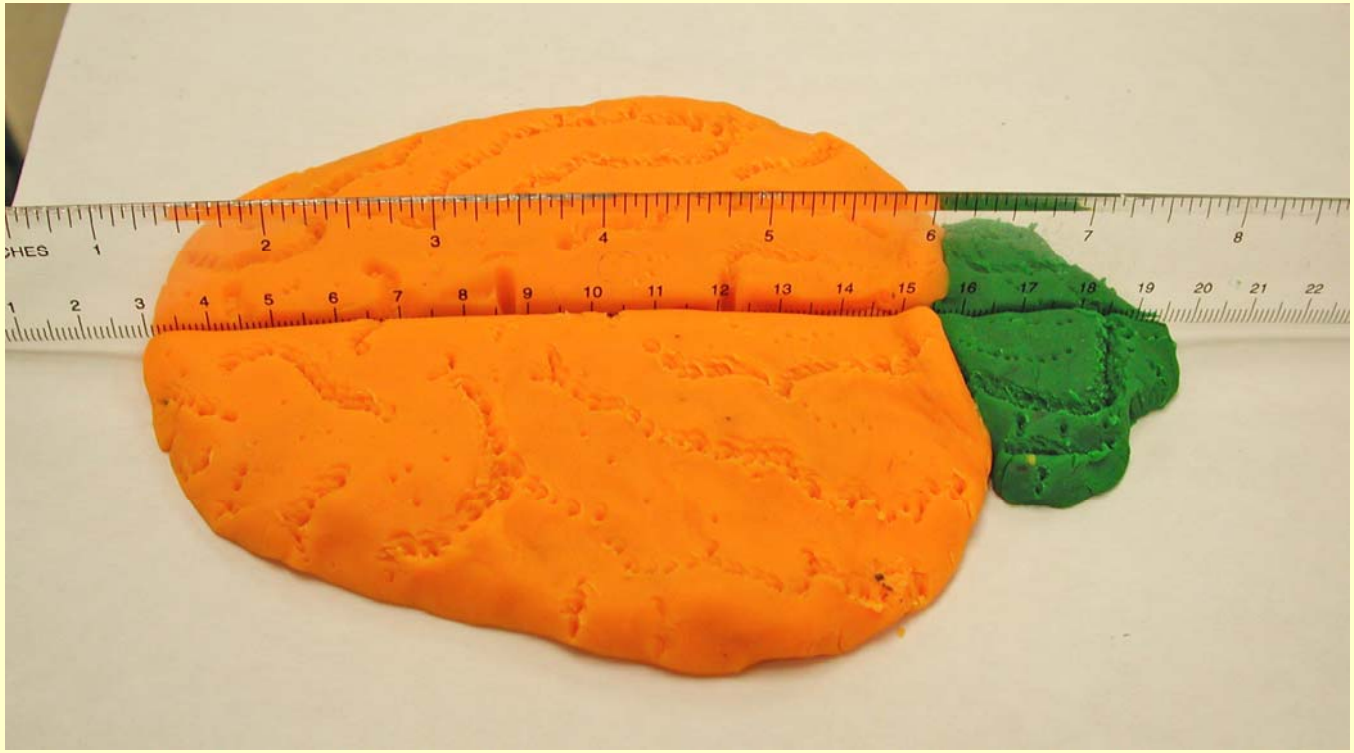
We will take three measurements of the diameter and average them


INCHES
cm
The first diameter measurement
A ruler works well- we can measure size in cm

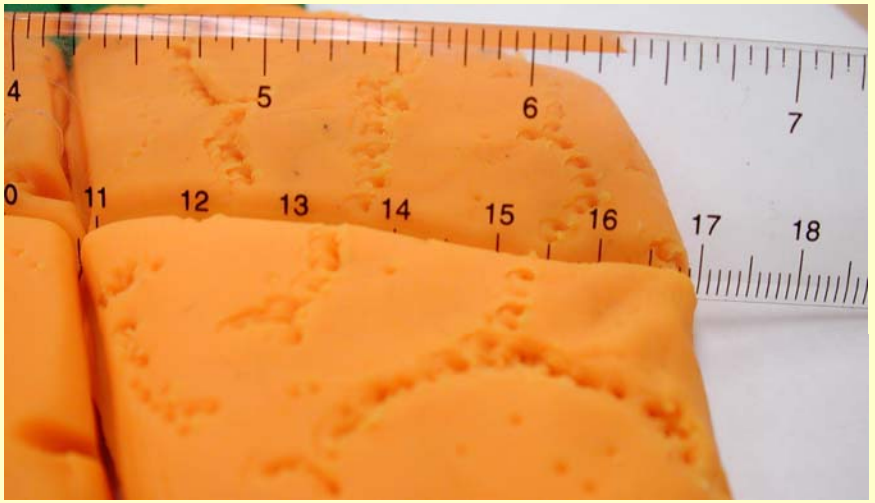
Diameter #1 in more detail

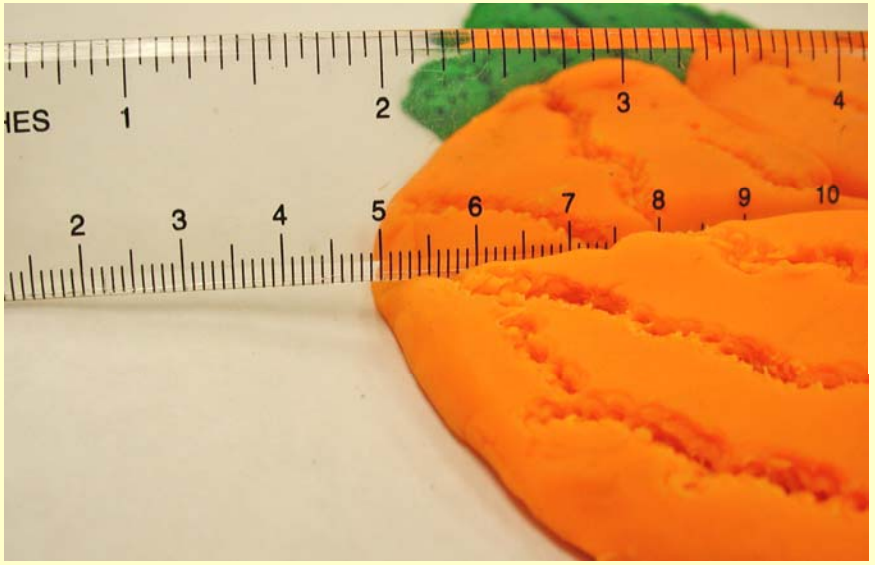




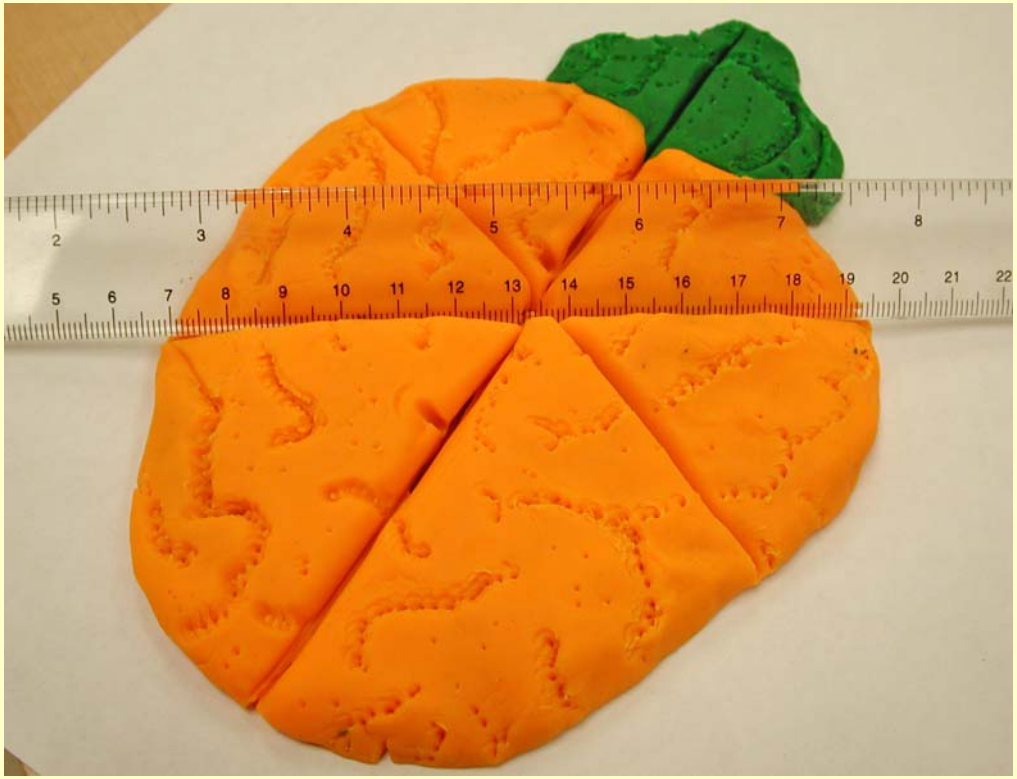


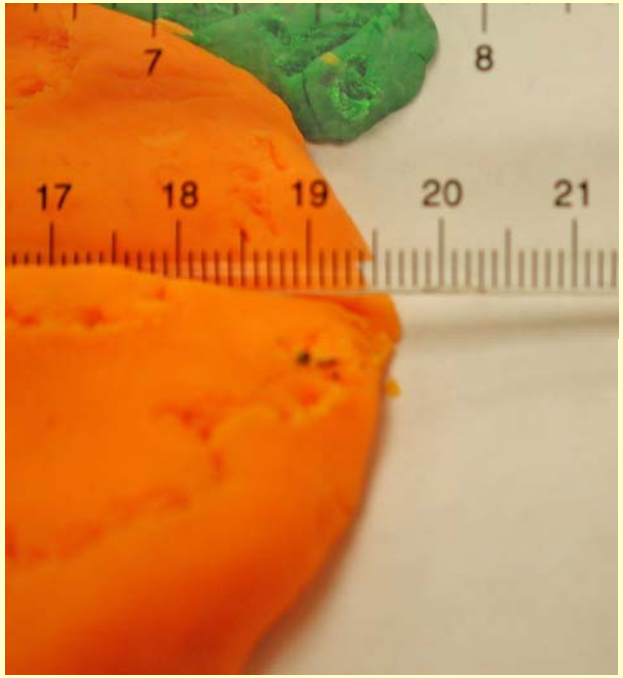
Diameter #2





Diameter #3



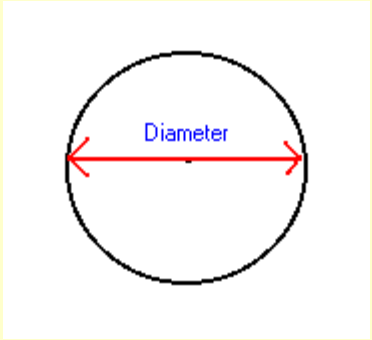


Circumference

$C = \pi d$

d is the

Diameter±Uncertainty



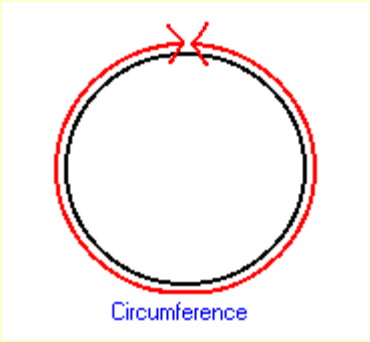
Or $d \pm e_d$

Therefore the Circumference

$C = \pi (d \pm e_d)$

$C = \pi d \pm \pi e_d$

$C = \pi d \pm e_c$



Calculating the Average Diameter and its' uncertainty

Diameter of the Carrot Brain

	Diameter (cm)	Uncertainty (±cm)		Instrumental Uncertainty (±cm)
1	13.2	0.6		±0.05
2	13.9	0.6		
3	12.7	0.6		
Ave	13.27	cm	Avg Diameter = AVERAGE(B4:B6)	
	Range =	13.9 - 12.7 =	1.2	cm
Uncertainty in each diameter =Range/2=.6 cm				
Uncertainty in the average Diameter (cm)				
	e_d =	Range÷2√3 =	D10/(2*SQRT(3))	
	e_d =	0.35		
Circumference (cm)				
	C = π*(Avg. Diameter)			
	C = PI()*B8 =	41.68		
Uncertainty in Circumference (cm)				
	$e_c = (\pi * e_d)$			
	e_c = PI()*C16 =	1.09		

The instrumental (or reading) error is so small compared to the range of the data, that it can be ignored!

When you multiply a number with uncertainty by a constant, the uncertainty is multiplied by the constant

- **Circumference (cm)**
- **C** = π*(Avg. Diameter)
- **C** = PI()*B8 =41.68
-
- **Uncertainty in Circumference (cm)**
- e_c **= (π*e_d)**
- e_c = PI()*C16 =1.09 (In Excel format)

Suppose now, we have a rectangle of length x and width y instead:

How do the uncertainty in the length (e_x) and the uncertainty in the width (e_y) affect the uncertainty in the area (e_A) ?

Should the uncertainties (each uncertainty is +/-) be

1. Added?
2. Subtracted?
3. Multiplied?

The answer is: none of the above.

How do errors combine?

Adding or subtracting would give an over-estimate or under-estimate.

Multiplying would not even be correct dimensionally. Absolute errors would have to be converted to percent or fractional errors to do that

There are many calculations that are less than the arithmetic sum, but we can show that the one we want is:

$$e_A = \sqrt{e_x^2 + e_y^2}$$

This is sometimes called the “Pythagorean Sum” for obvious reasons. We will refer to it as

QUADRATURE

It is left to you to verify that it yields a value less than

$e_1 + e_2$

The exceptions to the quadrature rule

- When we multiply a number with uncertainty by a fixed number, the uncertainty gets multiplied by the number

so the circumference of the brain was

$$C = \pi\,(13.27) \pm \pi\,(.35)\ \text{cm}$$

- When we multiply a number by itself, as in squaring the number, the uncertainty gets doubled

So for example $(5 \pm 0.1)^2\, cm^2 = 25 \pm 0.2\, cm^2$

Sample Calculations
Multiple Measurements of the Period

	A	B	C	D	E	F	G	H	I	J	K	L
1							**Sample Calculations**					
2							Multiple measurements of the period					
3												
4	Data			Calculations					Excel Formulae			
5						Result						
6		**Period T**		T_{avg}: (T1 + T2+ ….+T6)/6		7	s			Tavg	=AVERAGE(B8:B13)	
7		**(s)**										
8	Tmax----	9		Range:Tmax-Tmin		4	s			Range	=B8:B13	
9		7										
10		7										
11		6		Uncertainty in T		2	s		Uncertainty in T		=(B8:B13)/2	
12	Tmin-----	5		Formula: Range/2								
13		8										
14	T_{avg}	7		Uncertainty in T_{avg}		1	s		Uncertainty	in Tavg	=(B8-B13)/(2*SQRT(6))	
15				Formula: Range/(2*SQRT(6))								
16												
17						NOTE: An Excel formula entered in a cell is always preceded by an equal sign.						
18												